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AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

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SOME ASPECTS OF WATER CONSERVATION

By R. A. Sutherland,* Assoc. M. Am. Soc. C. E.

Synopsis

Development of water conservation in the past has frequently been made in a piecemeal and haphazard manner, resulting in a less effective and economical exploitation of rivers than could be obtained by a systematic and coordinated study of all the factors concerned. In this paper formulas are developed for studying, in a preliminary manner, the merits of various storages and dam sites. The formulas have been tested in practice and were found to give reasonably correct results in most cases. Methods are also given for studying the relative economy of different types of dam, where the site admits of alternative types. The economic limits of specific capacity, or water stored per cubic yard of masonry in the dam, are discussed by means of examples from practice.

The paper represents an attempt to reduce to general and simple terms those factors which admit of generalization, but it is recognized that each case has peculiarities which must be studied in the light of local conditions.

The original manuscript of this paper, containing many tables, curves, and illustrations, which are not published herein, may be found on file at the Engineering Societies Library.† An abstract that forms the nucleus of it was published in Australia in 1928.‡

Importance of Water Conservation

Water conservation, especially in countries where rivers show marked irregularity of flow, has become a question of such importance that Government departments have been set up to handle the whole question, particularly with regard to irrigation and water power. In addition, many private agencies or local authorities are actively undertaking conservation works in connec-

Note.—Written discussion on this paper will be closed in **January, 1931**, *Proceedings*.

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† Approved by the Senate of the Univ. of London for the degree of Doctor of Philosophy in the Faculty of Engineering.

‡ *The Commonwealth Engineer*, Vol. 15, No. 11, June 1, 1928, p. 419.

tion with irrigation, water power, or water supply projects. Immense sums of money have already been expended on such works, and with increasing population the necessity for the most economical and effective organization of all works for water conservation will become ever more insistent.

In most countries the systematic collection and recording of data with regard to water conservation has been entirely neglected, due probably to the fact that so many different authorities are represented in the field. Notable exceptions are the publications of the United States Bureau of Reclamation and other Federal Departments, and those of a branch of the Italian Ministry of Public Works ("Consiglio Superiore delle Acque").

The total storage for irrigation already developed or proposed by the U. S. Bureau of Reclamation is equal to 13 792 500 acre-ft. in 49 reservoirs. In California, power companies and municipalities have created for power purposes alone no less than 76 reservoirs with an aggregate storage of 1 724 402 acre-ft.*

Canada is richly endowed with water resources; the total available storage of 22 of the most important lakes and artificial reservoirs used for power purposes is 30 030 000 acre-ft.

South Africa, which geologically is a country ill-adapted to water conservation, has storages exceeding 750 000 acre-ft.

The States of New South Wales and Victoria in Australia are, in proportion to population, among the foremost in the conservation of water, which is mainly used for irrigation. The total storage for this purpose in New South Wales is about 1 900 000 acre-ft., or 0.84 acre-ft. per capita, while a further storage of 2 500 000 acre-ft. is proposed. Victoria has a total storage of 900 000 acre-ft. for irrigation, or 0.54 acre-ft. per capita, and an added quantity of 950 000 acre-ft. is proposed.

The State of Queensland, which is the most recently developed of the Eastern States of Australia, has, at present, storage equal to only about 30 000 acre-ft., while projects involving a total storage of about 3 000 000 acre-ft. are either proposed or being developed.

In Europe, the climate does not in general require, nor does the geological nature of the continent usually permit, the formation of large storages. Italy, however, has an immense number of storages of moderate size, totaling more than 675 000 acre-ft. in 86 reservoirs used for power, while works under construction will add a further 626 000 acre-ft. in 27 reservoirs. The largest of these reservoirs is that of the Tirso, with a capacity of 285 000 acre-ft.

Switzerland has many storages of small size, the total capacity of the 9 reservoirs which exceed 10 000 acre-ft. being 271 900 acre-ft.

An immense number of small storages exist in various countries for purposes of irrigation and water supply, but no complete list of these storages has ever been compiled. Some appreciation of the magnitude of the conservation of water for irrigation may be obtained from the fact that the estimated world area irrigated is more than 137 000 000 acres.†

* The data here given were up to date in 1928. For notes on large dams, see *"New Reclamation Era,"* January, 1930, "The Highest and Largest Dams in the United States and Abroad," by P. I. Taylor.

† *New Reclamation Era*, September, 1927.

The storage required per acre irrigated varies from zero to as much as 20 acre-ft. An average figure for projects under the U. S. Reclamation Bureau appears to be about 4 acre-ft. of storage per irrigated acre, while, in order to cope with the prolonged droughts occurring in Australia, a proposed plan there involves a storage of about 30 acre-ft. per irrigated acre. Taking a conservative all-round figure of only 0.5 acre-ft. of storage per irrigated acre, the world storage for irrigation would amount to about 69 000 000 acre-ft.

Future Development

Great as has been the progress already made in water conservation, it is only a fraction of that required to achieve the complete hydraulic development of the various countries of the world. Some countries are already so highly developed in this regard that the completion of their program of hydraulic development, if not already in view, will be so within the next ten or twenty years. Of such countries, Switzerland and Italy may be cited as examples. Italy has now (1930) reached such a stage that all the most promising storage sites have already been exploited, and a Government department is actively collecting data relating to the stream flow and potential storage sites of the remaining streams. Practically all the main rivers and streams have been accurately leveled, storage sites surveyed, and catchment basins defined. Other countries such as Brazil, Central Africa, Queensland, New Zealand, etc., have as yet developed only a small percentage of their hydraulic resources.

When it is realized that the present total world area irrigated is just a fraction of that for which water would be available with adequate conservation, and that in the field of hydro-electric power the present world development is only 25 000 000 h.p. out of an estimated total of about 440 000 000 h.p. available, it is considered safe to say that present water conservation is probably not more than 10% of that which will ultimately be required. Accurate data, however, are lacking on which to base any forecast of the rate of development.

In addition to storages required for irrigation and water power, an increasing number will be required also for water supply and flood control; but development on these lines depends so entirely on sectional requirements that it is usless to attempt any forecast of what may be required. It is interesting to note, however, that, in recent times,* considerable sums of money have been spent on flood control by the construction of detention basins. The works of the Miami Conservancy District in Ohio, of the City of San Antonio, Tex., and of Los Angeles County, California, are examples.

The investigation and development of water conservation have in the past been frequently haphazard, unscientific, and uneconomical. Common errors have been:

1.—The use of insufficient stream-flow data;
2.—The use of rainfall records and assumptions as to run-off without adequate data showing the relationship between them.
3.—An under-estimation of flood flows, and insufficient spillway provision;

* The Furen's Dam (1866) was the first large modern dam built for flood control. Until recently, large expenditure for this purpose alone was exceptional.

4.—Insufficient geological examination of reservoir basin and dam site;
5.—Incorrect design or faulty construction of dam; and
6.—Choice of site, and design of dam without regard to economic considerations or future development of other sites on the same stream.

Notation

The following notation is used in the paper:

a = area of a cross-section at a dam site, to any height, h.
A = total area of a cross-section at a dam site, to height, H.
w = capacity of a reservoir to any height, h_1, measured at the dam.
w_s = specific capacity. Other subscripts refer to the type of dam.
W = total capacity of a reservoir to height, H_1.
s = submerged area of a reservoir, corresponding to a height, h_1, measured at the dam.
l = width of a dam site at any height, h.
t = thickness of dam at any height, h.
t_1 = crest thickness of a dam with a trapezium section.
K = sum of face batters (ratio of horizontal to vertical) of a dam. Proper subscripts refer to type of dam.
K_R = sum of face batters of a dam with a trapezium section.
h = height of dam site to any elevation.
h_1 = depth of water at the dam.
H = total height of a dam of any type.
V = volume. Proper subscripts refer to type of dam.
V_R = volume of a dam with a trapezium section.
R_u = up-stream radius of an arch dam.
R_1 = up-stream radius of a constant-angle arch dam, at the crest.
R_2 = up-stream radius of a constant-angle arch dam, at the base.
R_h = up-stream radius of a constant-angle arch dam, at any elevation, h.
f = cylinder stress in an arch dam, in pounds per square inch.
ϕ = a subtended angle in an arch dam.

C_A = a coefficient (Equation (1)) $= \dfrac{a}{h^n}$.

C_w = a coefficient (Equation (2)) $= \dfrac{w}{{h_1}^m}$.

D = a coefficient (Equation (6)) $= \dfrac{l}{h^v}$.

n = an exponent (Equation (1)) $= \dfrac{\log a - \log C_A}{\log h}$.

m = an exponent (Equation (2)) $= \dfrac{\log w - \log C_w}{\log h_1}$.

v = an exponent (Equation (6)) $= \dfrac{\log l - \log D}{\log h}$.

J = a coefficient (Equation (20))
$$= \frac{0.217\,\phi}{n\,C_A\,H^{n-1}\sin\dfrac{\phi}{2}}\left[\frac{n\,R_1 + 2\,R_2}{n\,(n+1)\,(n+2)}\right].$$

J_A = a coefficient (Equation (14)) $= 0.434\,\dfrac{R_u}{L}\left(\dfrac{n}{n+1}\right)\left(\dfrac{\phi}{2\sin\dfrac{\phi}{2}}\right)$.

J_{CA} = a constant $= J\,n^2$.

Q = a constant (Equation (25)) = $L \frac{V_A}{V_g}$.
g = a subscript denoting gravity dams.
A = a subscript denoting arch dams.
$C A$ = a subscript denoting constant-angle arch dams.

Notes on River Sections and Profiles

Koechlin* has shown that for a river in a state of equilibrium, the shape of the bed is sinusoidal and depends only on the slope and the nature of the ground traversed. This shape he calls the "normal cross-section." In order that this section may be attained, the following conditions must be fulfilled:

(1) The river must have arrived at a state of equilibrium; that is, the longitudinal profile must be constant.
(2) The flow must be as constant as possible.
(3) The ground must be homogeneous.
(4) The banks must be bare.
(5) The river must be straight.
(6) The river, even at high flood, must be contained entirely within the bed.

Actually few of these conditions are ever satisfied simultaneously, so that the theoretical profile does not hold good in practice. The effect of departures from the conditions mentioned is examined by Koechlin in some detail.

Over certain limited tracts of river, all the conditions cited except Condition (2) may be fairly well fulfilled. This is generally the case in gorges or valleys suitable for the construction of dams, and, here, it might be expected that the sinusoidal shape of section might be most frequently found.

The area enclosed within a sine curve, can be shown to vary nearly as the $\frac{3}{2}$ power of the height measured from the crest of the curve. By computing areas carefully and plotting them against heights, on logarithmic paper, the writer obtained an index of 1.51. The cross-sectional areas of dam sites can generally be expressed closely in terms of height by means of an equation:

$$a = C_A h^n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The mean value of n for thirty-seven sites examined was found to be 1.54. This close agreement between theory and average practice appears to confirm the correctness of Koechlin's work.

It is a matter of common experience that the longitudinal profile of rivers is approximately parabolic in shape. Fig. 1(a), Fig. 1(b), Fig. 1(c), and Fig. 1(e) show the profiles of various rivers and illustrate the parabolic shape.† Land surfaces as a whole vary in slope in a similar manner.‡

In the vast majority of cases, a river flows through country of varying geological character, and the profile is far from regular. At certain points the river breaks through more or less well-defined ridges, the areas above these points constituting self-contained catchment areas. When the ridges are well defined, the river is usually in a gorge, and such a formation is frequently

* "Mécanisme de l'Eau," by René Koechlin, Vol. 1, 1924, p. 101.
† See, also, "Mécanisme de l'Eau," by René Koechlin, Vol. 1, 1924, Fig. 48, p. 136.
‡ "Géographie Physique," by Martonne, 1913, p. 49.

suitable both for a dam site and for an efficient storage basin. Each catchment above its respective gorge has its own characteristics, which depend on the geology of the region and a variety of other factors. The general land slope above each gorge will be parabolic. Hence, the profile of many rivers consists of a series of parabolas, the whole again forming roughly one large parabola.

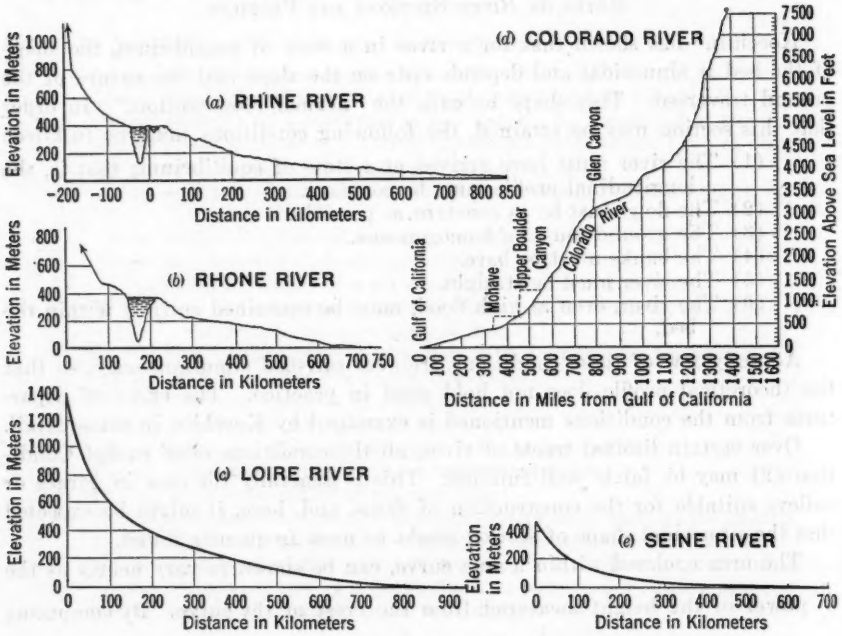


FIG. 1.—LONGITUDINAL PROFILES OF VARIOUS RIVERS.

Fig. 1(*d*) shows the profile of the Colorado River,* in which two major and several minor parabolas are readily distinguishable. The ideal case is shown in Fig. 2 in which the river is assumed to break through two successive ridges. The longitudinal profile then consists of three curved sections, each of approximately parabolic shape.

In the upper basin, the river has a relatively small flow but carries considerable gravel and many boulders, and, in general, the stream bed will be deeply eroded and the landscape scarred by numerous tributaries. The general nature of the cross-section therefore will be as shown by Section *A*.

In the second basin the stream bed will be wider and less deeply eroded, somewhat as shown at Section *B*, while even farther down the land may be almost level, and the river bed very wide, the normal dry-weather flow occupying only a small winding channel in a wide bed (Section *C*).

THE RESERVOIR

Suitable dam sites generally occur at points such as those marked *A* and *B* (Fig. 2); that is, some distance down the transition from one parabola to the

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), Fig. 13, p. 341.

next. An examination of the slopes above and below almost any storage dam will verify this fact.

From a consideration of the profile and cross-sections of Fig. 2 it will be seen that the storage obtained for a given height will generally be less for the upper reservoir shown than for the lower one. On the other hand, the rate of increase of storage with increasing depth of water will be greater. Hence, the reservoirs will have certain well-defined characteristics and may be classified accordingly. The classification adopted will be: (1) Gorge type; (2) hill type; (3) foothill and flood-plain type; and (4) lake type.

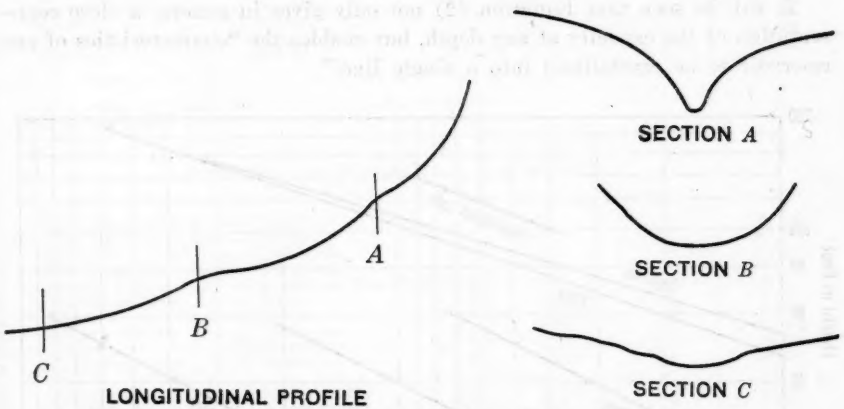


FIG. 2.—PROFILES AND SECTIONS OF AN IDEAL RIVER (DIAGRAMMATIC).

A classification such as this was proposed by W. H. R. Nimmo, Assoc. M. Am. Soc. C. E., when this paper was first discussed before the Institution of Engineers (Australia), but the criteria now adopted differ from those proposed by Mr. Nimmo.

Reservoir *A* would be of the first class, Reservoir *B* of the second, and a reservoir at a site, such as *C*, of the third class. The lake type of storage is a special case and may occur at any elevation. As the name implies, it is a reservoir formed either by artificially raising an existing lake, or by impounding water in what was once a lake or in a basin of similar general form.

CHARACTERISTICS OF RESERVOIRS

In 1911,* Capt. A. ff. Garrett, R. E., defined the capacity of a reservoir as a function of the depth, by expressing the area and capacity as series in ascending powers of the depth, and showed that the first three or four terms of such series sufficed to give quite accurate values of the quantities considered. Hence, he deduced rules for determining the capacity curve with the minimum amount of survey work. For this purpose Garrett's method is valuable. Later, Hillman† showed that the capacity of a reservoir can be expressed closely as an

* "The General Theory of the Storage Capacity and Flood Regulation of Reservoirs," Tech. Publication, Govt. of India, 1911.

† "The Capacities and Capacity Equations of Some Western Australian Dam Sites," *Transactions*, Inst. of Eng. (Australia), Vol. 11, 1921.

exponential function of the depth of water, but he made practically no use of this fact.

Fig. 3 represents a few of the large number of capacity curves which have been plotted by the writer on logarithmic paper, and shows that the relationship first suggested by Hillman is generally very close. When the capacity curve has been drawn as a straight line on logarithmic paper (with horizontal and vertical scales equal) the slope of this line will give the index of the height in the general capacity equation:

$$w = C_w h_1^m \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

It will be seen that Equation (2) not only gives in general a close representation of the capacity at any depth, but enables the "characteristics of any reservoir to be crystallized into a single line."

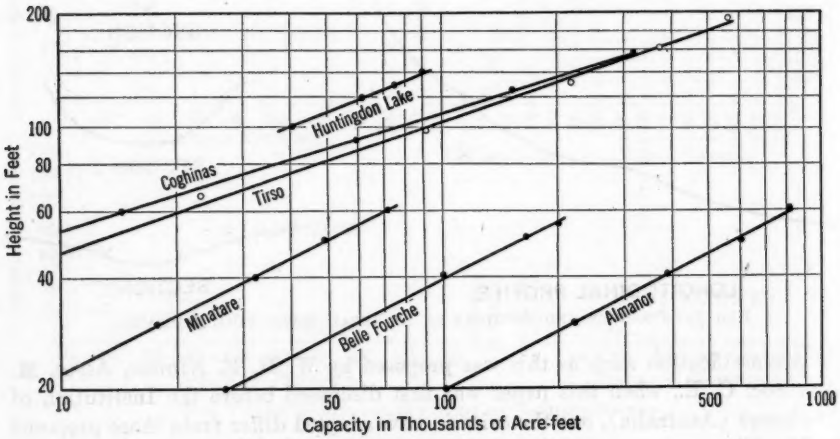


FIG. 3.—LOGARITHMIC CAPACITY CURVES.

In Table 1 a number of reservoirs have been classified into types, and the characteristics of each reservoir (as represented by C_w and m) have been tabulated. The basis of classification arbitrarily adopted is as follows:

	Values of m.
Lake type	1.0 to 1.5
Flood-plain and foothill type	1.5 to 2.5
Hill type	2.5 to 3.5
Gorge type	3.5 to 4.5

This classification is not intended to be hard and fast, since the different types merge into each other, but is a sufficiently definite basis for describing the general nature of any reservoir.

The constant, C_w, may be readily determined by taking the capacity from the straight line for a depth of 100 ft., then:

$$\log C_w = \log w_{100} - 2\,m \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The value of m is the ratio of maximum depth to average depth, the latter being defined as maximum capacity divided by maximum water-spread area.

TABLE 1.—Characteristics of Storage Reservoirs

Item No.	Name of reservoir.	Purpose.	m.	C_w.	Maximum depth, in feet.	Capacity, in acre-feet.
	Lake Type: m = 1.0 to 1.5.					
1	Great Lake	Power	1.04	29 500	35	1 157 000
2	Jackson Lake	Irrigation	1.13	13 330	39	789 000
3	Matese	Power	1.37	2 350	13.1	11 600
4	Strawberry	Irrigation	1.26	2 390	44	278 000
5	Vannino	Power	1.22	33.12	78.5	6 900
	Weighted mean (for capacity)		1.104			
	Arithmetical mean		1.204			
	Flood-Plain and Foothill Type: m = 1.5 to 2.5.					
6	Lake Almanor (1930)	Power	1.9	347	35	300 000
6(a)	Lake Almanor (future)	Power	1.9	347	59	700 000
7	Assuan	Irrigation	2.03	171.5	98	1 970 000
8	Belle Fourche	Irrigation	1.98	71.2	75	203 000
9	Boulder Canyon	Power and irrigation	2.35	9.07	.....	.....
10	Ceresole Reale	Power	2.32	0.325	134.3	26 900
11	Codelago	Power	1.66	9.43	79	13 100
12	Elephant Butte	Irrigation	2.39	10.75	193	2 637 000
13	Gibson	Irrigation	2.24	1.02	161	90 000
14	Huntington Lake	Power	2.38	0.695	140	88 700
15	Little Nerang	Water supply	2.24	0.1365	197	18 500
16	Lake Manchester	Water supply	2.03	3.7	70	21 100
17	Minitare	Irrigation	1.99	20.54	55	60 766
18	Mojave	Power and irrigation	1.76	1 330	.....	.....
19	Lake Spaulding	Power	2.43	0.0858	275	74 488
20	Sherburne Lake	Irrigation	1.65	63.4	73	75 000
	Weighted mean		2.2			
	Arithmetical mean		2.09			
	Hill Type: m = 2.5 to 3.5.					
21	Arrowrock	Irrigation	2.92	0.02848	249	280 000
22	Bon Accord	Irrigation	3.15	0.04134	75	32 000
23	Bull Run	Water supply	2.57	0.058	180	37 000
24	Lower Buller	Water supply	2.63	0.0324	57	.....
25	Burrenjuck	Irrigation	3.04	0.0773	200	771 641
26	Calitzdorp	Irrigation	2.77	0.01731	100	600 000
27	Coghinas	Power	3.18	0.3272	157	320 000
28	Coomera	Water supply	2.70	0.1592	170	175 000
29	Lake Cushman	Power	2.62	0.288	250	465 000
30	Hume	Irrigation	3.00	1.58	90	1 098 500
31	Kamanassie	Irrigation	2.50	0.310	100	31 100
32	Nepean	Water supply	3.41	0.000518	207	38 800
33	Roosevelt	Irrigation	3.04	0.0847	220	1 367 300
34	Shoshone	Irrigation	3.42	0.00433	220	460 000
35	Lower Stanley	Water supply	3.37	0.0273	120	290 000
36	Stony Gorge	Irrigation	3.05	0.0286	111	50 000
37	Tirso	Irrigation and power	2.87	0.1638	196	574 000
	Weighted mean		2.99			
	Arithmetical mean		2.78			

TABLE 1.—(*Continued.*)

Item No.	Name of reservoir.	Purpose.	m.	C_w.	Maximum depth, in feet.	Capacity, in acre-feet.
		GORGE TYPE: m = 3.5 TO 4.5.				
38	Avon	Water supply	4.25	0.000023	210	174 000
39	Upper Buller	Water supply	3.94	0.000131	60	1 340
40	Cataract	Water supply	4.11	0.0000915	150	77 000
41	Cordeaux	Water supply	4.33	0.0000165	170	76 000
42	Glenn Canyon	Power	3.97	0.00092	.....	.....
43	Grass Ridge	.....	3.64	0.01118	100	210 000
44	Mt. Edwards	Irrigation	3.56	0.00497	75	30 000
45	Nathan	Irrigation	4.35	0.001566	130	2 485 000
46	Pathfinder	Irrigation	4.28	0.0001376	202	1 070 000
47	Silverwood	Water supply	3.62	0.00075	70	3 600
48	Upper Stanley	Water supply	3.77	0.00317	110	159 754
49	Tarka	.....	3.61	0.00374	100	59 000
50	Van Ryneveld's Pass	.....	3.93	0.0009	100	32 500
51	Warragamba	Water supply	4.22	0.0000682	260	578 000
	Weighted mean		4.23			
	Arithmetical mean		3.97			

The ratio of mean depth to maximum depth has been given by Charles Slagg* for seventy-five different reservoirs. The maximum value of this ratio is 0.612, corresponding to $m = 1.64$, and the minimum value is 0.281, corresponding to $m = 3.57$, the mean values being 0.456 and 2.2, respectively. In Table 1, fifty-one reservoirs are arranged according to the writer's classification. The arithmetical mean value of m is 2.75 and the weighted mean, 2.87, for the fifty-one cases. Both these values are greater than the figure deduced from Slagg's table.

Certain of the reservoirs cited occur on the same river, and a study shows that their values of m are functions of elevation. The following reservoirs are arranged in order of descent of the river, and the values of m show a corresponding decrease:

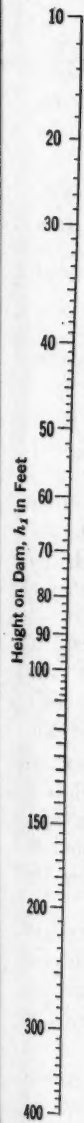
	Values of m.
Upper Buller River	3.94
Lower Buller River	2.63
Upper Stanley River	3.77
Lower Stanley River	3.37
Glenn Canyon, Colorado River	3.97
Boulder Canyon, Colorado River	2.35
Mojave Reservoir, Colorado River	1.76

Furthermore, the general deduction may be made that for reservoirs of similar type, C_w will vary usually with the catchment area.

REPRESENTATION OF RESERVOIR CAPACITY

Equation (2) may be represented by means of an alignment chart, such as Fig. 4, in which the outside scales are graduated for height and capacity, the

* "Water Engineering," by Charles Slagg, First Edition, 1888, p. 24, Crosby Lockwood and Son.



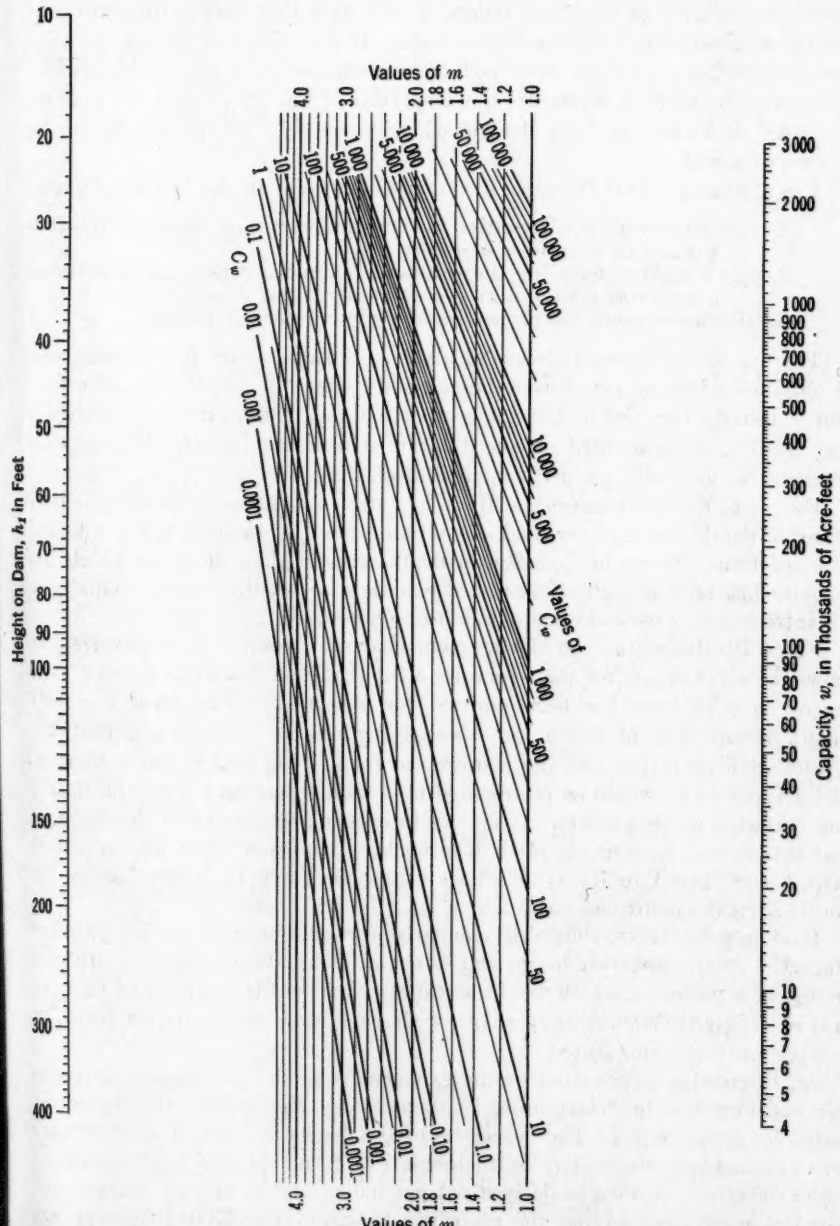


FIG. 4.—RESERVOIR CAPACITY DIAGRAM BASED ON THE EQUATION, $w = C_w h_1^m$.

central field representing various values of m and C_w, over a range such as occurs in practice. A moment's reflection will show that the C_w lines are concurrent at the point, 1, on the height scale. If the values of m and C_w have been obtained as previously described, or by computation, a point is defined in the central field which represents the properties of the reservoir, and by using this point as a fulcrum for a straight edge, the capacity at any height can be at once obtained.

The alignment chart for capacity (Fig. 4) is useful in the following ways:

1.—As a convenient alternative to the usual cartesian capacity diagram for one or a number of reservoirs.
2.—As a means of finding the values of C_w and m when the capacity of a reservoir is known at two or more depths.
3.—For comparing the properties of various reservoir basins.

The use of the logarithmic scales gives more satisfactory results than can be obtained with the usual cartesian capacity curve in which the lower portion is usually very flat and the upper portion very steep. Several reservoirs may readily be represented on one diagram without confusion, which cannot generally be done with the usual method of plotting.

For Use No. 2 intersecting lines are drawn connecting corresponding values of depth and capacity, and an optimum point of intersection is selected by inspection. Values of capacity for depths greater than those for which the capacity has been actually determined can then be readily found. This case not infrequently arises where a dam is to be raised.

Fig. 5 illustrates the use of alignment charts for comparing reservoirs. It shows the pivot points for the reservoirs cited (by Item No. 3) in Table 1 (the central field of lines has been omitted for clearness.) The ideal reservoir basins, irrespective of depth, are those lying on the upper margin of the constellation of points and the poorest are those lying on the lower margin. The average basin would be represented by a point lying on the central dotted line (inserted by judgment). It is very interesting to note from the diagram that this agrees very nearly with Kanthack's conclusion* that any reservoir basin better than Van Ryneveld's Pass (Item 50, Table 1) is satisfactory for South African conditions.

It should be noted that this comparison relates merely to the physical properties of the reservoir basin, and is not a final criterion of the usefulness or cost of a reservoir, which would be expressed generally in terms of cost per unit of regulated run-off or, perhaps, with very large reservoirs, in terms of cost per unit of water stored.

An interesting application of the exponential method of expressing reservoir capacity lies in determining mathematically the behavior of retarding basins for flood control. The properties of the flood wave and of the basin are both expressed approximately by mathematical formulas, and the reaction of one to the other can then be derived. A particular case of this general application was used for studying the retarding basins of the Miami Conservancy District† with great economy of time and effort.

* "The Principles of Irrigation Engineering," by F. E. Kanthack, 1924, p. 153.
† "Hydraulics of the Miami Conservancy District."

Submerged Area

The differentiation of Equation (3) with respect to depth gives the submerged area equation:

$$s = m\, C_w\, h_1^{m-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

A comparison of actual submerged areas for a number of cases, with the theoretical areas found from Equation (4), shows that this equation is less accurate than Equation (3). It is sufficiently accurate, however, to be used for preliminary purposes for determining the constants, C_w and m, from the areas at two or three levels only. Fig. 6 is convenient for this purpose. It is well known that for one reservoir proposal adopted, many may be examined and discarded, and the methods mentioned are quite suitable for this preliminary study.

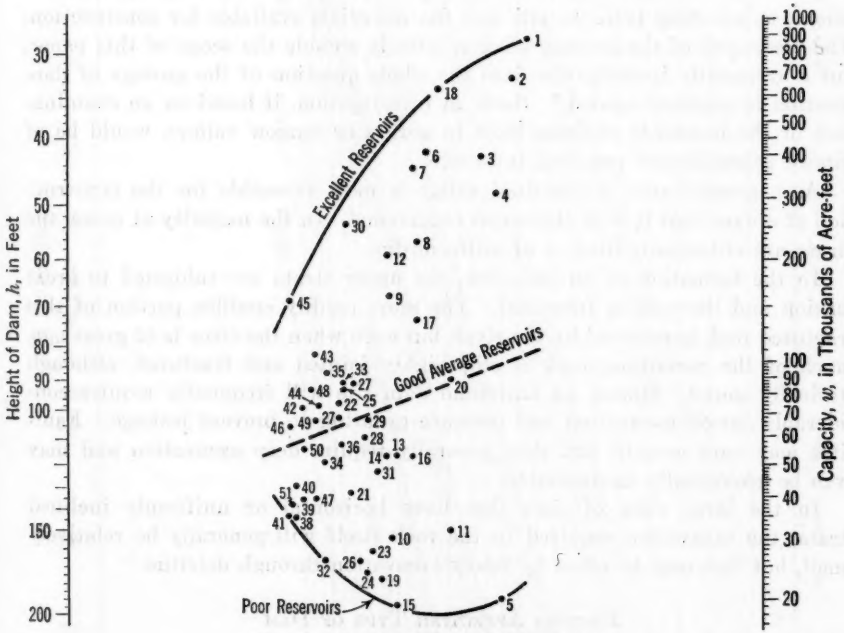


Fig. 5.—Comparison of Reservoirs.

It is interesting to note that the value of m has an important bearing on the effect of evaporation, which has sometimes to be considered very carefully in studying large storages in dry climates.

The effect of evaporation being to remove the top slice of water impounded, it is clear that, other things being equal, evaporation causes a much more serious loss in the case of reservoirs characterized by a high value of m, than with those having a low value of m. For long storage in dry climates, a low value of m, is, therefore, a desirable property for a reservoir to possess.

The Assuan and Elephant Butte Reservoirs are excellent examples, while the Nathan Reservoir is unfortunately handicapped in this respect by its

high value of m. To illustrate this point it will be sufficient to state that the evaporation losses for the Elephant Butte Reservoir over a period of twenty years would be equal to about 20% of the inflow, while for the Nathan Reservoir over a period of fifteen years these losses would be 40.5% of inflow. The depth lost annually is practically the same for each reservoir, so that the cases are comparable. The losses over a shorter drought period would be even more to the disadvantage of the Nathan Reservoir. It is unnecessary to add that sites suitable for the creation of such large storages are rare, so that there is not usually any opportunity of selecting the reservoir site with regard to its desirability or otherwise from the point of view of evaporation losses.

The Dam

The location and design of a dam depend primarily on geological considerations, as affecting both the site and the materials available for construction. The treatment of the geology of dam sites is outside the scope of this paper, but a systematic investigation into the whole question of the geology of dam location is urgently needed.* Such an investigation, if based on an examination of the hundreds of dams built in gorges or narrow valleys, would be of intense scientific and practical interest.

As a general rule, a synclinal valley is most favorable for the construction of a dam; but it is of the rarest occurrence. In the majority of cases, the strata are either anticlinal or of uniform dip.

In the formation of an anticline, the upper strata are subjected to great tension and the rock is fractured. The more readily erodible portion of this fractured rock is removed by the river, but even when the river is of great age, much of the remaining rock is still highly jointed and fractured, although statically sound. Hence, an anticlinal dam site will frequently require considerable cut-off excavation and pressure grouting to prevent leakage. Fault sites may vary greatly, but they generally require deep excavation and may even be geologically inadmissible.

In the large class of sites that have horizontal or uniformly inclined strata, the excavation required in the rock itself will generally be relatively small, but this may be offset by heavy excavation through detritus.

Factors Affecting Type of Dam

The selection of the most economical type of dam for any site, which geological examination has shown to be feasible, depends on a large number of factors such as, the purpose for which the storage is intended, economic considerations, flood conditions, the dimensions of the sites available, etc.

Purpose.—In certain cases the purpose of storage may influence the choice of a dam. Thus, the gravity dam is frequently preferred for water supply projects, because of its monumental proportions. In other cases, a hollow dam may be chosen, partly because it may afford housing for a power station or filter plant.

* A good sketch of the subject is that of Segré, "Cenni sugli studi geognostici applicati all'impianto di Laghi Artificiali", *Annali*, 1920; see, also, *Technical Publication No. 215*, Am. Inst. Min. and Metallurgical Engrs., "Geology and Engineering for Dams and Reservoirs."

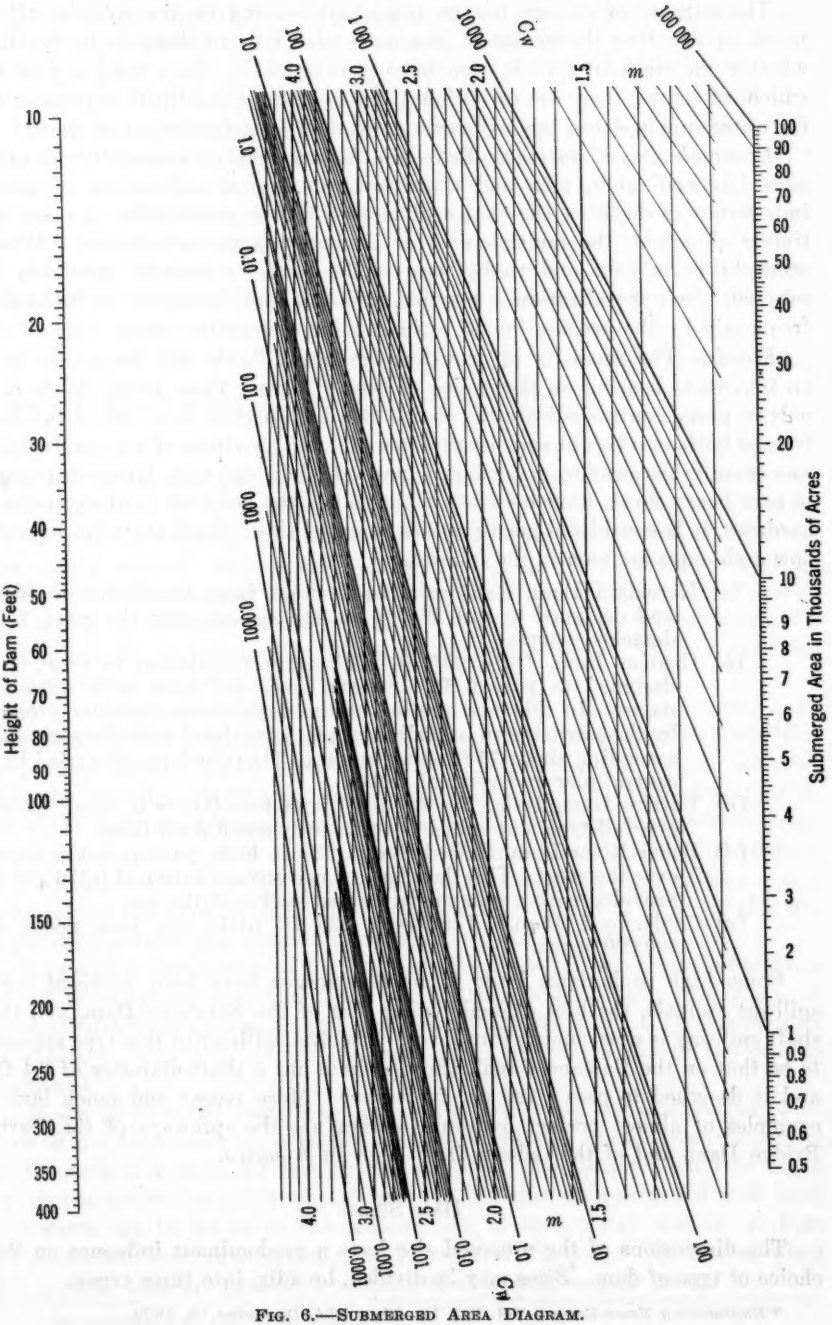


FIG. 6.—SUBMERGED AREA DIAGRAM.

The purpose of storage has an important bearing on the subject, not so much as affecting the choice of one particular type of dam, as in deciding whether the cheapest type is economically practicable. As a general rule (to which, of course, there are exceptions), the heaviest expenditure is permissible for water supply dams, less for power dams, and least for irrigation dams.

Economic Considerations.—Economic factors such as accessibility, cost of materials and labor, etc., and the return on capital outlay, are of prime importance in deciding the choice of dams. Where accessibility is poor, but timber abundant, the multiple-arch or Ambursen type is indicated. Where accessibility is poor and timber scarce, the earth or rock-fill type may be selected. No generalization is possible, and each case is treated on its merits, frequently by the preparation of estimates for alternative types.

Floods.—The necessity of providing for heavy floods will frequently have an important bearing on the choice of type of dam. Thus, heavy floods in a narrow gorge contra-indicate the choice of a high arched dam, unless the dam is to be built to a height sufficient to allow for the provision of a separate spillway or spillways away from the dam. Several fairly high arch dams constructed to pass heavy floods have been built, but unless the bed-rock is of exceptional hardness, it is considered good practice to avoid this. Good examples of overflow arch dams are seen in the following:

(*a*) Kerckhoff Dam, California, 94 ft. high from foundation to crest, and designed to spill 60 000 cu. ft. per sec. over the crest, with detached nappe.

(*b*) Cismon Dam, Italy, 140 ft. high from foundation to crest, and designed to pass 6 ft. of water over the crest with detached nappe. In this case erosion of the down-stream toe was avoided by the provision of a horizontal platform, level with the crest and extending about 75 ft. down stream. This platform is carried by a central pier and vertical arches.

(*c*) Turrite Dam, Italy, 130 ft. high from foundation to crest, of the overfall type, but subject to relatively small flood flows.

(*d*) Baker River Dam, United States, 233 ft. high, passing heavy floods over the crest. This dam has a down-stream batter of 0.376 and is provided with a substantial curved bucket at the toe.

(*e*) Calderwood Dam,* Tennessee, 230 ft. high, can pass floods of 200 000 sec-ft.

Some high arch dams built in narrow gorges have been provided with spillway tunnels, the best example being that of the Shoshone Dam, and the shaft spillway is now coming into favor. The first spillway of this type appears to be that on the Bassano Canal, Italy,† which has a shaft diameter of 8.4 ft. and is designed to pass 1 120 cu. ft. per sec. More recent and much larger examples of almost perfect hydraulic design are the spillways of the Davis Bridge Dam, and of the Gibson Dam, both in America.

Dam Sites

The dimensions of the proposed site have a predominant influence on the choice of type of dam. Sites may be divided, broadly, into three types:

* *Engineering News-Record*, Vol. 103, No. 25, p. 954, December 19, 1929.

† "Sfioratori a Pozzi," Mario Luiggi, 1917.

(1) Those which are very narrow at the bottom but which broaden considerably at the top (see Fig. 2, Section *A*).
(2) Those with **V**-shaped walls that are practically straight.
(3) Those that are approximately **U**-shaped.

It may be said at once that the first or convex type of site is exceedingly rare, three examples being the Morena, Pacoima, and Jogne Dams. The second or **V**-type is also quite infrequent, an example being the site of the Stevenson Creek Experimental Dam.* The majority of dam sites belong to the third or concave type.

Characteristic canyon profiles may be represented by parabolic curves, using the word, "parabolic", in its mathematical sense† as the shape of curve represented by the equation,

$$y^p = Q\,x^q \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

These curves may be represented by the general equation,

$$l = D\,h^v \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

It will be obvious that the shape of the curve depends only on the index, v, while the scale, or the width at any height for curves of the same shape, depends on the value of the constant, D. Hence, the dimensions of any dam site, if the natural surface were perfectly smooth, could be represented by Equation (6) by selecting the appropriate values of the constants. The convex type would be characterized by values of v greater than unity, the **V**-type by $v = 1$, and the concave type by values of v less than unity. The site need not be symmetrical to comply with Equation (6).

Any attempt to determine from the section of a site the constants in Equation (6) is generally unsatisfactory, because of the marked irregularities of natural surface which generally occur. The writer has found, however, that the equation derived by integrating Equation (6) with respect to h, affords a satisfactory means of representing the average dimensional properties of almost any dam site. Equation (1) which is used for this purpose will be called the "site area equation." In this equation the value of n is a criterion of the type of site. Thus, the convex, **V**, and **U**-types of site are typified by values of n greater than, equal to, and less than 2.

The writer has computed and plotted site areas for more than fifty dams of all descriptions, and has found that although marked irregularities of natural surface are present the hypothesis of Equation (1) is well supported in practice. Fig. 7 shows the site-area curves for only a few of the profiles examined.

It should be noted that in the case of arched dams, the site section to be used is not that taken on a vertical section across the gorge, since the whole of the arch dam does not rest on this section. The section should be taken on chords across the curve of the dam. In the case of a proposed arch dam, this curve can be drawn on the contour plan in such a way that the arch at all points will abut satisfactorily on the sides of the gorge. Of course, when the contour lines are parallel, a vertical cross-section may be taken.

* *Proceedings*, Am. Soc. C. E., May, 1928, Pt. III.
† Frost in "Curves and Curve Tracing."

It may be thought that the use of the site-area curve is a somewhat roundabout way of defining mathematically the properties of a dam site. The method, however, has the great advantage that when the area is plotted against height on logarithmic paper, the straight line which best represents the points may readily be drawn by inspection, and this cannot easily be done if width is plotted against height. As a method of representation, the site-area equation is, of course, equally as good as the site-width equation, from which it is derived by integration. The process is seen to consist practically in replacing the actual site by another theoretical site having the same general properties.

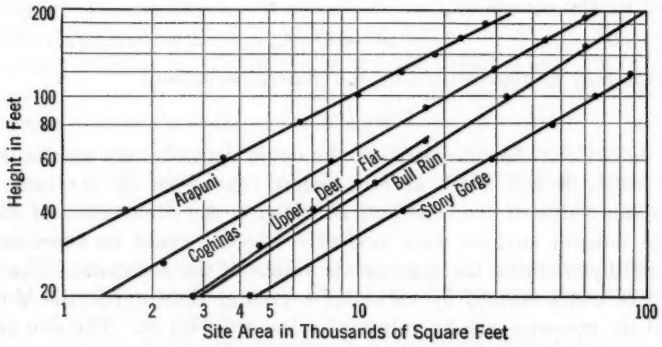


FIG. 7.—LOGARITHMIC SITE AREA CURVES.

Equation (1) may be represented graphically by means of an alignment chart (Fig. 8). This chart may be used to find C_A and n for any site by drawing straight lines between different points on the height scale, and corresponding points on the area scale, and selecting an average of the points of intersection. This point will define C_A and n.

VOLUME OF DAMS

Having seen that the dimensions of most dam sites may be approximately represented by a mathematical equation, it will be clear that the volume of dams built, or to be built, in such sites may be determined mathematically, since the cross-sectional dimensions of dams may also generally be represented by mathematical equations.

Gravity Dam.—The early profiles proposed by Rankine, Krantz, Harlacher, and others, which had curved faces, were dealt a severe blow by Atcherley* and the modern gravity dam is practically triangular in cross-section. Wegmann's† practical profiles, and those recently proposed by Koechlin‡ are in effect triangular (except for the additional thickness of the crest), and where variations do occur, the profile may be represented for the present purpose by an equivalent triangular profile.

* "On Some Disregarded Points in the Stability of Masonry Dams," 1904.
† "The Design and Construction of Dams."
‡ "Mécanisme de l'Eau," Vol. 2, 1926.

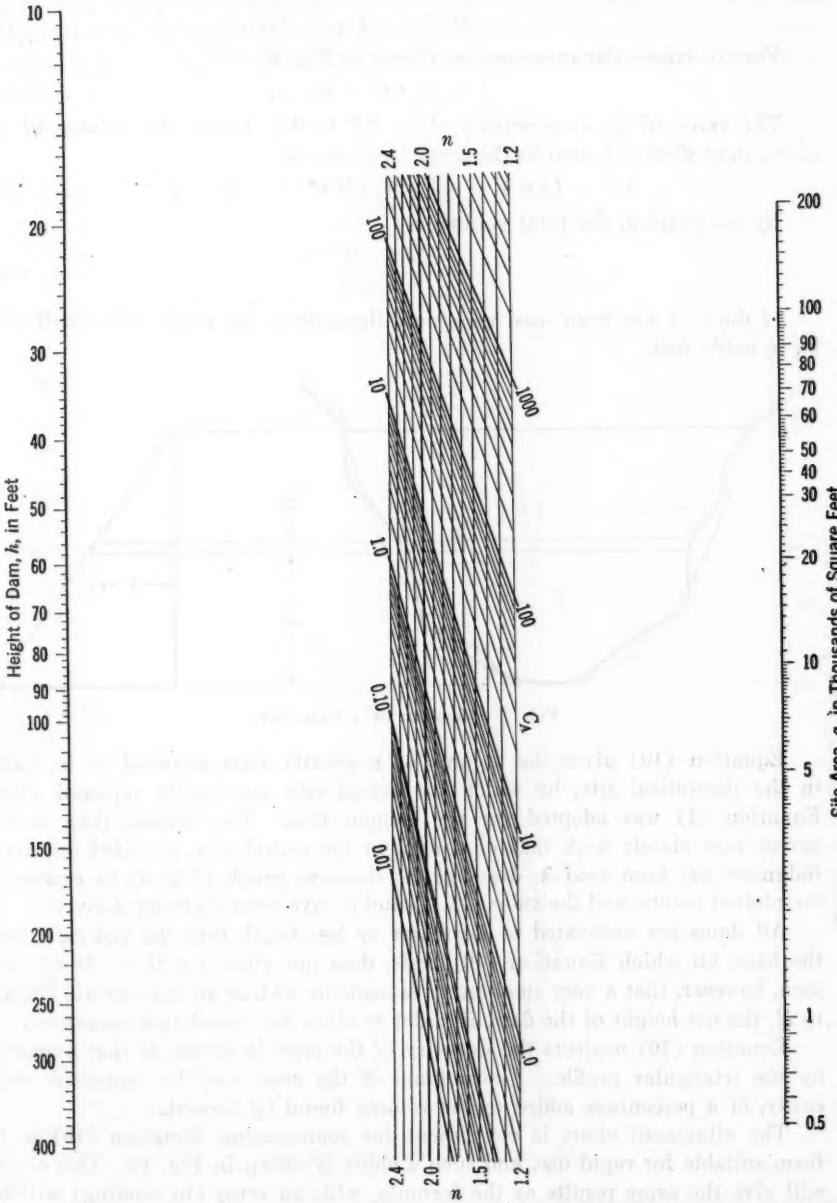


FIG. 8.—SITE AREA DIAGRAM FROM EQUATION (1).

Referring to Fig. 9 in which the actual site is shown in full lines and the theoretical site in broken lines, and differentiating Equation (1):

$$l = D\,h^v = n\,C_A\,h^{n-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

For the triangular cross-section shown in Fig. 9,

$$t = K_g\,(H - h) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The value of K_g is generally about 0.7 to 0.8; hence, the volume of an elementary slice of a dam in the theoretical site is:

$$d\,V = l\,t\,d\,h = n\,C_A\,K_g\,(H\,h^{n-1} - h^n)\,d\,h \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

By integration, the total volume is:

$$V_g = \frac{C_A\,K_g\,H^{n+1}}{n+1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

If the foot has been used as a unit throughout the paper, the result will be in cubic feet.

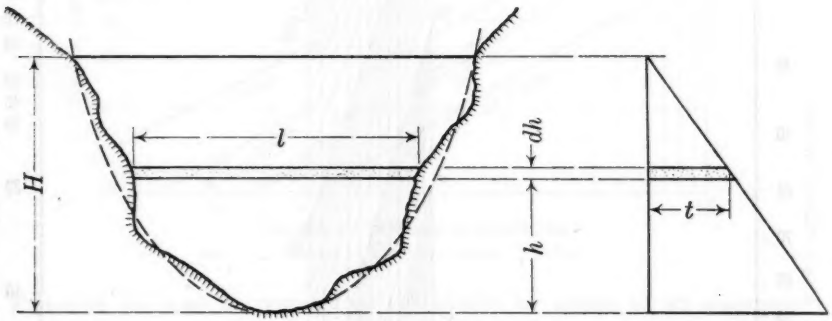


FIG. 9.—DIAGRAM OF A DAM SITE.

Equation (10) gives the volume of a gravity dam assumed to be built in the theoretical site, by which the actual site was tacitly replaced when Equation (1) was adopted for the computation. The volume thus found agrees very closely with that of a dam in the actual site, provided ordinary judgment has been used in drawing the site-area graph (Fig. 8) to represent the plotted points, and the values of C_A and n have been correctly derived.

All dams are excavated to a greater or less depth into the bed-rock, but the basis on which Equation (10) rests, does not allow for this. It will be seen, however, that a very small error is made by adding an appropriate height to H, the net height of the dam, in order to allow for foundation excavation.

Equation (10) neglects the masonry of the crest in excess of that required by the triangular profile. The volume of the crest may be computed very easily, or a percentage added to the volume found by formula.

The alignment chart is convenient for representing Equation (10) in a form suitable for rapid use, and such a chart is shown in Fig. 10. This chart will give the same results as the formula, with an error (in reading) within 2 per cent.

Fig. 10 is based on a value of K_g equal to 0.7 and includes 5% excess volume for crest thickness, this having been found by experience to be an

average value. The chart may be used as it stands to determine approximately the volume of a gravity dam of moderate height in a site for which the values of C_A and n have been determined. In other cases, as, for example, very high dams, or those in which allowance is made for a high degree of uplift, a preliminary design of cross-section is required to determine the appropriate value of K_g.

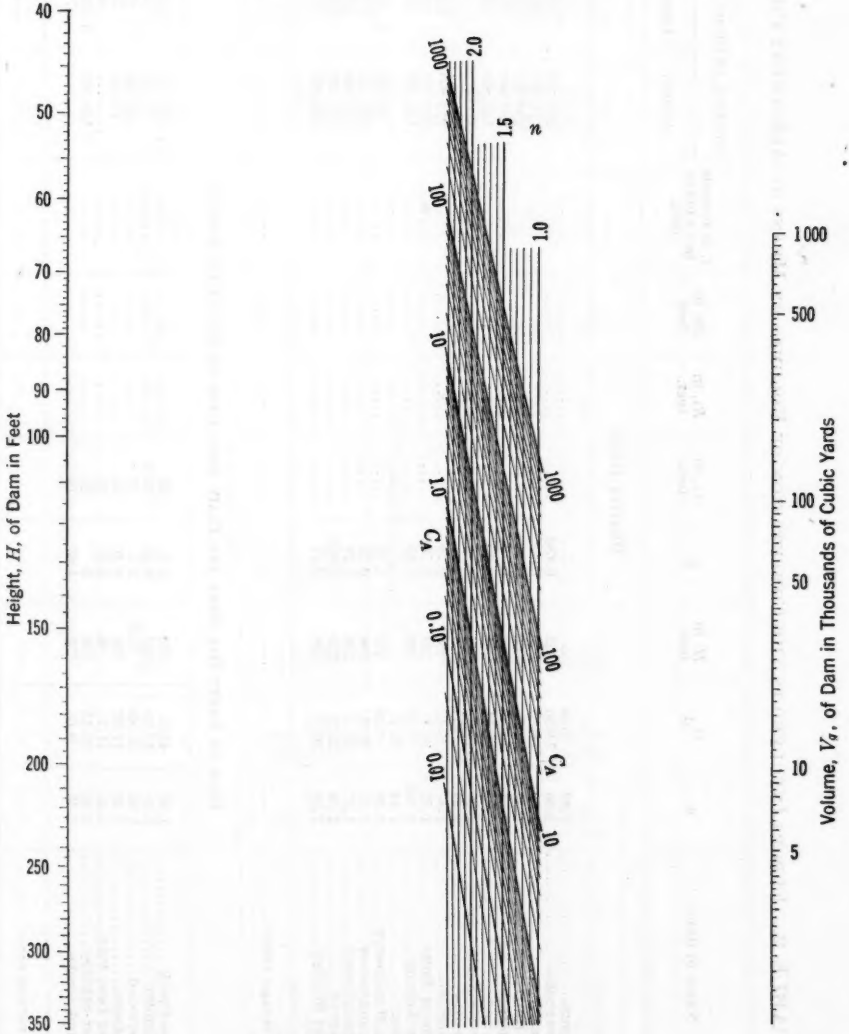


FIG. 10.—VOLUME OF GRAVITY DAM.

It may be stated at once that the writer does not suggest that the equation or chart should ever be used for an accurate computation of quantities, as for a contract, since the mensuration, although itself mathematically accurate,

TABLE 2.—ERRORS INVOLVED IN THE COMPUTATION OF VOLUMES BY MEANS OF ALIGNMENT CHARTS.

Item No.	Name of dam.	n.	C_A.	H, in feet.	K.	t_1, in feet.	R_1, in feet.	R_2, in feet.	f, in pounds per square inch.	Volume, in Cubic Yards.		Percentage of error.
										Actual.	Computed.	
					GRAVITY DAMS.							
1	Arapuni	1.89	1.59	.....	.....	.....	.....	.....	.....	.....	.....	.....
2	Arrowrock	1.66	14.36	323	0.680	.....	.....	.....	.....	585 165	591 000	1.01
3	Avon	1.60	15.17	226	0.70	.....	.....	.....	.....	214 000	200 000	6.5
4	Bull Run	1.52	30.5	200	0.74	.....	.....	.....	.....	230 000	222 000	3.48
5	Camarasa	1.57	10.43	299	0.827	.....	.....	.....	.....	283 000	286 000	1.05
6	Cataract	2.09	1.71	185	0.70	.....	.....	.....	.....	149 800	146 200	2.4
7	Ceresole Reale	1.30	140.4	.....	.....	.....	.....	.....	.....	.....	.....	.....
8	Coghinas	1.69	8.55	.....	.....	.....	.....	.....	.....	.....	.....	.....
9	Cordeaux	1.87	6.46	195	0.70	.....	.....	.....	.....	216 000	220 000	1.82
10	Elephant Butte	1.52	36.7	264	0.735	.....	.....	.....	.....	500 000	502 600	0.52
11	Little Nerang	1.764	7.88	200	0.65	.....	.....	.....	.....	165 000	156 700	.....
12	Nepean	1.44	35.7	.....	.....	.....	.....	.....	.....	.....	.....	.....
13	Lake Manchester	1.78	8.26	125	0.73	.....	.....	.....	.....	53 000	54 000	1.85
14	O'Shaughnessy	1.37	35.3	306	0.75	.....	.....	.....	.....	398 516	386 000	3 14
15	O'Shaughnessy	1.37	35.3	395	0.78	.....	.....	.....	.....	660 000	630 000	4.50
16	Roosevelt	1.53	25.2	275	0.600	.....	.....	.....	.....	342 325	346 302	1.15
17	Lower Stanley	1.56	30.3	150	0.67	.....	.....	.....	.....	155 000	110 000	.....
	Average error	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	2.5
					ROCK OR EARTH-FILL DAMS AND PLAIN ARCH DAMS OF TRAPEZIUM SECTION.							
18	Goose Creek	1.52	48.3	145	5.0	20	.....	.....	.....	1 000 000	1 059 700	5.97
19	Pathfinder	1.48	10.69	218	0.40	10.9	.....	.....	.....	60 210	57 800	3.65
20	Silverwood	1.56	26.85	72.5	3.5	12	.....	.....	.....	85 180	83 000	2.55
21	Shoshone	1.30	14.07	328	0.40	10	.....	.....	.....	78 576	72 300	8.00
22	Upper Deer Flat	1.41	41.7	66	5.0	20	.....	.....	.....	.....	89 180	.....
23	Upper Stanley	1.80	30.15	110	5	20	.....	.....	.....	1 130 000	1 136 200	0.55
24	Walnut Grove	1.45	9.06	110	1.07	10	.....	.....	.....	.....	479 000	.....
	Average error	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	4.14

TABLE 2.—(*Continued.*)

Item No.	Name of dam.	n.	C_A.	H, in feet.	K.	t_1, in feet.	R_1, in feet.	R_2, in feet.	f, in pounds per square inch.	Volume, in Cubic Yards.		Percentage of error.
										Actual.	Computed.	
					Plain Arch Dams.							
25	Gibson	1.67	13.7	195	0.28	.....	.....	.....	.....	.....	778 000	.....
26	Mt. Edwards	1.5	27.0	90	0.33	.....	.....	.....	.....	22 400	20 300	9.35
27	Melones	1.595	12.27	210	0.38	.....	.....	.....	.....	91 000	78 000	.....
28	Salmon River	1.33	44.8	220	0.55	.....	.....	.....	.....	.....	121 000	.....
29	Turrite	1.54	6.57	130	0.44	.....	.....	.....	.....	.....	10 100	.....
					Constant-Angle Dams.							
30	Bullards Bar	1.60	10.5	183	.....	.....	240	94	350	47 000	44 170	6.3
31	Coomera	1.468	23.39	170	.....	.....	290	140	300	.....	36 500	.....
32	Gerber	1.53	25.25	70	.....	.....	197	155	300	9 500	9 200	3
33	Jogne	2.28	0.175	170	.....	.....	129	90	.....	.....	.....	.....
34	King River	1.87	1.68	240	.....	.....	250	100	305	56 675	52 900	6.7
35	Nathan	1.68	10.43	205	.....	.....	380	200	335	110 000	110 000	.....
36	Pacoima	2.12	0.2924	375	.....	.....	320	115	290	160 000	149 000	6.9
37	Salmon Creek	1.68	11.34	168	.....	.....	331	147.5	330	52 000	53 000	1.92
38	Kerckhoff	1.53	20.4	108	.....	.....	.....	.....	.....	23 000	.....	.....
39	Lake Spaulding	1.99	1.466	275	.....	.....	440	250	420	192 000	165 000	1.50
40	Warragamba	1.51	21.8	310	.....	.....	.....	.....	.....	.....	261 000	.....
	Average error	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	4.38

is based essentially on an approximation, and no engineer would stake his reputation on an approximate method. The method is valuable, however, for many preliminary studies, and the use of the chart enables a volume to be found in a few minutes, which by the usual methods of calculation would require many hours' work.

In Table 2 are given the values of C_A and n for a number of sites as determined from the site area graphs, such as Fig. 8. These data, together with the actual height above foundation, and the total batter, are used to obtain the theoretical volume, which is compared with the actual volume.

The values of n and C_A for Item No. 2 of Table 2 refer to the developed up-stream face. A correction was made for arch curvature in this case by multiplying the computed volume by a factor,

$$\frac{\text{Up-stream radius} - \frac{1}{4}\text{ base width}}{\text{Up-stream radius}} = 0.92$$

Thus, an allowance is made for the reduced radius, measured to the center of gravity.

The computed volumes for Items Nos. 4 and 16 were multiplied by 1.03 and 1.06, respectively, to allow for curvature. Due to an added width of base, 64 000 cu. yd. were added to the volume of Item No. 14 as computed by the charts.

The actual volumes given for Items Nos. 11 and 17 include estimates of quantities in the spillways, access, bridges, etc., the amount of which was not known.

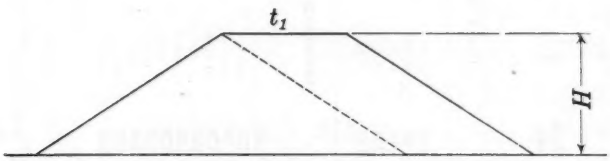


FIG. 11.—TYPICAL TRAPEZIUM CROSS-SECTION.

DAMS OF TRAPEZIUM CROSS-SECTION

The earth and rock-fill dam is almost always trapezoidal in cross-section. In some cases, one or more berms are provided and the batter may vary slightly from crest to base. In such instances, an equivalent trapezium section may be drawn. Gravity dams are also sometimes of trapezium cross-section. For example, the cross-section in Fig. 11 may be divided into two parts. One part is triangular, and the equivalent volume may be found from the same formula as for a gravity dam (Equation (10)), by inserting K_R for K_g. In this paper the batter is defined as the ratio of horizontal increment to vertical increment. The reciprocal of this ratio is sometimes referred to as the batter. The volume of the second part is simply the site-area to height, H, multiplied by t_1.

The total volume in cubic feet, therefore, is,

$$V_R = C_A \left(\frac{K_R H^{n+1}}{n+1} + t_1 H^n \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

The volume in cubic yards can also be readily found from the charts by taking $\frac{K_R}{0.735}$ times the volume of a gravity dam to height, H, from Fig. 10, plus $\frac{t_1}{27}$ times the site-area to height, H, from Fig. 8. In Table 2, Items Nos. 18 to 24, inclusive, have been prepared to furnish a comparison between the actual volume of a number of dams and the volume obtained by Equation (11). In the case of arch dams, a factor of 1.05 is introduced to allow for the additional length along the mean arc. For Items Nos. 19 and 21, the value of H is listed as 218 and 328, respectively. In computing the volumes by Equation (11), the values used were 223 and 338, respectively, in order to make allowance for excavation.

Plain Arch Dams

A plain arch dam with vertical up-stream face and uniform cylinder stress is theoretically of triangular section. The cylinder stress is the arch stress computed by the cylinder formula. As is well known, it may differ considerably from the stress that actually exists. The down-stream batter is uniform and is equal to,

$$K_A = \frac{62.4\ R_u}{144\ f} = 0.434\ \frac{R_u}{f} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

The volume may best be found by the same formula as for a gravity dam by inserting K_A for K_g in Equation (10) and multiplying by a factor to allow for curvature. The factor, 1.05, which corresponds to a mean subtended angle of about 60°, may be used for this purpose. For other values of the mean subtended angle the factor may be read from the curve in Fig. 12.

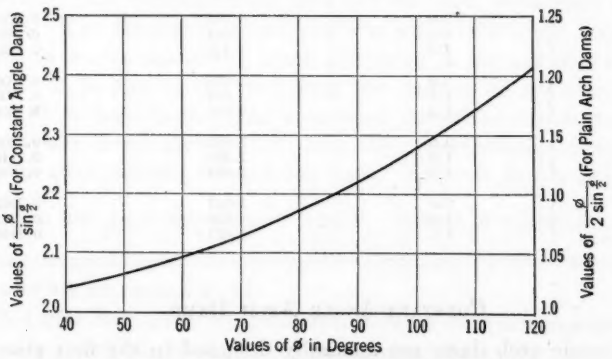


Fig. 12.—Curvature Factors for Arch Dams.

In Table 2, Items 25 to 29, inclusive, show volumes for five arch dams. In all cases, 5% has been added for additional material in the crest. Another 5% has been added in all cases to take account of curvature. The quantity given as actual volume in Item No. 27 includes the volume of the bucket on the down-stream side of the dam.

From the fact that the use of the formula for the volume of a gravity dam has been recommended for finding that of an arch dam, it must not be assumed

hat this volume varies directly as C_A, or as the $(n + 1)$ power of the height. This is not so. A formula can be derived as follows: Take a crest angle of 110° and write,

$$R_u = 0.613\ n\ C_A\ h^{n-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Inserting this value of R_u in Equation (12) and the resulting value of K_A in Equation (10), and also introducing a curvature factor, $\frac{\phi}{2 \sin \frac{\phi}{2}}$, the result is,

$$V_A = \frac{0.266 \left(\frac{n}{n+1}\right) C_A^2\ H^{2n}}{f} \times \frac{\phi}{2 \sin \frac{\phi}{2}} = \frac{J_A\ C_A^2\ H^{2n}}{f} \quad \ldots\ldots (14)$$

To determine the values of J_A in terms of n for different crest angles, several "type" designs were developed for a crest span of 600 ft. and a height of 100 ft. (the values used are immaterial). The values of the curvature factor were obtained by weighting the value of the ratio, $\frac{\text{arc}}{\text{chord}}$, at different points by the volume per foot of height at that point and thus obtaining the weighted mean. Values of this curvature factor and of J_A are given in Table 3

TABLE 3.—VALUES OF CURVATURE FACTORS AND J_A.

Crest angle, in degrees.	n.	$\frac{\phi}{2 \sin \frac{\phi}{2}}$.	J_A.
120	2.0	1.048	0.1750
	1.6	1.064	0.1642
	1.2	1.122	0.1532
100	2.0	1.036	0.1953
	1.6	1.047	0.1826
	1.2	1.086	0.1676
80	2.0	1.025	0.2300
	1.6	1.032	0.2140
	1.2	1.054	0.1936
60	2.0	1.014	0.2934
	1.6	1.018	0.2721
	1.2	1.029	0.2440

CONSTANT-ANGLE ARCH DAMS

Constant-angle arch dams are generally designed in the first place by the use of the cylinder formula, modifications being then made on the basis of more accurate and frequently involved analyses. In any case the cylinder stress is readily determinable.

It will be assumed that the up-stream radius varies linearly from crest to base, an assumption which is generally valid.* It will also be assumed that the cylinder stress and subtended angle are uniform from crest to base. Actually, this is not always true,* but average values may be chosen to represent,

* Due to Mr. Nimmo, in his discussion previously mentioned.

fairly well, the actual value. In the case of a preliminary design, the cylinder stress is taken to be uniform throughout.

Now, the thickness at the height, h, is,

$$t = \frac{0.434}{f} R_h (H - h) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

in which,

$$R_h = R_2 + \frac{h}{H} (R_1 - R_2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

Substituting this value of R_h in Equation (15):

$$t = \frac{0.434 (H - h)}{f} \left[R_2 + \frac{h}{H} (R_1 - R_2) \right] \ldots\ldots\ldots\ldots (17)$$

The length of arc is $\frac{n C_A h^{n-1} \phi}{2 \sin \frac{\phi}{2}}$. The volume of an arch element at a height, h, is, therefore,

$$d V = \frac{0.434\, n\, C_A\, \phi\, h^{n-1} (H - h) \left[R_2 + \frac{h}{H} (R_1 - R_2) \right]}{2 f \sin \frac{\phi}{2}} d h \ldots\ldots (18)$$

Integrating between $h = 0$ and $h = H$ and reducing:

$$V_{CA} = \frac{0.217\, n\, C_A\, \phi\, H^{n+1} \left[\frac{n R_1 + 2 R_2}{n (n + 1) (n + 2)} \right]}{f \sin \frac{\phi}{2}} \ldots\ldots\ldots (19)$$

Theoretically, the constant-angle dam has the same subtended angle from crest to base. The most economical angle is about 133°, but comparatively wide variations do not cause very much alteration in the quantity of material in the arch. It is due to this fact that the method of analysis leading to Equation (19) is legitimate. The subtended angle for constant-angle arch dams is generally about 110° to 120° over the upper portion of the dam, the angle falling away rapidly toward the base. For use in Equation (19), a value of 85° to 95° gives satisfactory results. Values of the factor, $\frac{\phi}{\sin \frac{\phi}{2}}$, for various angles are given in Fig. 12.

Table 2, Items Nos. 30 to 40, inclusive, shows a comparison between actual quantities for a number of dams and those obtained by Equation (19). In Item No. 32, the writer added 15 ft. for foundations and reached an assumed value of H, equal to 70 ft. Likewise, for Items Nos. 35 and 36, 10 ft. were added to the net height to allow for foundations. For Items Nos. 37 and 39, 5 ft. were added for foundations. The heights of these four items were further increased 5% to make allowance for the crest. The actual volume listed for Item No. 39 includes approximately 24 500 cu. yd. of extra material in the gravity base. This dam (Lake Spaulding) was partly constructed as a gravity dam and was later altered to a constant-angle type. The concrete in

the constant-angle dam is, therefore, about 167 500 cu. yd. The computed volume of Item No. 40 is as presented by Mr. Nimmo.*

A comprehensive formula for the volume of a constant-angle arch dam can also be found in much the same way as Equation (14) was derived for the volume of an arch dam. Three "type" designs were prepared on the following basis:

Height, in feet	100
Crest span, in feet	800
Crest angle, in degrees	116
Cylinder stress, in pounds per square inch	300

n = 1.6, 2.0, 2.4, respectively.

The up-stream radii were made to reduce linearly from the crest to one-fifth of the height, at which point the radius was taken as R_2. Furthermore, the radii were naturally made as small as possible without undercutting on the up-stream face. From these trial designs, the factors involving radii and angle in Equation (19), can be expressed in terms of n and of crest span, on which alone the radii and angles depend, under the assumptions stated.

In Equation (19) let, $0.217 \dfrac{\phi}{\sin \dfrac{\phi}{2}} \left[\dfrac{n R_1 + 2 R_2}{n (n + 1) (n + 2)} \right]$ be made equal to $J \times$ the span, or $J \times n C_A H^{n-1}$, in which, J is a function of n only.

Inserting this value in Equation (19),

$$V_{CA} = \frac{J n^2 C_A^2 H^{2n}}{f} = \frac{J_{CA} C_A^2 H^{2n}}{f} \quad \ldots\ldots\ldots\ldots (20)$$

which is similar to Equation (14). For practical purposes, J_{CA} may be taken as 0.147. Equation (20), therefore, may be reduced to,

$$V_{CA} = \frac{0.147 C_A^2 H^{2n}}{f} \quad \ldots\ldots\ldots\ldots (21)$$

which is a very simple expression for the volume of a structure of relatively complicated design.

Equation (21) may be solved by the use of the alignment chart, Fig. 13. For this purpose, f is assumed as 300 lb. per sq. in. For other values of f the volume can be adjusted proportionately. To allow for crest thickness increase the volume from 2 to 5 per cent. The error introduced by using this equation is not greater than that by Equation (19), and the alignment chart introduces only a small reading error.

Use of Volume Formulas

Equations (10), (11), (14), and (20) have been evolved for gravity, rock-fill (or trapezoidal dams of any type), arch, and constant-angle dams, respectively. They have been found by test to give errors in most cases within 5%, as compared with the actual volume of constructed dams, or the carefully computed volume of proposed dams. For many preliminary purposes these equations, or the charts derived from them, therefore, may be used to estimate

* In his discussion previously mentioned.

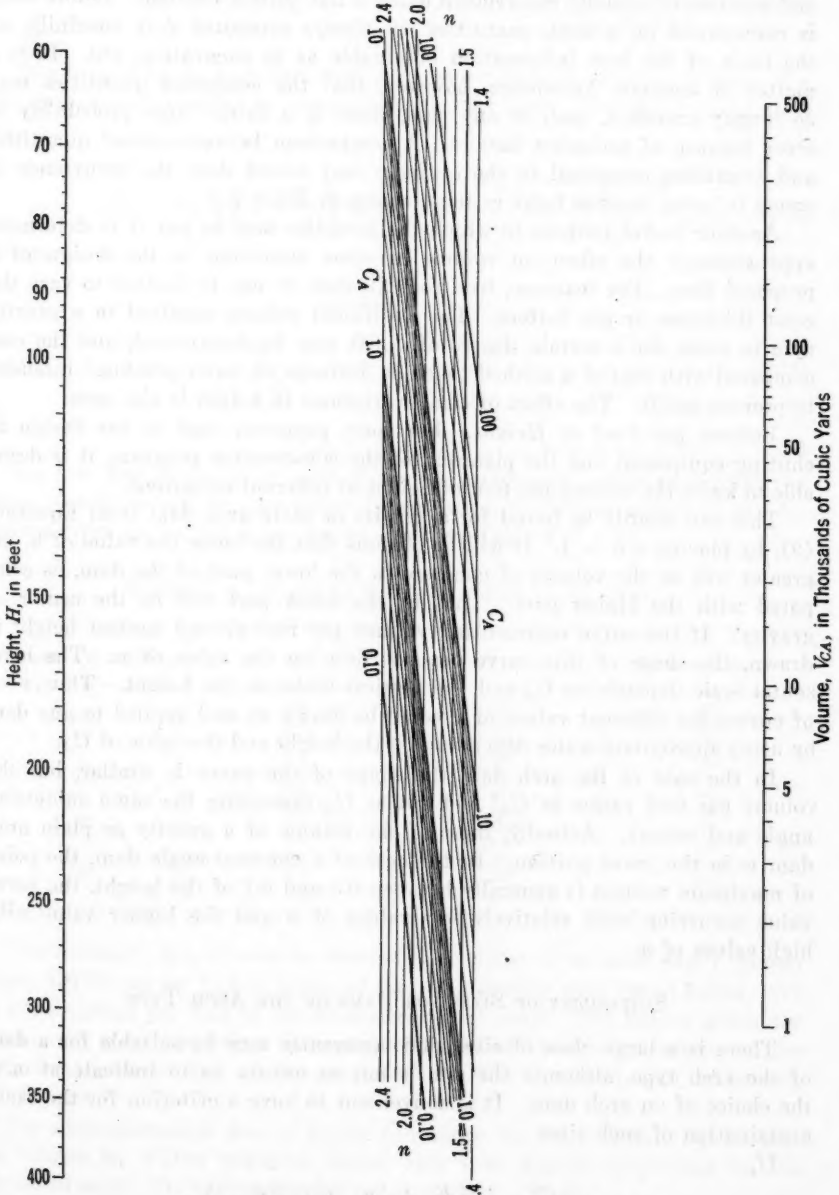


FIG. 13.—VOLUME OF CONSTANT ANGLE ARCH DAM.

roughly the volume and cost of a proposed dam. In this connection it should be stated that the comparison made between volumes computed by the formulas and volumes of actually constructed dams is not quite a fair one. Before work is commenced on a dam, quantities are always computed very carefully, on the basis of the best information obtainable as to excavation, etc. It is a matter of common knowledge, however, that the computed quantities may be largely exceeded, and, in any case, there is a fairly large probability of error because of unknown factors. A comparison between actual quantities and quantities computed in the ordinary way would show the occurrence of errors in many cases as large as those shown in Table 2.

Another useful purpose to which the formulas may be put is to determine approximately the effect on volume of some alteration in the design of a proposed dam. For instance, in a rock-fill dam it may be desired to vary the crest thickness or the batters. The additional volume required in a gravity dam to allow for a certain degree of uplift may be determined, and the cost compared with that of a method (such as drainage or water-proofing) intended to prevent uplift. The effect of small variations in height is also seen.

Volume per Foot of Height.—For some purposes, such as the design of chuting equipment and the planning of the construction program, it is desirable to know the volume per foot of height at different elevations.

This can readily be found for a gravity or plain arch dam from Equation (9), by placing $d\,h = 1$. It will be obvious that the lower the value of n, the greater will be the volume of concrete in the lower part of the dam, as compared with the higher part. That is, the lower part will be the center of gravity. If the curve representing volume per foot plotted against height is drawn, the shape of this curve depends only on the value of n. The horizontal scale depends on C_A and the vertical scale on the height. Thus, a set of curves for different values of n could be drawn up and applied to any dam by using appropriate scales depending on the height and the value of C_A.

In the case of the arch dam the shape of the curve is similar, but the volume per foot varies as $C_A{}^2$ and not as C_A (assuming the same subtended angle and stress). Actually, most of the volume of a gravity or plain arch dam is in the lower portion. In the case of a constant-angle dam, the point of maximum volume is generally between 0.5 and 0.7 of the height, the lower value occurring with relatively low values of n and the higher value with high values of n.

Suitability of Sites for Dams of the Arch Type

There is a large class of sites that apparently may be suitable for a dam of the arch type, although the site is not so narrow as to indicate at once the choice of an arch dam. It is convenient to have a criterion for the rapid examination of such sites.

If,

$$J_A = 0.434\,\frac{R_y}{L}\left(\frac{n}{n+1}\right)\left(\frac{\phi}{2\sin\frac{\phi}{2}}\right)$$

Equation (14) may be written,

$$V_A = 0.434 \frac{R_u}{L} \left(\frac{C_A^2 H^{2n}}{f}\right) \left(\frac{n}{n+1}\right) \left(\frac{\phi}{2 \sin \frac{\phi}{2}}\right) \ldots\ldots\ldots (22)$$

The volume of an arch dam may then be determined in terms of that of a gravity dam and relative to the theoretical crest span. For this purpose note that,

$$V_G = C_A K_G \left(\frac{1}{n+1}\right) H^{n+1} = \frac{n K_G \left(\frac{1}{n+1}\right) C_A^2 H^{2n}}{L} \ldots\ldots (23)$$

Hence,

$$\frac{V_A}{V_G} = \frac{0.434 \frac{R_u}{L} \frac{\phi}{2 \sin \frac{\phi}{2}}}{f K_G} L \ldots\ldots\ldots\ldots (24)$$

or,

$$V_A = V_G Q_A L \ldots\ldots\ldots\ldots (25)$$

Let $f = 300$ lb. per sq. in. and $K_g = 0.7$, and insert in Equation (24) the values already found for the other terms; then successive values of Q_A for various crest angles and spans are as listed in Table 4.

TABLE 4.—VALUES OF Q_A IN EQUATION (25).

Crest angle, in degrees.	n.	Q_A.	Maximum span.
120	2.0	0.125	800
	1.6	0.127	787
	1.2	0.134	746
100	2.0	0.140	715
	1.6	0.141	710
	1.2	0.147	680
80	2.0	0.1646	608
	1.6	0.1654	603
	1.2	0.169	591
60	2.0	0.209	477
	1.6	0.210	476
	1.2	0.212	472

The constant, Q_A, is seen to express the volume of an arch dam (cylinder stress, 300 lb. per sq. in.) in terms of that of a gravity dam (total batter, 0.7) as a percentage per foot of the theoretical crest span. The figures given for "maximum span" represent the spans which would make the volume of the arch dam equal to that of the gravity dam. The practical span, of course, will be less.

For a constant-angle dam, a similar procedure can be followed, but because the volume is, within practical limits, only very slightly dependent on the subtended angle, the expression derived is simpler. Thus,

$$\frac{V_{CA}}{V_G} = \frac{100 J_{CA} (n+1) L}{210 n} = Q_{CA} L \ldots\ldots\ldots\ldots (26)$$

Values of Q_{CA} for use in this equation are listed in Table 5. The practical use of these results (Table 5) is to determine rapidly for any given site and height of dam, what degree of economy may be effected by the use of an arch as against a gravity dam. The arch dam will be required to show considerable economy in material for the following reasons: (1) The design and setting out of the dam is more complicated; (2) the cost of form work per cubic yard is higher, since the faces are curved and frequently warped and the sections are less massive; (3) the concrete is frequently made either richer in cement or with a less quantity of large rock added; (4) greater care is generally required in construction; and (5) the cost of spillways and accessory works may be higher since the arch dam is less conveniently adapted for passing floods over the crest.

TABLE 5.—VALUES OF Q_{CA} IN EQUATION (26)

n.	Q_{CA}.	Maximum span.
2·4	0.1055	958
2.0	0.1054	959
1.6	0.1138	878
1.2	0.1288	776

Table 6 has been prepared to show the relative suitability of various sites on which dams of the plain arch or constant-angle arch type have been constructed, or for which they have been proposed. The table is based on the following assumptions:

1.—The cylinder stress in all cases is 300 lb. per sq. in.
2.—The correct values are given for the crest angle where it is known. Other values are assumed.
3.—In Items Nos. 1 to 10, inclusive, the dams have a vertical up-stream face.
4.—In Items Nos. 11 to 27, inclusive, the up-stream radii at other points than the crest are made as small as possible without undercutting.
5.—The gravity dam used for comparison has a value, $K_g = 7$.

Table 6, therefore, compares, on a uniform basis, the merits of the sites only, and has no reference to existing or proposed dams on the sites. This is considered the most reasonable method of comparison, since the design of the actual dams may vary considerably from the bases assumed, because of a variety of factors, such as the strength of rock, the predilections of the engineers, or local peculiarities of the site, and it would be difficult to make just allowance for all these factors.

Items Nos. 8, 9, 10, and 22 to 27 inclusive, contain data relating to sites at which actually other types of dam are constructed or proposed, but where dams of the arch or constant-angle arch type, under certain circumstances, might be suitable.

It appears evident that, for economy, the volume of an arched dam should not exceed 80% of that of a gravity dam, on the assumptions mentioned. In other words, the theoretical crest span should not exceed 80% of the maximum

permissible span. It must be remembered, however, that favorable circumstances such as very resistant rock may influence the design of an arched dam to a much greater extent than in the case of a gravity dam. The profile of the latter may be varied only between comparatively narrow limits, no matter how excellent the site, but an arch dam may be designed so as to use more effectively the strength of the foundations and of the material of construction. The Gibson Dam exemplifies this fact, and it represents no doubt the first step in the direction of broadening the scope of arch dams by a more rational system of design and technique of construction.

TABLE 6.—RELATIVE SUITABILITY OF DAM SITES AS REGARDS ECONOMY OF MATERIALS.

Item No.	Name of dam.	n.	C_A.	Height, in feet.	Theoretical crest span, in feet.	Crest angle, in degrees.	Maximum span, in feet.	Volume expressed as a percentage of the volume of a gravity dam.
	PLAIN ARCH DAMS.							
1	Pathfinder	1.48	10.69	218	209	80	600	34.8
2	Shoshone	1.30	14.07	328	104	65	510	20.4
3	Gibson	1.67	13.7	195	782	120	792	98.6
4	Mt. Edwards	1.50	27.0	90	385	80	600	64.2
5	Melones	1.595	12.27	210	470	110	750	62.7
6	Salmon River	1.33	44.8	220	353	100	690	51.2
7	Turrite	1.55	6.57	130	140	80	604	23.2
	Mean	1.49	.....	....	....	....	....	50.7
8	Camarasa	1.57	10.43	299	422	100	708	59.7
9	Coghinas	1.69	8.55	180	520	100	712	73.0
10	Nepean	1.44	35.7	210	542	100	700	77.5
	CONSTANT-ANGLE ARCH DAMS.							
11	Bullard's Bar	1.60	10.5	183	384	116	877	43.8
12	Coomera	1.468	23.39	170	379	116	845	44.8
13	Gerber	1.53	25.25	70	368	116	860	42.8
14	Jogne	2.28	0.175	170	287	116	972	29.6
15	King River	1.87	1.68	240	370	116	935	39.5
16	Nathan	1.68	10.43	205	653	116	895	73.0
17	Pacoima	2.12	0.2924	375	479	116	970	49.4
18	Salmon Creek	1.68	11.34	168	620	116	895	69.2
19	Kerckhoff	1.53	20.4	108	374	116	860	43.5
20	Lake Spaulding	1.99	1.466	275	758	116	955	79.4
21	Warragamba	1.55	21.8	310	618	116	855	72.1
	Mean	1.75	.....	....	....	....	....	53.4
22	Camarasa	1.57	10.43	299	422	116	870	48.5
23	Coghinas	1.69	8.55	180	520	116	900	64.6
24	Nepean	1.44	35.7	210	542	116	840	87.4
25	Gibson	1.67	13.7	195	782	116	895	57.8
26	Mt. Edwards	1.50	27.0	90	385	116	853	45.2
27	Melones	1.595	12.27	210	470	116	876	53.7

MOST ECONOMICAL SIZE OF INSTALLATION

For every installation involving storage there exists a size of storage which gives the most economical result in terms of cost per second-foot of regulated outflow.* By means of the procedure outlined in this paper, and following

* "The Most Economical Size of a Hydro-Electric Installation," by A. B. Mead, *New Zealand Journal of Science and Technology*, 1921.

the general lines of the method previously used by the writer,* the curve of specific cost (total cost of installation per second-foot of regulated outflow), may be found very readily for any particular case, and the most economical size of installation determined.

Whether the installation adopted will be of the size indicated by considerations of lowest specific cost depends on a large number of local conditions, which must be considered for each particular case. In general, there will be a tendency to push the size somewhat beyond the point of lowest unit cost, although the installation may be developed in stages.

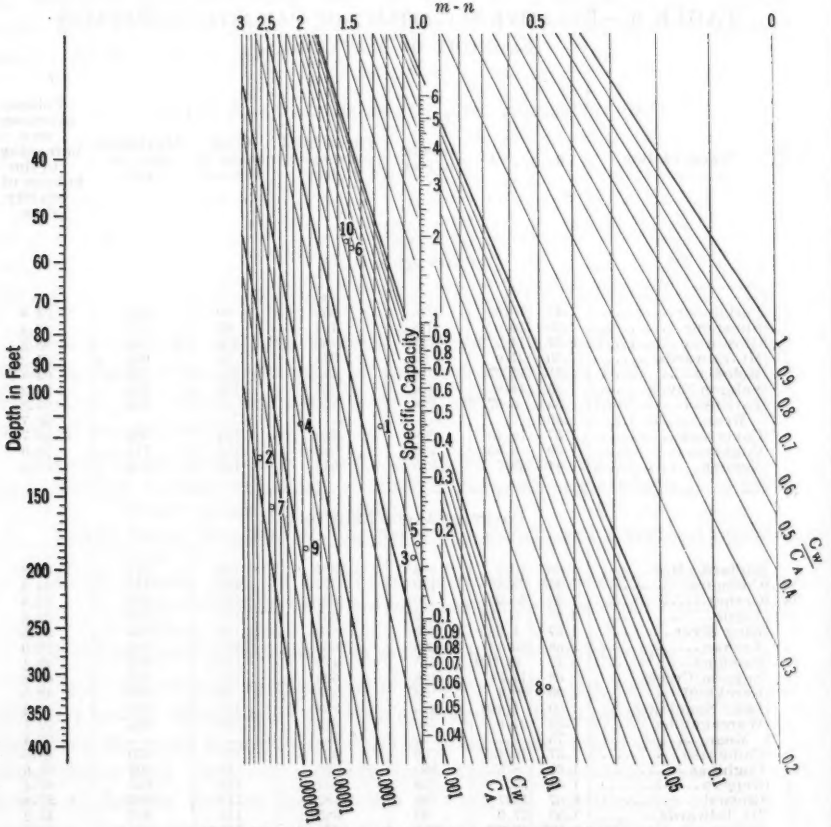


FIG. 14.—SPECIFIC CAPACITY OF GRAVITY DAMS.

The method is useful in the study of any proposed plan, but is not intended to constitute a hard and fast rule by which the size of installation is to be fixed. It may be stated, however, that many economically and technically unsound projects would never have been adopted had such a method of study been in general use in the past.

* "The Economic Height of a Dam," *New Zealand Journal of Science and Technology*, 1923, Vol. 6, No. 1, pp. 36-39.

TABLE 7.—COMPUTATIONS TO DETERMINE SPECIFIC CAPACITY.

TABLE 7.—COMPUTATIONS TO DETERMINE SPECIFIC CAPACITY.

Item No.	Name of dam.	Depth, in feet.	C_w.	C_A.	C_x.	m.	n.	$m-n$.	$m-2n$.	Stress, in pounds per square inch.	W_S, IN ACRE-FEET PER CUBIC YARD.	
											From chart.	Actual.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
					GRAVITY DAMS.							
1	Arrowrock	249	0.2848	14.36	0.00198	2.92	1.66	1.26	.....	.....	0.54	0.48
2	Avon	210	0.000023	15.17	0.00000152	4.25	1.60	2.65	.....	.....	0.80	0.81
3	Bull Run	180	0.058	30.5	0.0019	2.57	1.52	1.05	.....	.....	0.16	0.16
4	Cataract	150	0.0000915	1.71	0.0000535	4.11	2.09	2.02	.....	.....	0.60	0.51
5	Ceresole Reale	134	0.325	140.4	0.00232	2.32	1.30	1.02	.....	.....	0.18	0.154
6	Coghinas	157	0.03272	8.55	0.00383	3.18	1.69	1.49	.....	.....	2.8	3.37
7	Cordeaux	170	0.0000165	6.46	0.00000256	4.33	1 87	2.46	.....	.....	0.27	0.35
8	Little Nerang	197	0.1365	7.83	0.0174	2.24	1.76	0.48	.....	.....	0.076	0.11
9	Nepean	207	0.000518	35.7	0.0000145	3.41	1.44	1.97	.....	.....	0.195	0.185
10	Roosevelt	220	0.0847	25.2	0.00336	3.04	1.53	1.51	.....	.....	4.0	4.0
.....	Mean	174	.....	.....	.....	.....	1.64	.....	.....	.....	0.96	1.02
					ARCH DAMS.							
11	Coghinas	157	0.03272	8.55	0.000447	3 18	1.69	.....	−0.20	300	5.15	.....
12	Gibson	161	1.02	13.7	0.00542	2.24	1.67	.....	−1.1	600	1.36	1.15
13	Nepean	207	0.000518	35.7	0.000000406	3.41	1.44	.....	+0.53	300	0.21	.....
14	Mt. Edwards	75	0.00497	27.0	0.0000068	3 56	1.5	.....	+0.56	300	2.3	1.36
15	Pathfinder	202	0.0001376	10.69	0.0000012	4.26	1.48	.....	+1.32	122	17.1	17.84
					CONSTANT-ANGLE ARCH DAMS.							
16	Coghinas	157	0.03272	8.55	0.000447	3.18	1.69	.....	−0.2	300	6.0	.....
17	Coomera	170	0.1592	23.39	0.000291	2.70	1.47	.....	−0.24	300	3.2	4.85
18	Gibson	161	1.02	13.7	0.00543	2.24	1.67	.....	−1.1	600	0.78	.....
19	Mt. Edwards	75	0.00497	27.0	0.0000068	3.56	1.5	.....	+0.56	300	2.8	.....
20	Nepean	207	0.000518	35.7	0.0000004	3.41	1.44	.....	+0.53	300	0.25	.....
21	Lake Spaulding	275	0.0858	1.47	0.0397	2.43	1.99	.....	−1.55	420	.....	0.45

Specific Capacity

Specific capacity will be defined as the acre-feet of reservoir capacity per cubic yard of material in the dam. By means of formulas developed herein, the specific capacity can be determined approximately for gravity, arch, and constant-angle arch dams. The results are useful in enabling an opinion to be formed as to the economic nature of any storage proposal, or for comparing quickly the merits of different reservoirs and sites.

Gravity Dams.—Let $h_1 = 174$, $H = 194$, and $n = 1.64$ (average values from Table 2). Then Equations (2) and (10) can be modified by the insertion of these quantities, and combined to form the following formula for specific capacity of a gravity dam:

$$w_{SG} = 72.9 \frac{C_W}{C_A} h_1^{m-n-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

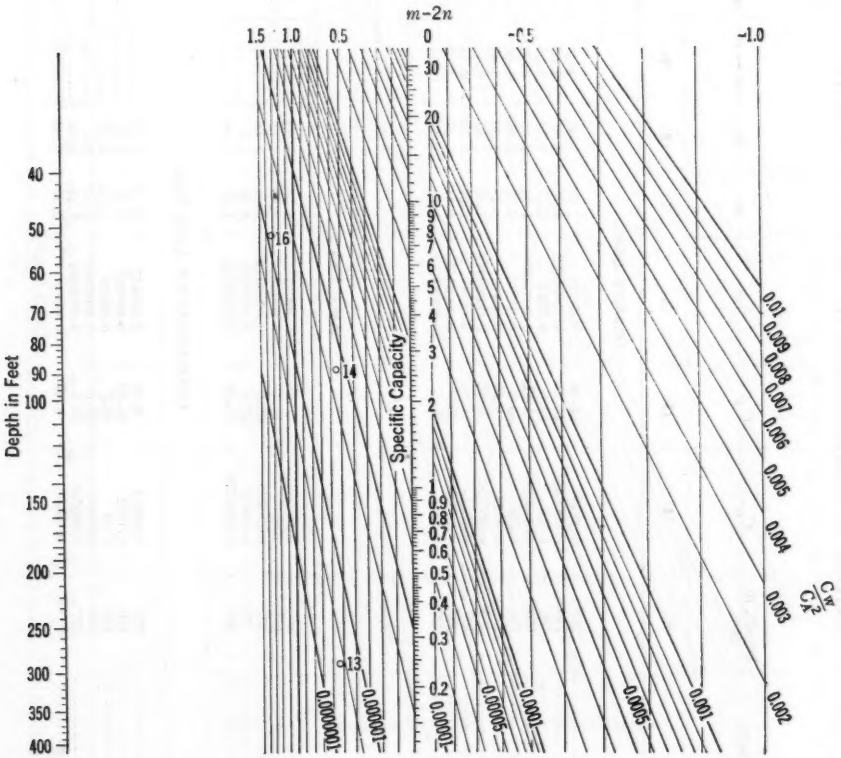


Fig. 15.—Specific Capacity of Arch Dams.

This formula will give reasonably correct results for dams of about the height mentioned, when the depth of excavation below the river bed is relatively unimportant, but it may be considerably in error for low dams, or for sites at which heavy excavation below the river bed is involved.

Equation (27) has been represented as an alignment chart in Fig. 14, and a comparison of the results obtained from the chart with actual values is shown in Table 7, Items Nos. 1 to 10, inclusive. No allowance has been made for the fact that in some cases the batter differs somewhat from the value of 0.7, on which the formula is based. Discrepancies are due to this variation in K_g, and to variations of n, height, and difference between height and depth, from the values assumed. The "actual" specific capacity for Items Nos. 5, 6, and 9 (see Column (13), Table 7) are estimated. Values of the quantity, $\frac{1}{n+1}$, vary from 0.322 at Item No. 4 to 0.435 at Item No. 5. In solving Equation (27) for values in Column (13) a mean value of 0.380 was used. Errors due to this cause alone may therefore be equal to about 20 per cent.

Arch Dams.—The assumptions necessary to derive a simple formula for specific capacity for gravity dams have been seen in Table 7 to lead to only approximate results. In the case of arch dams the variables are even more numerous, and the degree of approximation, therefore, is even less close. Let $\phi = 90°$, $n = 1.50$, $h_1 = 200$, and $H = 220$. Then, as before, Equations (2) and (14) may be combined to form an equation for specific capacity, in acre-feet per cubic yard, as follows:

$$w_{SA} = 30\,000 \frac{C_W}{C_A{}^2} h_1{}^{m-2n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

This formula is represented graphically in Fig. 15, and a comparison of results obtained by its use with the actual values of specific capacity is shown in Table 7, Items Nos. 11 to 15, inclusive. Unfortunately, the number of arch dams for which particulars are available regarding both the storage and the site is rather limited. Table 7, therefore, shows also the values of specific capacity for certain other sites in which dams of the arch type might be constructed (see Items Nos. 11 and 13). The values for Items Nos. 11 to 15, inclusive, in Column (12), were corrected for stress only.

Constant-Angle Arch Dams.—Let $n = 1.75$, $h_1 = 210$ ft., and $H = 230$ ft. Then, as before, the expression for specific capacity for a constant-angle dam may be written:

$$w_{SCA} = 38\,000 \frac{C_W}{C_A{}^2} h_1{}^{m-2n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

This formula is represented in Fig. 16 and the results obtained are given in Table 7, Items Nos. 16 to 21, inclusive.

It is also interesting to compare values of the specific storage for actual dams which have been built for different purposes, irrespective of the type used. In Figs. 17 and 18 only masonry dams have been included, since for dams built of this material the relative costs of storage can be compared roughly on the same basis. These diagrams may be taken as showing broadly values of the specific capacity which have been found economic for the purposes named, although naturally a number of local factors will alter the economic limits in each particular case. The upper and lower limits for specific storage are such that, for storages of 10 000 to 100 000 acre-ft., a much

lower specific storage (and, hence, higher unit cost) is permissible for water supply purposes than for irrigation or power.

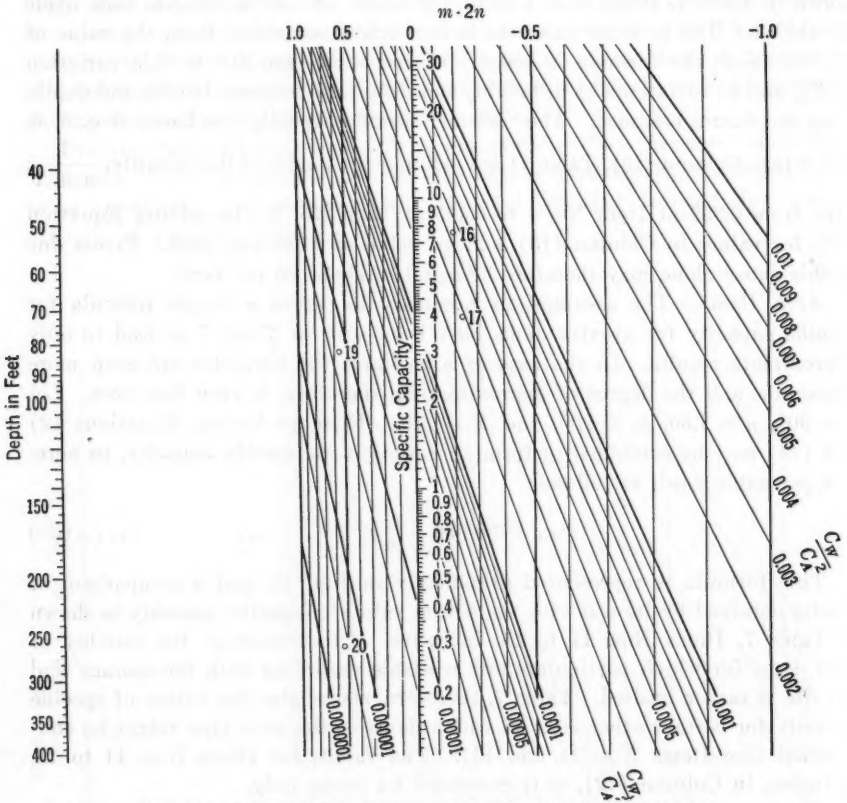


FIG. 16.—SPECIFIC CAPACITY OF CONSTANT-ANGLE ARCH DAMS.

It is usually desirable that "specific capacity," should increase with the height of dam; in other words, that, for gravity dams, m should exceed $n + 1$, and, for arch dams, m should exceed $2n$; but in practice this is quite frequently not so, as will be seen from the values given in Tables 1 and 2. When these conditions are not fulfilled, it would theoretically be cheaper to build two or more dams of less height instead of one; but suitable sites are generally lacking.

AVERAGE COST OF STORAGE

A number of writers have quoted figures for the cost per acre-foot of various storages. Thus, Wilson* quotes unit costs for a large number of storages, mainly for irrigation, in different countries. The reservoirs are arranged in order of increasing magnitude, and with inceasing capacity the unit cost tends to fall. Exceptions are due to the reservoirs having been con-

* "Manual of Irrigation Engineering," 1893, Table XI.

structed in different countries, at different times, and under a variety of different conditions.

A better method of stating "unit costs" is by means of the volume of dam per acre-foot of storage. Actually, the cost of a dam is not strictly proportional to the volume, but it is approximately so. By this means, the physical merits of various storages may be compared, irrespective of the time at which they were completed or the economic conditions of the country.

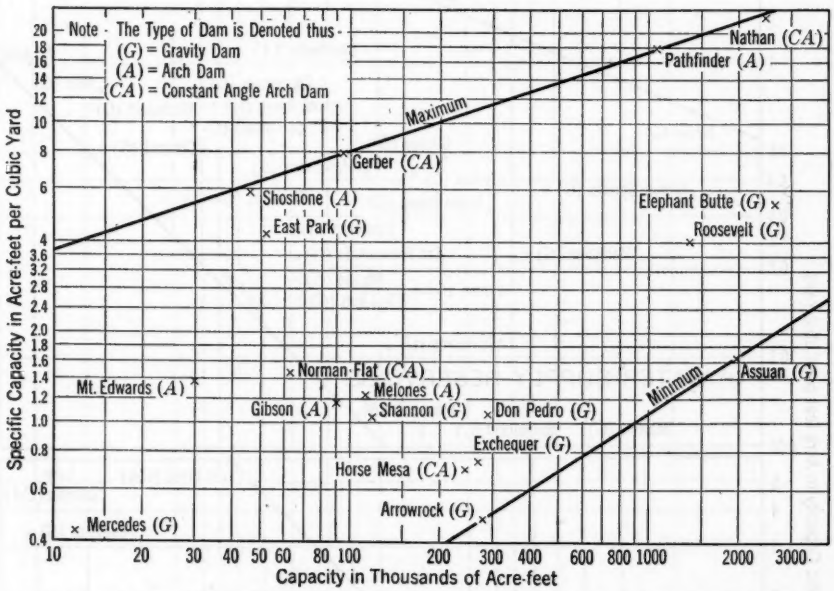


FIG. 17.—COMPARISON OF SPECIFIC CAPACITY FOR DAMS BUILT FOR IRRIGATION PURPOSES.

The various types of dam, such as masonry, earth-fill, and rock-fill, are then comparable in each group, but not between groups without introducing factors depending on the relative costs of the materials of construction.

CONCLUSION

It is the writer's hope that this outline, although "sketchy" in parts, will point the way to a more scientific treatment of the many aspects of water conservation. Some at least of the factors which have been treated in the past by rule-of-thumb or trial-and-error methods have here been shown to be susceptible of graphical or mathematical treatment.

Many agencies are engaged in taking stock of water resources in various countries, and the use of the methods outlined will enable development to be planned in the most economical manner.

ACKNOWLEDGMENTS

The writer wishes to acknowledge the use of information culled from many sources. Some of these are cited in footnotes, but much matter noted during

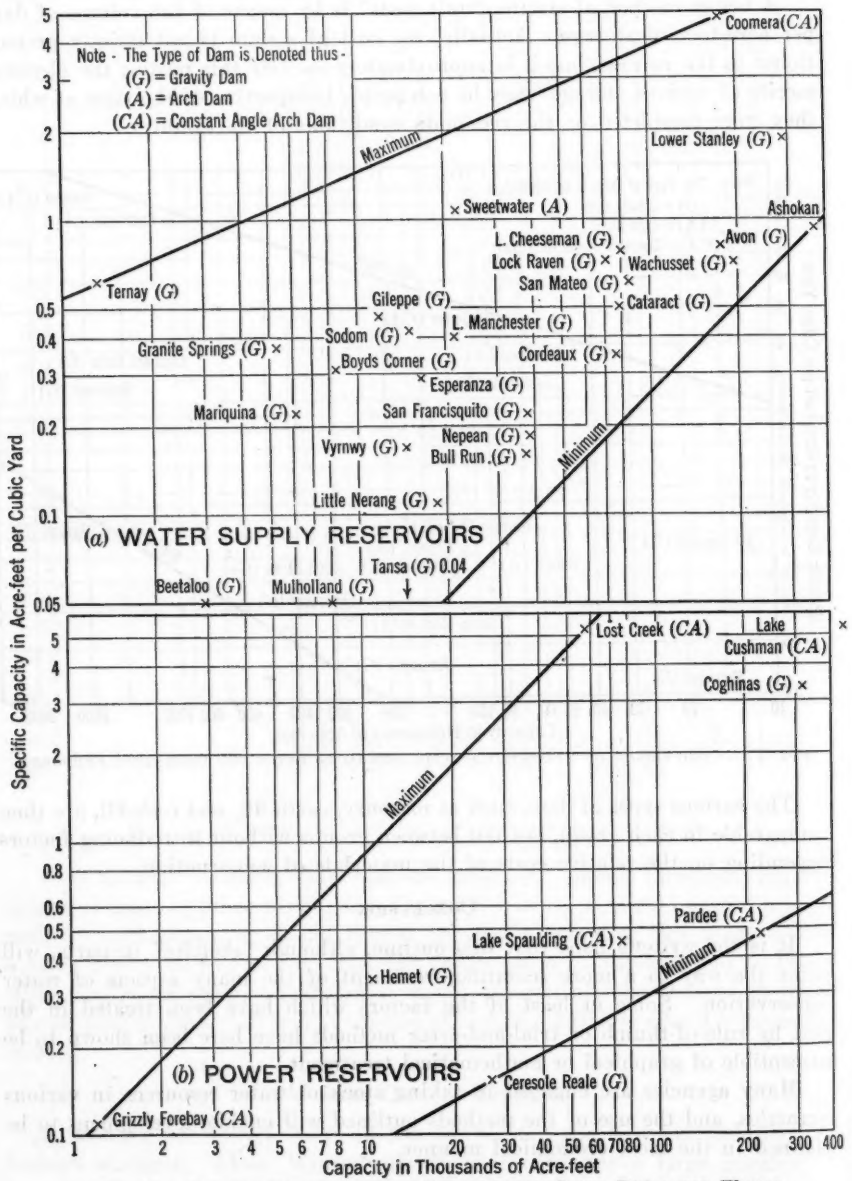


FIG. 18.—COMPARISON OF SPECIFIC CAPACITY FOR DAMS BUILT FOR WATER SUPPLY AND POWER RESERVOIRS.

the course of his professional work, from current and past technical literature, from information privately supplied by friends, notably W. H. R. Nimmo, Assoc. M. Am. Soc. C. E., and from other sources, has been incorporated.

He also wishes to thank A. F. Partridge, Commissioner for Irrigation and Water Supply, Queensland, for permission to use information relating to proposals in Queensland, and R. J. Spowart, Jun. Am. Soc. C. E., for much valuable assistance in the preparation of diagrams and with the computations.

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PAPERS AND DISCUSSIONS

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CONSTRUCTION OF LA OLA PIPE LINE, IN CHILE

By W. B. Saunders,* M. Am. Soc. C. E.

Synopsis

The purpose of building the pipe line described in this paper was to supply water to the Metallurgical Plant of the Andes Copper Mining Company, at Potrerillos, Chile. The line is 32.3 miles long and the pipe varies in diameter from 22 to 36 in.

The unique and outstanding features are as follows:

1.—The fabrication of riveted steel pipe in the field.
2.—The unusual method of field transportation for construction materials.
3.—The construction of a number of inverted siphons under very high pressure heads.
4.—The rapidity with which construction work was accomplished and the low unit costs obtained under difficult conditions.

Introduction

After several years spent in the development of its mining properties, and in experimental work on the metallurgy of its ores, the Andes Copper Mining Company, in January, 1925, started the construction of its Metallurgical Plant. The schedule called for the completion by January 1, 1927, of that part of the plant to be used for treating sulfide ores.

One of the major problems in connection with this project was to secure an adequate supply of water for use in the metallurgical processes adopted. The problem was difficult, because of the scarcity of water, the character of the country, and the high altitude of the plant.

The estimated requirements were 29 cu. ft. per sec., of which about 9 cu. ft. could be reclaimed.

Potrerillos is about 596 miles south of the northern boundary of Chile, and about 74.5 miles east of the Port of Chañaral. (See Fig. 1.)

Note.—Written discussion on this paper will be closed in January, 1931, *Proceedings*.

* Hydr. Engr., Andes Copper Min. Co., Barquito, Chañaral, Chile.

About 2 miles south of Chañaral, the Andes Copper Mining Company maintains its own port—Barquito—where its ocean-cargo handling facilities are located, a steam power plant of 30 000 kw. capacity, a warehouse, the Chilean Custom House, the railway terminal, and living quarters for employees engaged in port activities.

The Chilean Government maintains and operates a railway from Chañaral to Pueblo Hundido, a village 36 miles east of Chañaral. The Andes Copper Mining Company operates over this railway, under an agreement with the Government. It has constructed its own line between Pueblo Hundido and Potrerillos, a rail distance of 57.8 miles.

FIG. 1.—REGION OF LA OLA PIPE LINE.

Potrerillos, the site of the Metallurgical Plant, is the terminus of the La Ola Pipe Line, described in this paper. The mine is connected with the Metallurgical Plant by an electrified railway, 6.2 miles in length.

DESCRIPTION OF COUNTRY

The Continental Divide of South America is formed by the main range of the Andes Mountains which parallels the Pacific Coast; the summit is only from 100 to 150 miles inland. This range of mountains is the highest in the Western Hemisphere, the peaks reaching more than 22 000 ft. in altitude.

From a point about 300 miles south of Chañaral to beyond the northern boundary of Chile, the Humboldt Current (extremely cold), touches the western coast of South America and tempers the climate of that area. This cold-water area coming in contact with cold air currents from the high mountains and with cold ocean winds from the south, creates a physical condition that eliminates all possibilities of any great precipitation of moisture. As a consequence Potrerillos and the entire area surrounding it, is almost entirely devoid of rainfall. This area is known as the Atacama Desert, and receives a smaller amount of precipitation than the Sahara Desert.

This arid condition has produced a marked effect on the country. Incomplete erosion has left a topography of extreme roughness with considerable variation in elevation. The entire area is practically devoid of vegetable growth, and consequently geological formations are strikingly exposed.

The Metallurgical Plant is at an altitude of about 9 500 ft. and the mine at about 10 500 ft. The plant and the mine are both in what might be classed

as the foothills of the main Andes Range, the summit of which is about 50 miles farther east.

The area between Potrerillos and the Continental Divide is rough in the extreme; it is cut by "quebradas", or canyons, thousands of feet deep, which have created large areas inaccessible to any form of transportation other than by saddle or pack animals. Water holes are few and far between, and the entire area is incapable of supplying food of any character for either man or beast.

The La Ola Pipe Line was constructed within this area during 1925 and 1926, under the direction of the writer.

Water Supply

There is only one possible supply of water lying within a reasonable distance and capable of furnishing a sufficient quantity for the metallurgical requirements. This is a small stream, known as Rio La Ola, about 25 miles east of Potrerillos. The Ola River traverses a large volcanic "pampa," or plain, for a distance of about 9 miles. This pampa, practically surrounded by mountains, is about 125 miles long by 10 miles wide, and lies at an elevation ranging from 11 500 to 13 000 ft.

The river has its sources in a series of "vegas", or meadows, which include a large number of springs and water-holes; the supply is probably from an artesian flow originating in the high mountains where there is sometimes heavy precipitation in the form of snow. The "vegas" cover an area of several thousand acres and are almost entirely surrounded by a water-bearing formation. (See Fig. 2.)

The Ola River flows north above the surface for a distance of about 9 miles and then gradually disappears into the loose sands and gravels of which the pampa is formed. The indications are that from its point of disappearance it follows an underground seepage course and again rises about 12 miles farther north in Pedernales Lake, which is at the most northerly end of this pampa. Pedernales Lake is a salt bed with an area of about 115 sq. miles.

Necessarily any diversion of water from the Ola River must be accomplished at some point within the 9 miles where the water flows above the surface. Only one site was found that was suitable for the construction of a diversion dam. At this place, the formation of the pampa changes from loose sand and gravel to a conglomerate which has the characteristics of a very fair grade of concrete. Through this formation the river cut a narrow channel, leaving a good site for a low diversion dam. This formation appears for only a short distance up stream and down stream from the dam site, and seems to form a cut-off or barrier that throws all the Ola River water to the surface at this point.

The Spanish word, "ola," means ocean wave or billow; so this stream very probably received its name from the fact that the water is slightly salt or brackish to the taste. It does not contain sufficient salt to be unpalatable, however, and it was used throughout the construction period for camp drinking water and construction purposes. An analysis of the water is given in Table 1.

During the period of investigation of the Potrerillos ore deposits, a weir was constructed in the Ola River a short distance below the present dam site (see Fig. 3), and when work was started in 1925, flow records were available extending from September, 1915, to November, 1920. Since February, 1925, continuous records of flow have been kept. (See Fig. 4.)

TABLE 1.—ANALYSIS OF RIO LA OLA WATER.

Analysis.	Parts per million.	Probable combination.	Parts per million.
Ca	147.0	SiO_2	64.0
Mg	43.0	FeO	8.0
Na	661.0	$NaNO_3$	48.0
SiO_2	64.0	NaCl	1 646.0
FeO	8.0	$MgCl_2$	122.6
NO_3	35.0	$CaSO_4$	405.2
Cl	1 089.0	$Ca(HCO_3)_2$	112.5
SO_4	286.0	$Mg(HCO_3)_2$	68.6
HCO_3	141.9	.....	.....
Total	2 474.9	Total	2 474.9
Residue	2 502.0	.....	.....

The 5-year hydrograph which was available at the time the pipe line was designed, shows the following characteristics:

1.—The average flow for the 5-year period was 27.87 cu. ft. per sec.;
2.—The minimum flow occurred in December, 1917, with a monthly average of 21.44 cu. ft. per sec.; and,
3.—The maximum flow occurred during July, 1916, with a monthly average of 55.50 cu. ft. per sec.

The 5-year hydrograph was considered to be too short a period on which the base calculations but it indicated that the stream was one of fairly constant flow. This is reasonable when it is considered that practically all the water comes from underground sources and is subject to only slight variations, due to precipitation on the drainage area. The principal variations in flow are due to freezing during the winter months of June, July, and August, and to evaporation during the summer months.

Variations from freezing are characterized by daily peaks caused by the vegas freezing at night and thawing during the day. This variation often reaches 40% of the average flow of the stream; but it is easily controlled by the pondage up stream from La Ola Dam.

Climatic conditions in this area are such that a maximum of evaporation may be anticipated. The atmosphere is absolutely dry, the sun shines practically every day of the year, the air is brilliantly clear, there is no foliage and no grass, except in the vegas, and the wind reaches a velocity of about 20 miles per hour on practically every day of the year. During the warmer months, from November to April, these conditions have a marked effect on the flow of this stream, and it is believed that almost the entire seasonal variation is due to evaporation.

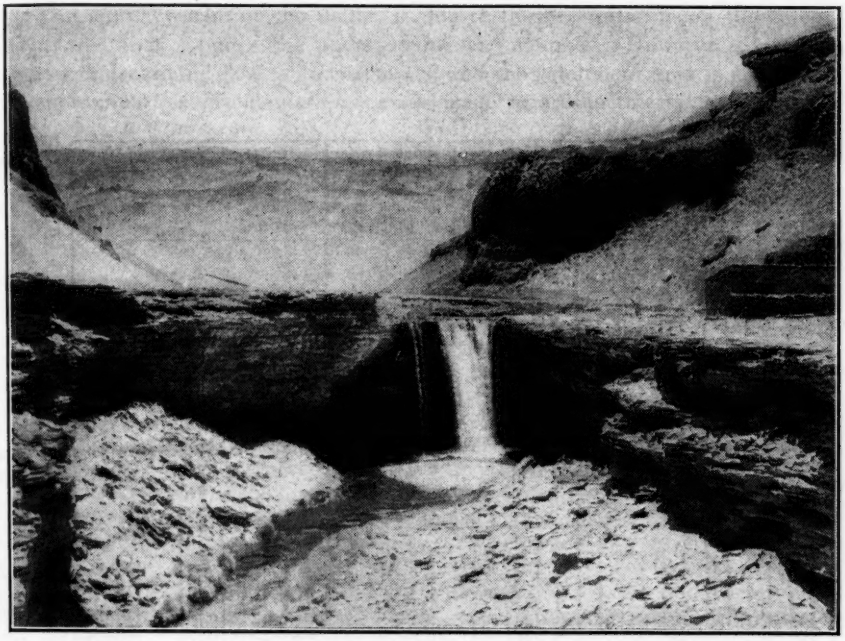

FIG. 2.—JUNCAL FALLS IN THE DRAINAGE AREA ABOVE LA OLA RIVER. THIS WATER DISAPPEARS BEFORE IT REACHES THE RIVER.

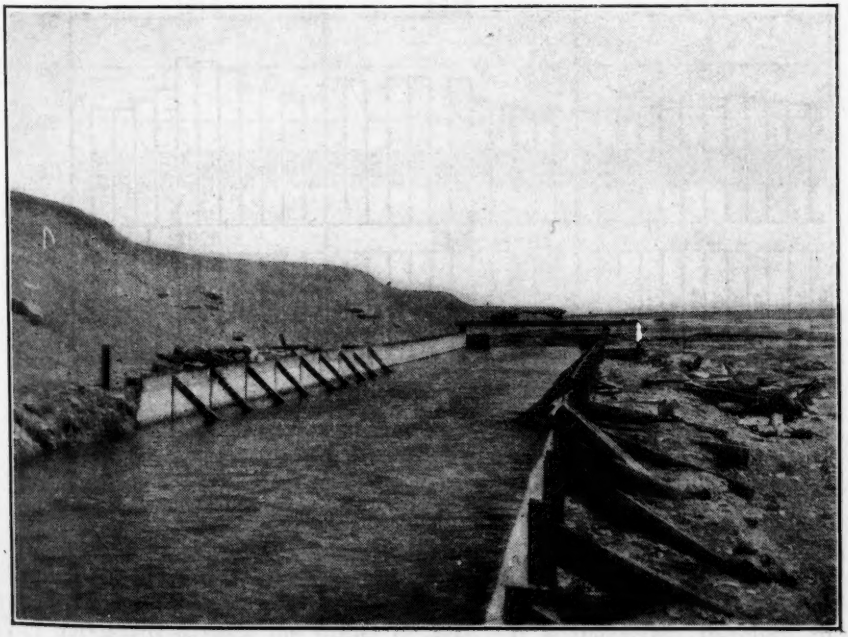

FIG. 3.—WEIR FOR MEASURING FLOW OF STREAM ABOVE LA OLA DAM.

The annual variation, no doubt, is due to the precipitation on the catchment area that supplies the underground watercourses. However, since the rate of underground flow is undetermined and the catchment area is unknown, no approximations or calculations can be made regarding this factor.

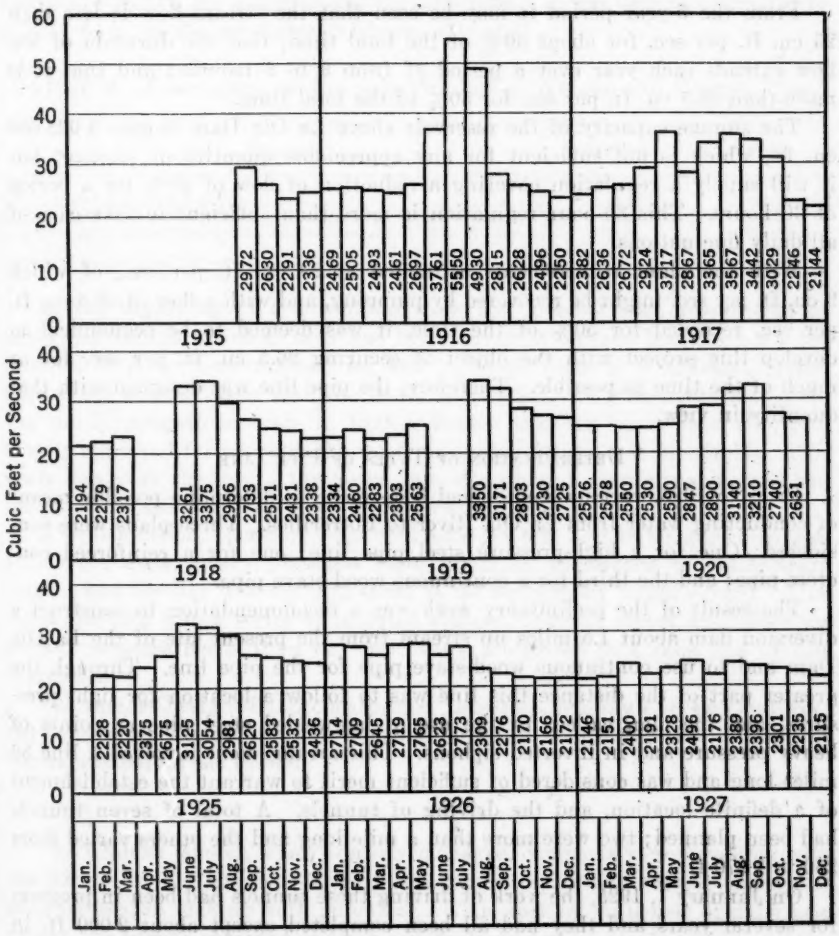

FIG. 4.—HYDROGRAPH OF LA OLA RIVER.

An excellent demonstration of this variation is shown by the hydrograph for 1927 (Fig. 4). During the early part of July of that year an unusually heavy snowfall occurred on the pampa and on the mountains surrounding it (see Fig. 1). The average depth of snow was more than 3.0 ft. on the pampa and undoubtedly reached a depth of many feet on the western slope of the main Andes Range; yet the month of June, just prior to this storm, shows a greater run-off than any following month of that year. A similar snowfall had not occurred since the winter of 1919. It is impossible, therefore, to

forecast accurately the annual variations of flow. Hence, it was necessary to assume, from past performance, that the stream has a fairly consistent and constant flow and that it is not subject to sudden great increases or decreases.

From the 5-year period it may be seen that the stream flow is less than 25 cu. ft. per sec. for about 30% of the total time; that the duration of low flow extends each year over a period of from 3 to 6 months; and that it is more than 26.5 cu. ft. per sec. for 50% of the total time.

The storage capacity of the reservoir above La Ola Dam is only 1 025 000 cu. ft., which is not sufficient for any appreciable quantity of storage; but it will supply a regulation covering a reduction of flow of 40% for a period of 30 hours. This 30-hour regulation is more than sufficient to take care of all daily fluctuations.

With the plant requirements estimated at 29 cu. ft. per sec., of which 9 cu. ft. per sec. might be recovered by pumping, and with a flow of 26.5 cu. ft. per sec. recorded for 50% of the time, it was deemed to be economical to develop this project with the object of securing 26.5 cu. ft. per sec. for as much of the time as possible. Therefore, the pipe line was designed with that quantity in view.

Determination of Types of Pipe Line

Prior to 1925, various studies had been made to determine possible means of conducting water from La Ola River to Potrerillos. Three plans were considered: One for a high-pressure steel pipe line; one for a reinforced concrete pipe; and the third for a continuous wood-stave pipe.

The result of the preliminary work was a recommendation to construct a diversion dam about 1.5 miles up stream from the present site of the La Ola Dam and to use continuous wood-stave pipe for the pipe line. Through the greater part of the distance this line was to follow a location for light pressures, and the plan called for the use of lap-welded steel pipe at points of heavy pressure and in inverted siphons. This arrangement involved a line 39 miles long and was considered of sufficient merit to warrant the establishment of a definite location, and the driving of tunnels. A total of seven tunnels had been planned; two were more than a mile long and the others varied from 250 to 1 270 ft.

On January 1, 1925, the work of driving these tunnels had been in progress for several years and they had all been completed except about 2 000 ft. in Pedernales Tunnel which has a total length of more than 1 mile.

During the prosecution of this work, a considerable length of 6-in. wood-stave pipe failed. It had been laid in soil having a chemical content similar to that to be traversed by the Ola line. This resulted in a decision to use steel in the construction of the new line. The difference in ocean freight and customs duties on fabricated pipe and steel plates made it necessary to manufacture as much of the pipe as possible on the site of the work. This is illustrated in Table 2.

Factors recommending the use of riveted pipe in preference to electrically welded pipe were: (1) The class of labor available; (2) the speed of fabrica-

tion that might be maintained; (3) the quality of workmanship to be obtained with unskilled labor; and (4) the joining of pipe in the field where electric power would not be available. In order that the life of the pipe would be equal to the requirements of the plant, and in order to construct pipe of sufficient rigidity, for handling and transportation, it was deemed advisable to specify $\frac{5}{16}$ in. as the minimum thickness of metal.

TABLE 2.—COMPARISON OF COSTS BETWEEN LAP-WELDED AND RIVETED PIPE.

Pipe.	Per foot.	Per ton.
Riveted pipe assembled on job	$ 4.33	$ 84.20
Light-weight lap-welded joint pipe purchased in United States	8.20	167.50
Heavy-weight lap-welded joint pipe purchased in United States	19.15	172.90

At the beginning of construction work the problem was encountered of using the tunnels which already had been driven at a grade laid down for the location of a wood-stave pipe line, and to adapt them to a hydraulic grade that would be consistent with the use of steel pipe. To complicate this problem the investigations made in 1925 indicated that the upper dam site, previously approved for use with the wood-stave pipe design, was unsuitable. This made necessary the use of the lower dam site where a previous structure had failed. This site was very close to the grade of the tunnels.

A bold location, therefore, was adopted. As few horizontal angles as possible were used; heavy pressures were taken in preference to unnecessary length; and the line was made as short as the topography would permit. This threw the line into a series of inverted siphons, some of which are under far greater pressure heads than any previously attempted in a pipe line of this character.

The line, as designed and constructed, therefore, is unique in two features: (*a*) It involved the use of heavy siphons; and (*b*) the greater part of the pipe was fabricated on the ground.

AVAILABLE LABOR

The Chilean laborer, when properly managed, compares very favorably with the laborer in the United States; but to obtain efficiency, certain expedients must be used in making work an object. With work on an hourly or daily rate of pay the results obtained are not satisfactory; but when placed on a piece-work system, or "contrato", the Chilean laborer works steadily and efficiently.

With these characteristics in view, methods of handling labor on the Ola Pipe Line were revised to suit the needs of the situation. Much of the work was done on a strictly piece-work basis. Often a "capataz", or foreman, would be given a contract and he, in turn, would divide his profits with the laborers. Special contracts were entered into with men or groups of men. In other cases bonuses were paid for certain work accomplished.

A separate and important class of labor on this work were the cartmen or "carreteros" who drove the mule carts which were used exclusively for trans-

portation of pipe and supplies. These men are almost a distinct class, and their work is a profession among them, which is handed down through generations in the same family. They consider themselves superior to the ordinary laborer, and will not do his work. They are a tough, hardy set of men who provide their own meals on the road and sleep under their carts at night, regardless of the weather.

There are few rough-work, or form, carpenters in Chile, but high-grade carpenters are rather plentiful. They are especially adapted to cabinet-making or work of a similar nature. They are slow, but they turn out a splendid product.

Mechanics are not plentiful, and it became necessary to recruit men from the better grade of laborers and train them as riveters. These people are naturally quick and skilful with their hands, so that by confining them to one class of work and putting that work on a contract basis, they would quickly become efficient.

The higher class of engineers and foremen on this work were in some cases contract men brought from the United States, and the others were secured in Chile.

The money values quoted in this paper are reduced to United States currency at the various rates of exchange existing throughout the period that the work was in progress. For practical purposes 8.25 pesos equal $1.00. The average 8-hour daily wages paid throughout this entire work were as follows:

Blacksmiths and helpers	$1.41	Machinists and helpers	$1.69
Boiler-makers and helpers	1.61	Masons and helpers	1.44
Carpenters and helpers	1.47	Miners: Hand	1.28
Chauffeurs	1.38	Miners: Machine	1.54
Clerks	2.66	Painters and helpers	1.40
Cooks and helpers	1.53	Pipe-fitters and helpers	1.85
Electricians and helpers	1.74	Rodmen and levelmen	1.71
Engineers and draftsmen	8.36	Teamsters and stablemen	1.50
Foremen	4.52	Tinsmiths and helpers	1.43
Harness makers and helpers	1.32	Watchmen	1.36
Iron workers and helpers	1.53	Welders and helpers	1.69
Laborers	1.22	Wheelwrights	1.48

The average wages, including bonus on contracts, were $1.57 U. S., or 12.95 Chilean pesos.

When work began on the Ola Pipe Line there was a decided scarcity of labor, due to the fact that all the work, in and about Potrerillos, was being started at that time. The ultimate requirement for the Company's entire project was to be about 10 000 men. The isolated location, slow transportation facilities, and incomplete organization in the beginning made the mobilization of labor forces slow and difficult. The quality of labor secured in the beginning was also of a low order. There was no choice except to take all who came, many of whom were drifters, and men who had no intention of doing a day's work. This, combined with the high altitude which affected many, the severe climatic conditions, and lack of proper clothing, made the turnover of labor reach as high as 40% each month at the beginning of the work.

There was a continual demand for more labor on this work from its start until September, 1926, when the first necessary reduction of forces was begun. The maximum number of men on the job was reached in April, 1926, at which time the payroll showed 1 310 employed. Shortly after that date an epidemic of influenza made a considerable reduction in the forces, and that number was not reached again.

Pipe Assembly Plant: Steps in the Process

All riveted pipe and special shapes were fabricated at the assembly plant, which was located in the bottom of Pasto Cerrado Canyon where the pipe line intersects the Potrerillos Railway (see Fig. 1). This is the lowest point on the entire line and is about 2 000 ft. lower than the outlet.

The canyon is about 650 ft. wide at the bottom and has a gradient of about 4 per cent. The walls are 1 800 ft. high on the north side and 2 500 ft. on the south. The Potrerillos Railway runs along the northern edge of the canyon bottom, and a spur was run at the end of the plant in such a way that, by utilizing the 4% grade, tracks were dropped from this spur so that all materials could be transported through the assembly plant by gravity.

The plant equipment consisted of layout benches, punches, shears, drill-press, forges, rolls, air compressors, oil rivet-heaters, assembly platforms, dipping-tanks, and the necessary trackage, trolleys, and derricks for handling the materials through the plant.

Twelve distinct operations carried the steel plates through the plant and finished them into pipe of approximately 20-ft. lengths. Each operation was performed by men specially trained for that particular work. Practically every step was done on a piece-work basis and the gangs of men were so proportioned that the work ran through the plant at about an equal speed for each operation. The itemized costs given in Table 3 are the averages for the eighteen different classes of pipe manufactured.

Receipts of Materials.—All steel plates and other materials were received on railway cars, dropped into the assembly plant yard, and "spotted" at the proper unloading point. Many cars of plates contained loads of mixed sizes and thicknesses. Laborers were kept busy sorting this material but the size of the gang varied with the requirements. These men would work at any time, night or day, when cars were received for unloading. The pay for unloading and sorting plates and other material was from 40.00 to 60.00 pesos ($4.85 to $7.25) per car, depending on the class of material to be unloaded. (See Table 3, Item 4.)

Layout Work.—The second process consisted of transporting plates on push-cars from the stock piles to the layout tables and there marking them for the punches.

For straight pipe, pilot holes were first made with screw punches from a template which squared the plate. Later, these were used to hold the plate in proper position in the gang punches. This was done by drift-pinning the plate through the pilot holes to a corresponding hole in a spacing table erected in front of each punch. These pilot holes were placed in the regular position of rivet holes and, later, fell into the line and spacing of the regular punching.

(See Table 3, Item 5.) The price paid for handling and pilot-punching plates for straight pipe was 0.60 peso per plate.

Special layout work for curved pipe, valve connections, flange connections, expansion joints, and special length pipe was not done by contract or piece work.

Punching and Bevel Shearing.—From the layout tables, plates were handled over chain-block trolleys to the two 62-in. gang punches. (See Fig. 5.) These punches had a capacity to make thirty-six $\frac{7}{16}$-in. holes through a $\frac{7}{8}$-in. plate when evenly spaced, or as many as nine $\frac{13}{16}$-in. holes through $\frac{3}{8}$-in. plate.

All the pipe designs were adapted to using male and female plates with no taper in the plate sections; double riveting was specified in the longitudinal seams and single riveting in the circular seams. Rivet spacing was designed to use the same size and spacing of rivets in both the longitudinal and circular joints so that one type of plates could be completely punched with one setting. One gang punch was always assigned to male plates and the other to female plates of the same type; and they were set so that the holes would correspond on the circular seams of the two plates. This method of punching proved to be fast and accurate, and scarcely any reaming of holes was necessary, except at the scarfed corners of the plates where the accuracy of the holes had been upset in the process of scarfing.

Four men could handle and punch about 8 plates per hour with this equipment. (See Table 3, Item 6.) Only plates $\frac{1}{4}$ in. thick, and more, were bevel-sheared for caulking. These plates went directly from the punches to the rotary bevel shears for cutting.

Scarfing.—From the punches and shears the plates were carried by trolley to the two scarfing forges. These were equipped with electrically driven blowers, and were placed so that each would heat one corner of the plate to be scarfed. After heating, the plates were drawn directly from the fires to two anvils where the scarfing was done by hand. Four men would scarf about 9 plates per hour at the rate of 0.50 peso per plate. (See Table 3, Item 7.)

Bending Ends of Plates.—Before being turned through the rolls it was necessary to bend the end of the plates to conform with the pipe radius; otherwise, the rolls would leave a flat section at each end of the plate. For doing this work, forms were constructed with the proper bend, upon which the plates were held by clamps. The ends were bent by hammering the plates cold over these forms with 12-lb. hammers. Four men could bend both ends of the plates at the rate of about 9 plates per hour. The piece-work price paid for this work was 0.50 peso per plate. (See Table 3, Item 8.)

Rolling.—The plant was equipped with a 5-ft., motor-driven, reversible bending roll for rolling plates to the pipe radius. (See Fig. 4.) One operation through these rolls would bend the lighter plates to the proper radius, the heavier plates requiring two or three operations to bend them to shape. A force of four men could handle and roll an average of 9 plates per hour at the rate of 0.50 peso per plate. (See Table 3, Item 9.)

Assembling.—From the bending rolls the plates were rolled by hand to the assembly platforms. These platforms were constructed in such a manner that

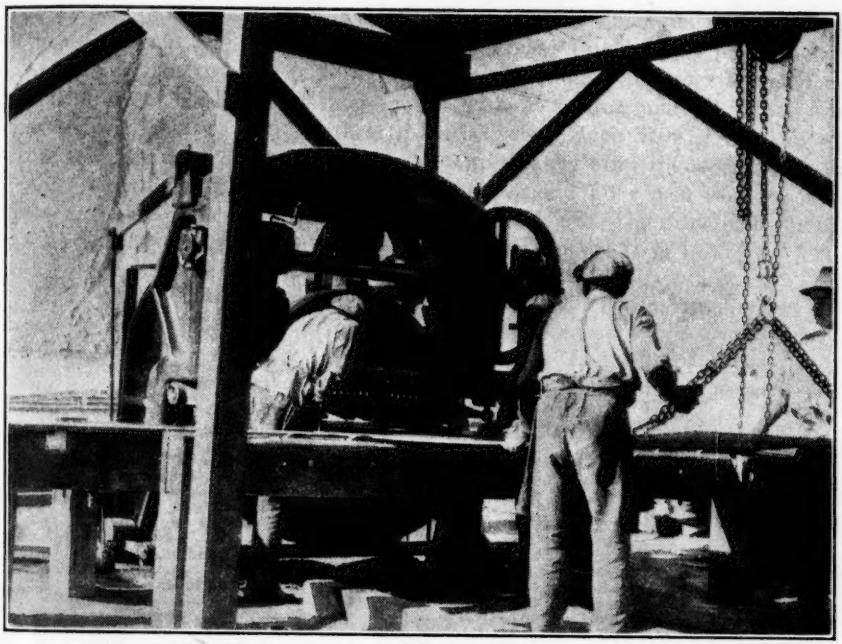

FIG. 5.—PUNCHING PIPE PLATES WITH GANG PUNCH.

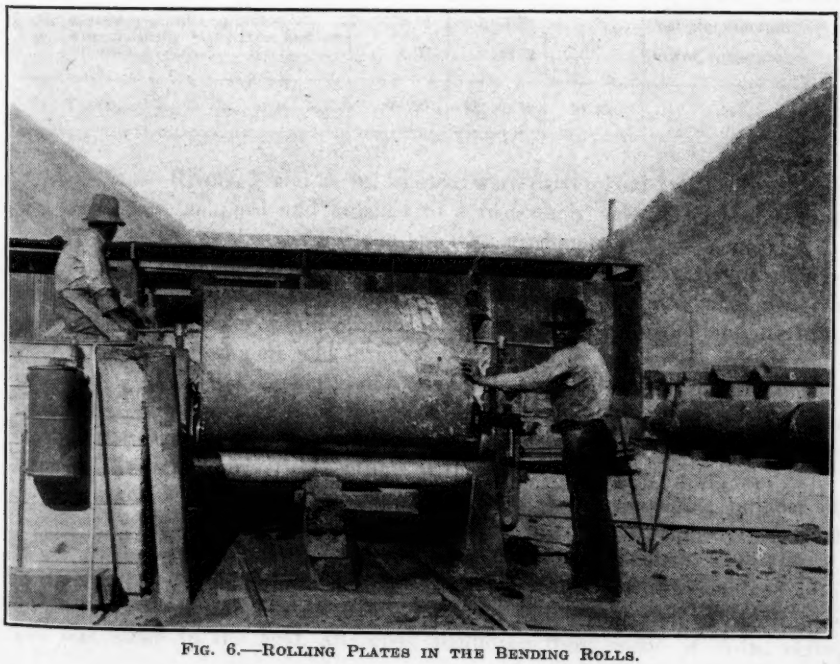

FIG. 6.—ROLLING PLATES IN THE BENDING ROLLS.

the pipe would rest on a system of disk rollers, on which the pipe could be turned without being moved from the platforms. Pipe of four plate lengths, or a laying length of 19 ft. 5 in., were assembled and bolted together on these platforms. Circular joints were made without belling and all joints were assembled, metal to metal, with no paint between. Such little reaming as was necessary at the scarf corners was done immediately after the assembly of the pipe, and this cost was included in the unit price. Four men would average the assembly of 1 pipe made up of 4 plates, per hour. The price paid for this work was 6.50 pesos per pipe of 4 plates. (See Table 3, Item 10.)

TABLE 3.—MANUFACTURE OF RIVETED STEEL PIPE IN THE FIELD.

Item No.	Steps in manufacture.	Cost, U. S. Dollars.		Man-hours per mile.	Remarks.
		Per ton.	Per mile.		
1	Flat plates (F. A. S. Chanaral)	$42.50	$11 560.00	.....	.....
2	Customs duties	6.30	1 713.60	.....	.....
3	Delivery to job	7.50	2 040.00	.....	Lighterage, dockage and railroad freight.
4	Unloading and sorting	0.18	48.96	177	.....
5	Layout work	0.33	89.76	326	.....
6	Punching and bevel shearing	0.44	119.68	544	.....
7	Scarfing	0.28	76.16	484	.....
8	Bending ends of plates	0.28	76.16	484	.....
9	Rolling	0.28	76.16	484	.....
10	Assembling	0.87	236.64	1 088	.....
11	Riveting	7.04	1 914.88	5 712	Cost includes rivets.
12	Caulking and welding	0.32	87.04	340	.....
13	Inspection	0.09	24.48	245	.....
14	Dipping	7.37	2 004.64	272	Cost includes asphalt and fuel.
15	Assembly plant equipment	6.16	1 675.52	.....	Cost and erection.
16	Miscellaneous supplies and expenses	4.26	1 158.72	.....	Power, tools, etc.
	Totals	$84.20	$22,902.40	10 156	.....

Riveting.—Rivets, $\frac{3}{8}$ and $\frac{7}{16}$ in. in size, were driven cold and by hand, and the heads were snapped and finished to a reasonably smooth button. Those more than $\frac{7}{16}$ in. in size were driven hot with air hammers.

All rivets were bucked up from the inside of the pipes by means of a bucking iron held in place by a wooden lever. The weight of the man who sat on the end of the lever would provide the proper pressure to the bucking iron.

Air for rivet hammers and forges was provided by two motor-driven compressors with a total capacity of 550 cu. ft. per min. A gasoline-driven compressor of 250 cu. ft. capacity was used as a stand-by for times when no electric power was available. Rivets were heated in a series of oil forges distributed through the riveting sections of the plant.

The riveting gangs consisted of three men, and the average number of rivets driven was 65 per hour per gang. The prices paid ranged from 7.00 to 9.00 pesos per hundred rivets, according to size. (See Table 3, Item 11.)

Caulking.—All joints between plates heavier than $\frac{3}{16}$ in. were caulked with air hammers. In the beginning of the work and before the first hydraulic test was made in the field, all pipes, including those made of $\frac{3}{16}$-in. plate,

were caulked at the scarfs. The first hydraulic test showed scarf caulking to be ineffective and many pin-leaks developed. Thereafter, all scarfs were electrically welded, which completely eliminated this defect. Caulking and welding were done on a "per day" wage basis because inspection of this class of work is difficult. (See Table 3, Item 12.)

Inspection.—General inspection of the work was made by the foremen in the plant and special inspection of rivets, caulking, punching, and assembly was made by men who also kept accounts on piece-work contracts. Test pipes were also made, with bulkheads in the ends, and these were subjected to hydraulic pressure to twice their working capacity. (See Table 3, Item 13.)

Dipping.—After they were riveted and inspected, all pipes were numbered and marked to show the type and were then placed in a storage yard. From this yard they were rolled by hand to the dipping tanks as they were required for use.

The two dipping tanks, with a capacity of four pipes each, were constructed of steel plates, surrounded with adobe. These tanks rested upon furnaces built of fire-brick; they were equipped with odometers and were heated by oil burners.

The pipe dip used was a high-grade asphalt. When this was heated, a special flux was added to an amount of about 2% of the asphalt. The temperature of the dip was maintained at from 375° to 400° Fahr. It has a flash-point of about 425° and caught fire twice, due to carelessness of the furnace operators. The first fire resulted in the loss of practically all the mastic in the tank as well as the pipe that was in the bath. Prior to the second fire a steel cover for the tanks had been constructed, and with this the fire was quickly smothered.

The pipe was dipped cold and allowed to remain in the tank about 20 min. to bring the temperature of the metal up to the temperature of the bath. This resulted in a very smooth, shiny coating, about $\frac{1}{32}$ in. thick, which had an excellent bond to the metal, was not brittle in cold weather, not too soft in hot weather, and withstood well the rough handling received in transportation. After almost four years of use this coating has now (June, 1930) resisted the abrasive and other effects of the water and is in excellent condition.

The two dipping tanks were served by a derrick which was used for placing the pipes in the tanks, for removing them, and for standing them on the drying platform. The pipes were held over the tanks until the drip had stopped and were then stood on end to cool. As soon as they reached the temperature of the air they were ready for use, because this dip required no further drying. The labor of dipping was paid for at the rate of 2.00 pesos per pipe. (See Table 3, Item 14.)

Special Pipe.—All special pipes were laid out and fabricated to fit the particular point for which they were made. Curves were limited to a maximum of a 5° angle for a single joint between 5-ft. plates; hence a length of pipe, of four plates, was enough for any angle up to 15 degrees. Angles of less than 5° were made in one joint. Angles between 5° and 15° were made in three equal joints, and those of more than 15° were extended to as many joints as were necessary to cover the angle. Field surveys and excavations followed

these rules, so that when the Special Plant received an order for a curve of a certain degree angle, no further information was necessary to show how that pipe should be laid out to fit the ground.

Special lengths of pipe were required for making closures between sections of pipe-laying and for fitting curves of high degree to the ground. These had to be specified, together with the type of ends, whether male or female, and had to be turned out at the plant in a short time to prevent delays in the pipe-laying. Holes for air and drain valves were cut out with acetylene torches and all saddles were caulked after riveting. Expansion joints were manufactured in the United States and shipped to Chile with blind companion flanges. These flanges had to be punched to fit the type of pipe to which they joined, and were riveted at the plant, to one plate length of pipe.

The fabrication of "specials" was carried on in a manner similar to that of straight pipe except that the special department was equipped with a double-end punch and shear for cutting and punching curved joints, and with acetylene welders and torches for cutting and welding. All "specials" were tagged as they passed through the dipping tanks, and as soon as they cooled from the dip they were lettered with white paint with their line station numbers.

TABLE 4.—SUMMARY OF PIPE LINE COSTS.

	Item.	UNIT COSTS, IN DOLLARS (8.25 PESOS).			
		Materials and supplies.	Labor.	Expenses.	Total.
	Cost of Pipe, Including Specials:				
1	Assembly per foot	$ 2.88	$ 0.85	$ 0.27	$ 4.00
2	Dipping per foot	0.24	0.09	0.00	0.33
3	Total, Items 1 and 2	$ 3.12	$ 0.94	$ 0.27	$ 4.33
4	Total, Item 3, in dollars per ton	$60.60	$18.40	$ 5.20	$84.20
	Cost of Field Transportation:				
5	Per ton-mile	$ 0.37	$ 0.32	$ 0.08	$ 0.77
	Cost of Excavation and Back-Filling:				
6	Total per linear foot	$ 0.08	$ 0.61	$ 0.02	$ 0.71
7	Total per cubic yard	0.17	1.28	0.04	1.49
	Cost of Tunnel Driving (1 942 ft.):				
8	Total per linear foot	$13.72	$12.34	$ 0.54	$26.60
9	Total per cubic yard (3 100 cu. yd.):	8.59	7.73	0.34	16.66
	Cost of Tunnel Lining:				
10	Total per linear foot (7 196 ft.)	$ 2.87	$ 3.88	$ 0.38	$ 7.13
11	Total per cubic yard (5 300 cu. yd.)	3.83	5.27	0.52	9.68
	Cost of Pipe Laying:				
12	Total cost per foot	$ 0.12	$ 0.84	$ 0.09	$ 1.05

Approximately 8% of the total pipe fabricated was in "specials." In Table 4, showing a summary of pipe-line costs, "Expenses" include electric power, railway freights, railway passages, and various other small items.

Rate of Progress.—Activities began at the "Sifon Assembly Plant" in March, 1925, but such work as was possible at that time was confined to the construction of camp buildings, and the laying out and grading of the assembly plant and yard. At first, the shortage of labor and material retarded progress.

It was not until July, 1925, that the first of the equipment was received, and it was nearly the end of August before sufficient equipment had been received and erected to start assembly work on the pipe.

The total period of manufacture extended from August, 1925, to August, 1926. In that time 131 200 ft. of riveted pipe were fabricated, or an average of 360 ft. per day. This rate required the use of 75 plates and the driving of about 8 100 rivets per day.

The maximum rate of fabrication was reached in April, 1926, when the monthly average was 640 ft. per day, which required the daily use of 130 plates and the driving of 15 000 rivets.

Transportation

The largest single item of labor cost for the construction of the Ola Pipe Line was that of transportation, and except for the cost of pipe material, it was the largest item of the entire work.

Those who are unfamiliar with South American and Chilean conditions can hardly realize the tremendous undertaking required to transport the 22 040 tons of materials and supplies that were used.

The Assembly Plant was the freight-receiving station for all materials and supplies for the entire job. At this point the general store house, and the yards with sufficient capacity for handling all materials, were located. It was 28 miles, by pipe line, from the intake at La Ola Dam, and 4.3 miles from the outlet of the pipe at Potrerillos. The distances by rail were 83.9 miles from the Port of Barquito and 12.4 miles from Potrerillos. As previously stated, the plant was built at the bottom of a deep canyon and at the point of lowest elevation on the entire pipe line, which meant an uphill haul from there to all parts of the line. The location chosen, however, was the best possible because railway facilities were necessary for incoming materials, and this location was at the only intersection of the pipe line and the railway.

Existing roads, which in reality were little more than trails, extended from the Assembly Plant down Pasto Cerrado Canyon to Jardin, a distance of 4.3 miles, and from there zigzag roads had been constructed up the canyon sides, one leading south to Potrerillos and the other going up the opposite canyon wall, and thence to Pedernales and La Ola. From the Assembly Plant to the rim of the canyon the vertical distance is 1 700 ft., and the haul by road is 12.4 miles, of which 3.7 miles is a switch-back road on a 10% grade. This haul is a full day's work for a mule cart. The construction of the inverted siphon across Pasto Cerrado Canyon required the construction of inclined tracks parallelling both legs. It was decided, therefore, to build the north leg track with the purpose of using it for hoisting all pipe and supplies to the top of the canyon wall.

This inclined railway was built with a vertical lift of 1 600 ft. in a horizontal distance of 2 950 ft. The average slope was 27° and the maximum slope, 46 degrees. The roadbed was cut through limestone and conglomerate throughout the entire distance. The lower end of the track began at a sloping trestle about 328 ft. long which made an overhead crossing over the railway and landed at the top of an almost vertical cliff of limestone. The track had a gauge of 39.37 in., which corresponded with the gauge of the Potrerillos Railway. The rails were laid on ties in the usual manner, but were anchored at each rail-joint to an eye-bolt grouted into the rock of the roadbed. On the trestle the ties were drift-bolted to the stringers.

This incline was equipped with a 10-ton electric hoist having a drum, 5 ft. in diameter, which carried 3 800 ft. of 1-in. wire rope. Flanged cast-iron rollers, 2 ft. long and mounted in bronze bearings, were placed at all points where the loaded cable would drag.

The car used on this incline was 20 ft. long and had a capacity of 12 tons. Its traction pull was from the center through a bridle that could be raised and lowered at the cable end to conform with the irregular grade of the track and to prevent lifting the front wheels of the car from the rails.

The hoist had a rope speed of 200 ft. per min. and the average trip, up or down, consumed about 20 min. With loading and unloading, a round trip of the car required about 1 hour. For much of the time this car ran night and day and handled as much as 2 314 tons in 1 month.

All the materials for the construction of the La Ola Dam, 27.9 miles of pipe line, the Montandon Power Station, tunnel lining, the stock feed, and camp supplies for all the work at the dam end of the line, passed over this incline.

Sifon Alto Camp was at the head of this incline railway on a sloping mesa or ridge about 0.62 mile wide which drops away to the north into another canyon of about the same depth as Pasto Cerrado. At this camp there was ample room for a large storage yard for pipe, corrals for mules, shops for blacksmiths, cart repairmen, and harness makers, and for the necessary camp buildings for housing the men.

Sifon Alto was headquarters for all the animals used on the job and for all cartmen. All cart and harness repairs were made here and all feed for the animals passed through this camp. The maximum number of animals in service was more than 600. All water for animals and men had to be hoisted to this camp from the Assembly Plant which was supplied with water from a pipe line constructed to serve the railway.

From Sifon Alto the cart road followed a ridge for about 4½ miles at about a 4% grade; then it meandered and zigzagged along side hills at about a 10% grade for a distance of 7½ miles to the top of Piñon Pass which has an elevation of about 13 000 ft. From Piñon Pass the road first drops and then rises a second time to Llanta Pass in a distance of about 2½ miles, and then drops abruptly to an elevation of about 11 250 ft. at Pedernales Camp, which is at the bottom of a deep valley between two mountain ridges.

Pedernales and Llanta Tunnels were driven through these two ridges. They were each more than 1 mile in length and were the destination of much trans-

ported material. The camp consisted of a staff house, bunk houses, store, warehouse, mess house, first aid station, corral, and blacksmith shop. Water in sufficient quantities to supply the corrals and camp, was available from shallow wells. A slaughter house was also located at this camp where all meat for the upper end of the line was butchered. Beef and mutton provided their own transportation to this point.

From Pedernales Camp the road followed down the course of Pedernales Canyon for about 3¾ miles, on about a 2% grade, and from there it led to the open La Ola Pampa, which is a flat valley 6 to 12 miles wide, with a sand and gravel floor. This pampa rises at about a 1% grade toward La Ola Dam and the intake of the pipe line. About 15.5 miles of this pampa had to be crossed to reach the dam and no attempt was made to maintain roads because the character of the country was such that carts could break new tracks as fast as the old ones became impassable. The pipe line was located along the western edge of this pampa and gradually reached the bounding ridge of mountains as the grade dropped away. The cart roads, therefore, followed the pampa as far as practicable, and then turned in to accessible points on the pipe line, with no attempt to follow any established trails.

The Chilean mule cart, which was used exclusively on the work of transportation of pipe and other materials, is at first glance a very inefficient piece of equipment. However, it has been developed through generations of use, and proved in this instance to be well adapted to the bad roads, steep grades, deep ruts, and sharp turns over which material had to be hauled. It is a heavy vehicle, with two wheels about 6 ft. in diameter, and a gauge of 5 ft. (See Fig. 7.) The tires are 5 in. wide and 1 in. thick. The hubs are of cast steel, and the axles are made of 3-in square metal. The car is so constructed that the load is balanced equally, fore and aft, on the axle. At the front and rear are two swinging props on which the cart is balanced while being loaded or when the team is resting on the road. A brake-beam with large wooden brake shoes operates against the rear of the wheels by means of a screw stem and large wing nut called a "mariposa," or butterfly. To operate this brake the driver has to run behind the car and turn up the screw. The brake is powerful and can be set at any tension desired.

When in motion the cart and load are balanced by one mule that is hitched between rigid shafts. This mule's harness consists of a heavy halter, a collar with chain traces hooked to the front of the shafts, and a padded saddle with a heavy cinch to which the shaft ends are bound in a manner to balance the cart by the weight of the mule. The mule also wears a heavy breeching which is attached forward to the shafts so that he is hitched firmly forward, back, up, and down. He acts as a flyball for the entire unit and is either carrying a heavy load on his back or is scratching for a toehold. The drivers are remarkably clever in balancing their loads, however, and the outfit performs better than might be expected. The mules are hitched three abreast to these carts, one on each side of the shaft mule.

The harness of each of the other mules in the team consists of a collar and chain traces which are held up by a padded saddle. The traces of the

FIG. 7.—THE NOON STOP AT REFRESCO.

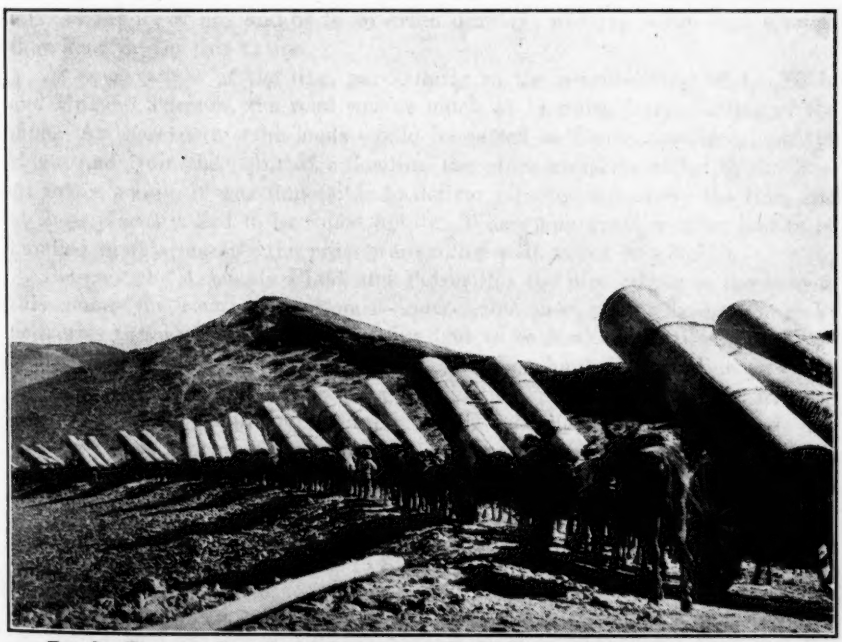

FIG. 8.—TRAIN OF PIPE CARTS RESTING ON PINON PASS. ELEVATION, 13 000 FEET (3 962 METERS), AND THE HIGHEST POINT ON THE ROAD.

wheel mules are hitched to single-trees and no eveners are used. The traces of the lead mules are hitched to the traces of the mule behind at the saddle. No bits are used, and no lines, except a jerk-line to the lead mules' halters which is pulled to stop the team or when making a turn. The driver rides the left-hand wheel mule on a high saddle and drives his team with a whip. This hitch is ingenious and very flexible, providing for quick changes and additions to a team, and can be strung out to any number of animals. On this work as many as twenty-four mules were used on one hitch.

A driver will ordinarily handle six or seven animals and the best drivers, nine. In transporting pipe, with a load of about 5 500 lb., in addition to the weight of the cart, one driver and one helper were used for a 9-mule team. On this work it was necessary to have one man to handle the brake while the other drove the team.

Pipe carts were constructed with a rack in front, wide enough to carry three 36-in. pipes side by side. This rack raised the front end of the pipes and held them in a slanting position with the rear end high enough to clear the irregularities of the road. The pipes were bound to the carts with chains tightened with turnbuckles and were held from slipping back by chains hooked into the ends. The pipes, being about 20 ft. long, extended over the wheel mules and driver and produced such a teetering motion that an extra shaft was added to the right side of these carts so that two mules could be used in the shafts to balance the load.

Each cart carried its ration of hay and feed for the entire trip which, on the longest end of the haul, took four days to complete. The mules were allowed 15½ lb. of hay and 6½ lb. of grain per day; working seven days a week, they kept fat on this ration.

At some points of the line, particularly in the neighborhood of La Tabla and Huacho Tunnels, the road was as much as 1¼ miles from the line of the pipe. At these points the loads would be carted as far as possible along the ridges and from the point of unloading the pipes would be rolled to the line. At many points, it was impossible to deliver pipe by cart above the line, and at these places it had to be rolled uphill. Where any great number had to be handled in this manner the pipes were rolled with ropes and blocks.

Between the Assembly Plant and Potrerillos the pipe traverses the canyon side across the roughest and most inaccessible part of the line. To make deliveries throughout this section, pipe had to be loaded on railway cars and transported part of the distance by rail, and then hauled to the line on carts. In order to do this, 4⅓ miles of new, side-hill road had to be constructed through a steep and difficult country. This road had one main entry to the railway where the greater part of the pipe was taken from the cars, hoisted to the top of a ridge about 100 ft. high, and there loaded on the carts with a steam derrick. From this entry the road followed the top of a long ridge at a grade of about 10%, to a point above the pipe line where it branched in both directions, going at a level grade to Sifon Sur at one end and to San Augustine Canyon at the other.

San Augustine Canyon presented the most difficult problem for the delivery of pipe of any point on the line. A double siphon crossed this Canyon and the

ridge between the two outside legs was inaccessible to any form of transportation except pack animals. It would have been necessary to construct about 3 miles of difficult road to reach the central ridge. The problem of delivering pipe to this section was solved by the use of a low two-wheeled cart. The pipe was balanced on the cart and was pulled to the top of the ridge with a hand winch. It required the work of 2 men for 16 hours to hoist 1 pipe to the top. The vertical lift was 325 ft. in a horizontal distance of about 500 ft. About 1 000 ft. of pipe were transported in this manner and it took about 30 days to make the delivery.

Trains of pipe and supply carts (see Fig. 8) were started each morning from Sifon Alto under the supervision of a leader who had complete charge of the train en route. These leaders were men who had been advanced from drivers, were thoroughly competent and reliable, proud of their responsibility, and who would take their trains through the trip and without delay under almost any circumstances. Each day's trip was laid out for the drivers, and if they completed that distance they were paid a bonus of 25% more than their regular pay. Regardless of the reason, the driver, unless he completed his trip, received no bonus. This made them care for their carts, harness, and mules, and assume entire responsibility for their outfits. The result of this bonus system was that very few drivers would fail to finish their trips.

The average day's haul, up grade and with a load, was about 15½ miles. Returning, down grade and with no load, the average was about 25 miles per day.

The haul to La Ola Dam and the upper end of the line required four days for the round trip. From La Cabra Tunnel the time was cut to three days and from Perro Muerto Tunnel, to two days. Train leaders would start from Sifon Alto at 7:00 A. M. with as many as twenty-four carts and would arrive at Pedernales at about 5:00 P. M.

Pipe carts were unloaded in the field by throwing off the binding chains and sliding the pipe to the rear until the ends rested on the ground. This would throw them out of balance so that the cart could be driven out from under them. Table 4, Item 5, includes the construction of 4.34 miles of road, the maintenance of 19.88 miles, the purchase of carts and harness, all maintenance of equipment, the loading, unloading, and operation of the incline car, and all regular transportation expenses.

Trenching and Back-Filling

In the high altitudes where the Ola Pipe Line was constructed an extreme daily variation of temperature exists. To prevent freezing and to reduce expansion and contraction it was decided that the pipe would be placed in a trench and covered throughout its entire length, except across the Montandon Siphon, where it would be supported on piers and anchors.

On the Ola Pampa, where the line is exposed to extreme weather and violent winds, it was planned to cut a trench of sufficient depth to place the pipe entirely below the surface of the ground. In less exposed locations and at lower altitudes the depth of the trench was planned to be equal to about two-

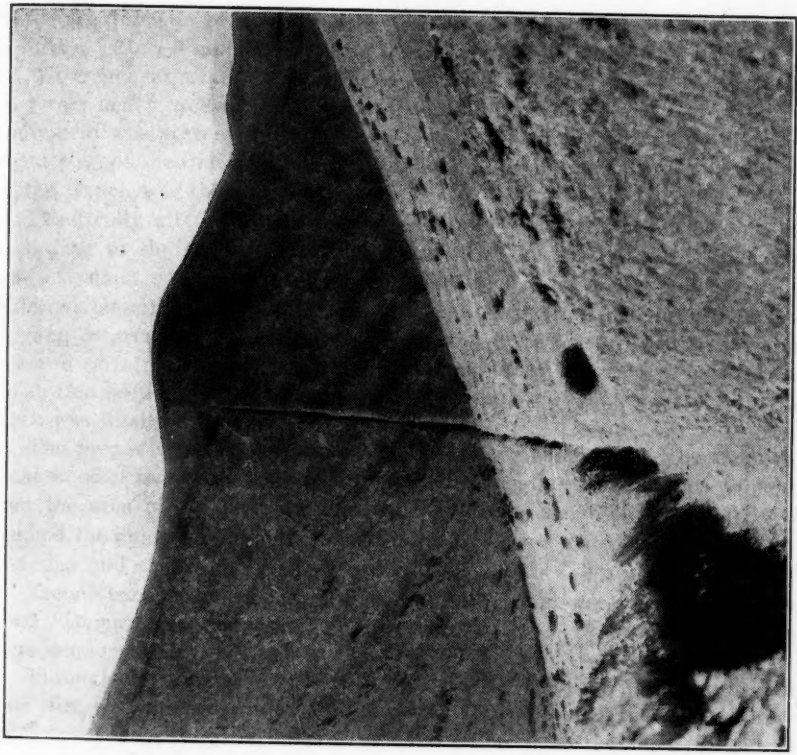

FIG. 10.—PIPE TRENCH BETWEEN PINONCITO AND LA TABLA TUNNELS.

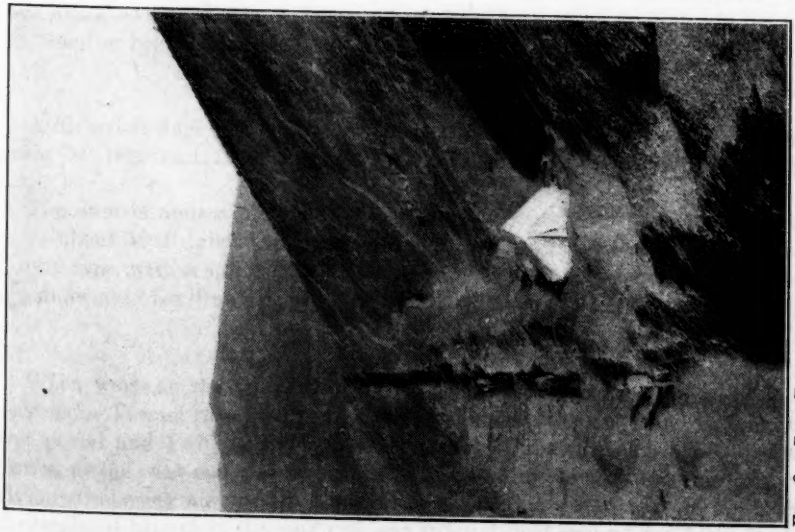

FIG. 9.—PIPE TRENCH EAST FROM PEDERNALES TUNNEL.

thirds the diameter of the pipe, and the excavated material was to be used for covering. Figs. 9 and 10 illustrate conditions along parts of the line.

All trench excavations were made by hand. The material was such that only very heavy excavating equipment would have sufficed, and the use of such equipment was prevented by the roughness of the country. The ground along about 50% of the line required blasting before it could be excavated, and much of the trench was through solid rock.

Practically all the excavation was done by contract, the men being paid according to the classification and quantity of material moved. The earth was classified as earth, hardpan, or solid rock, and measurements of the different classifications were made after the excavation had been completed. A gang or group of men would be given a contract for excavating the trench between certain stations on the line. They would be required to excavate and finish that section, and would receive no liquidation of their contract until the work was finally finished.

The proper width and shape of the trench was maintained by the use of light wooden templates which were marked along the sides in such a manner that the area of the cross-section could be read from them direct. By this method the quantities and classification of excavated materials could easily be recorded and checked.

Trench excavation was started in July, 1925, and was finished in August, 1926. During the greater part of this time two gangs of from 60 to 100 men, were employed.

Through the greater part of the work the back-filling was done by hand. The dirt was tamped under the invert of the pipe with shovels to prevent settling. Where the pipe followed side-hills, the fill on the down-hill side was maintained with dry rubble walls. On the pampa where the pipe was laid below the level of the ground, the top-fill was placed with teams and scrapers. The costs are shown in Table 4, Items 6 and 7. These items include the cost of constructing rubble walls.

La Ola Dam

A diversion dam was constructed in the La Ola River during the summer season of 1925 and 1926, between the months of October and April. (See Fig. 1.)

The dam is approximately 160 ft. long between abutments and raises the water about 10 ft. above its natural level. It is a concrete structure of the gravity type, with a spillway 65 ft. long, an outlet chamber and head-gate, and one sluice-gate for drawing off sediment from the reservoir.

Tunneling

When work on the Ola Pipe Line was begun in March, 1925, 1 942 ft. of Pedernales Tunnel remained to be driven. This section was 1 190 ft. from the west portal and 1 670 ft. from the east portal of the tunnel. The completed portion at the west end was through a hard blue limestone which stood up well without timbering and required no lining to make it water-tight. The section of completed tunnel at the east end was through bad ground for 1 640 ft., but

ended in hard granite. The timbering in the first 1 640 ft. was in bad condition, which necessitated the lining of that section to make it safe. In order that the driving of the remainder of this tunnel should not conflict with the lining work, it was decided to complete all the driving from the west face.

The construction program called for the completion of the tunnel driving by January 1, 1926. In order to do this it was necessary to drive 220 ft. per month, with an average haul of more than 2 000 ft. for the material broken.

The section of tunnel was 6.56 ft. square, which allowed room for the operation of only one drill at the face. Two portable gasoline-driven air-compressors, one of 250 cu. ft. and one of 175 cu. ft. capacity, were installed at the west portal to provide air for drilling and for running a drill sharpener and forge. The efficiency of these compressors was only about 60%, because of the high altitude.

Ventilation was provided by a gasoline-driven blower, and fresh air was conducted to the face through canvas vent tubes; these were extended as the driving progressed. Broken rock was hauled by mules in 1-yd., side-dump mine cars.

Three 8-hour shifts were worked at the face. Holes were drilled to a depth of 5 ft. It required two shifts to drill a round of holes, to blast them, and to remove sufficient material to permit further drilling.

Chilean labor, on a contract basis, was used exclusively. The Company furnished the equipment, tools, air, timber, and powder, and paid 150.00 pesos per meter of tunnel completed ($5.53 per ft.), including all labor for drilling, drill sharpening, shoveling, and timbering.

Work was started March 1, 1925, by hand drilling. It was not until May 1 that the compressors were installed and ready for operation. During September, 1925, a fault plane was intersected which opened up quite a heavy flow of water. The ground was badly shattered and the tunnel opening was difficult to maintain. For about 820 ft. the tunnel had to be heavily timbered and, later, lined with concrete. This condition slowed up progress, but the tunnel was finished on December 29, 1925, two days ahead of schedule, and with a perfect closure.

The greatest progress made in any one month was during August when 376 ft. were driven. The total cost is shown in Table 4, Items 7 and 8.

Tunnel Lining

The final location of the Ola Pipe Line with the necessary adjustments of grade to the completed tunnels, threw Pedernales and Llanta Tunnels below the hydraulic grade of the line, and, consequently, they are under a light hydraulic pressure.

Practically the entire length of Llanta Tunnel and 2 890 ft. of Pedernales Tunnel were in bad ground, composed of a shattered limestone through which were sloping layers of soapstone. Some water seeped into these tunnels; the timbering was old, and several falls of rock had occurred in Llanta Tunnel which had blocked the entries and flooded it for almost its entire length.

As a concrete lining seemed better adapted to inspection and maintenance than the process of running the pipe through these tunnels, it was decided to

line the bad sections. On account of the small size of the tunnels it was necessary to use concrete forms that would not obstruct the passage in any manner to prevent the placing of concrete, or the removal of forms from the completed sections. To accomplish this, forms were developed that needed no center bracing, and that left the tunnel bore entirely open between the form ribs. This was done by making the ribs in three sections, consisting of an invert section and two side sections.

A concrete floor was first laid to grade, and upon this, while the concrete was still green, the invert ribs were placed, held in position by bracing against the roof, and then lagged. Near each end of these invert ribs were holes which held a ¾-in bolt. The bolts were fitted with nuts and O. G. washers which were embedded in the invert concrete when the forms were filled. Upon removal of the invert forms the bolts would be tapped lightly and then unscrewed, leaving holes with the nuts and washers embedded in the concrete. After the invert concrete was set and the invert forms were removed, the side forms were erected and held in place at the bottom by screwing back the bolts through corresponding holes in the upper ribs. The two side ribs were held in position at the top by a wedge, which when dropped would loosen the ribs for removal. The lagging for the upper ribs was built up as the sides were filled with concrete. The top lagging was put on in 5-ft. sections, and the concrete was shoved and spaded back from the end until the top was thoroughly filled.

The concrete in these tunnels was reinforced with a ⅝-in., round, reinforcement placed on 12-in. centers both ways and short ⅝-in., round, bars were used for bonding at the construction joint between the invert and the sides.

Sand and gravel were available at the portals of nearly all these tunnels, but had to be separated and screened by hand. Water for concrete was available from the inside except for about one-half the east end of Pedernales Tunnel. For this work the water had to be hauled about 2 miles up a 15% grade. The cement was carted from Sifon Alto to Llanta Tunnel and to Pedernales Camp, and packed from there on burros to both ends of Pedernales Tunnel.

All the concrete for the latter was mixed by hand inside the tunnel and shoveled from the mixing platforms directly into the forms. The concrete for Llanta Tunnel was mixed in a steam mixer at the east portal, and hauled by mules in end-dump cars to the forms.

All the work was done on a contract basis and was paid for at the rate of 80.00 pesos per lin. m. ($3.00 per ft.) This price included the removing of timber, cleaning up the tunnels, screening sand and gravel, placing and removing forms, cutting and placing reinforcement, and mixing and placing concrete.

The progress of the work was more rapid in Llanta Tunnel where the concrete was mixed outside. In Pedernales Tunnel a delay was caused by bad air from the setting concrete. This was before the tunnel had been holed through, and there was no ventilation until a canvas vent-tube and a blower were installed.

The 1 640 ft. of tunnel lining in Pedernales, East, was started in May, 1925, and completed in August. During July, 853 ft. were completed, which was the best monthly average on that work.

The Llanta Tunnel lining was started in August and completed in November. The length of tunnel lined was 4 308 ft., and the best run was made in November, during which 1 801 ft. were finished.

Pedernales, West, was lined in January and February, 1926, the best run being made in February when 1 014 ft. were completed by the 20th of that month.

The moist cool air of these tunnels retarded the setting of the concrete to some extent; but it contributed in producing a splendid lining.

Due to the cheap labor and to the fact that salvaged steel transmission-line cable was used for reinforcement, for which no charge is included, the costs of this tunnel lining are very low. These costs include all labor and material, but do not include transportation. (See Table 4, Items 10 and 11.)

Pipe Laying

The laying and jointing of the pipe in the field were the final steps in the construction of La Ola Pipe Line, and required the co-ordination of all the work up to this stage of the construction. This co-ordination included preparation of the ground, the receipt of materials and supplies (including expansion joints, saddles, air and drain valves, and those sections of lap-welded pipe which were purchased in the United States), the proper order of assembly, and the transportation of material from the Assembly Plant.

The pipe-laying work extended over so much territory that, even when material was ready for shipment from the Assembly Plant, it often would take two days, or more, to make deliveries to the points where the materials were needed. This part of the work included the laying of eighteen different classes of riveted pipe, fabricated at the Assembly Plant, and seventeen different types of lap-welded pipe, purchased in the United States. Much of the latter was pipe that had to be joined in its proper order because each joint had been manufactured to fit its special location. The delivery of this pipe from the United States was erratic, and a delay in the receipt of a few sections held up the work of laying for about two months. This co-ordination proved to be the most difficult problem with which the field organization had to deal.

Pipe-laying started at the intake end of the line with 36-in. riveted pipe. The first part of the work was across the flat pampa where there were no horizontal and very few vertical curves. The system of laying adopted was to rivet together several 20-ft. sections of pipe outside the trench, and to join such a section to the last that had been laid. As shown in Fig. 11, the end of the finished section and the section being joined to it were held up in the trench with tripods and chain-blocks until the joint had been riveted, and then were lowered into place with the chain-blocks. It was found that a long string of this pipe had sufficient flexibility to permit the use of this method without putting undue strain on the rivets. After riveting and caulking, all field joints were painted inside and outside with hot mastic.

FIG. 11.—LAYING PIPE ON LA OLA PAMPA; SHOWING METHOD OF SUPPORTING PIPE WHILE MAKING FIELD JOINTS.

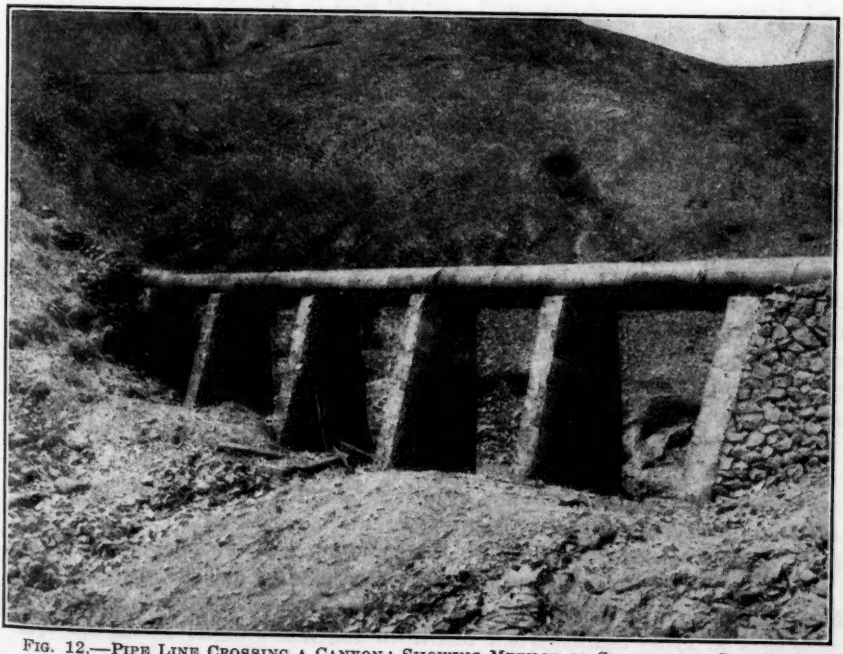

FIG. 12.—PIPE LINE CROSSING A CANYON; SHOWING METHOD OF SUPPORT FOR DRAINAGE.

In the beginning of this work more than 12½ miles of pipe were laid before the expansion joints and valves were received. Consequently, the laying was done in sections leaving openings at points where the expansion joints were to be placed. Saddles for air and drain valves had been received in time to be connected to the riveted pipe at the time of fabrication, and the valves were bolted in place after the pipe was laid. Inability to install expansion joints and valves, at the time the first pipe was laid, delayed the hydraulic tests on the line until the construction had reached Pedernales Tunnel, a distance of 14¼ miles from the intake.

The first test included the section between La Ola Dam and La Cabra Tunnel, a distance of about 5 miles. This disclosed no breaks in the line and no faulty pipe, but showed a large number of pin-leaks, particularly at the points where the plates had been scarfed and lapped. After this test all the pipe assembled at the plant was electrically welded at the scarfs and much greater care was used in caulking all field joints.

In the rougher country it became necessary to decrease the number of lengths of pipe connected together outside the trench, and it was found that, in very rough sections and on steep grades, better progress could be maintained by handling only one length of pipe at a time.

When expansion joints were finally installed it was found that the openings, which had been left between pipe sections, varied considerably in length, due to expansion and contraction. In practically all cases of installing expansion joints special lengths of pipe had to be provided to produce a proper fit. It was by this method, and at expansion joints, that all closures were made between different sections of pipe laying. Four gangs were necessary to keep this part of the work up to schedule.

Two types of lap-welded pipe were used: (*a*) Bell and spigot pipe with leaded joints was used in sections of lower pressure; and (*b*) Van Stone flange-joint pipe was used at points of high pressure on the line—in general, for pressures greater than 400 lb. per sq. in. Angles in this latter pipe were made either with curved sections for the higher degree or with wedge-shaped "Dutchmen" for the low-degree angles. As much of this pipe was laid in the Montandon Siphon, where the slopes exceeded 45°, the fitting of the joints required much care and accuracy.

Each second joint was supported on a concrete pier (see Fig. 12), and anchors were spaced about 164 ft. apart, with an expansion joint near each anchor. Reinforced concrete anchors were used where solid-rock footings were available. The reinforcement was extended into the rock foundations by being grouted into drill holes, and concrete anchors of the gravity type were used where solid-rock footings were not available.

A special cross-casting, equipped with high-pressure drain-valves, was installed at the invert of Montandon Siphon where the working pressures were 1 000 lb. per sq. in. This drain is equipped with two high-pressure 10-in. valves in tandem, which are interconnected by a by-pass pressure relief. Two men can operate them by hand under the full pressure head.

When hydraulic tests were put on the lap-welded pipe sections it was found that the leaded joints would not withstand the combined effect of expansion

and contraction of the pipe, and the high working pressures. The first test caused a joint on the south leg of Montandon Siphon to open and four lengths of pipe blew out. Two of these went to the bottom of the siphon 1 500 ft. below the break. This was followed by several breaks on the north leg of the siphon.

These leaded joints were designed with only a small bead of lead in the spigot end to prevent movement. An analysis of the trouble disclosed the fact that the movement of the pipe from expansion and contraction had sheared this bead of lead, and subsequent movements had worked it loose in the joint. The high pressures then blew it from the joint. This analysis showed that any process that might be used to hold the lead in place would remedy the difficulty. It was found that the quickest and simplest method was to construct reinforced concrete collars. Three rings of $\frac{3}{8}$-in. reinforcement were made, one to lie in front of the lead, one behind the bell of the joint, and the third around the bell. These three rings were laced together with heavy wires of sufficient cross-section to withstand a stress equal to that exerted on the lead joint. This reinforcement was held in place by pouring a concrete collar around it. About 1 100 of these collars were made to protect leaded joints. To date (June, 1930), none of these joints has broken and only one has shown necessity for repair.

Only four of the Van Stone joints in the entire line showed leaks, and these required only a slight taking up on the bolts to make them tight. No leaks have developed at these joints to date (June, 1930).

Pipe laying was started on November 22, 1925, and the last pipe was laid November 21, 1926. The average rate of laying was 3 280 ft. per week; the maximum laid in any one month was 6.52 miles, or 1 150 ft. per day; and the maximum laid in one day was 1 312 ft. The cost items are shown in Table 4, Item 12.

Montandon Power Station

The original construction plans and estimates made for the Ola Pipe Line did not contemplate the construction of the Montandon Power Station, although it had been considered. It was only after the arrival of the construction forces in Chile, and the financial possibilities of the plant being presented by them, that the construction of this plant was authorized.

The idea of locating a hydro-electric plant in the middle of a desert is at first thought rather fantastic, and would not be practical unless the water was brought to it free of cost. In this case 1 250 ft. of head were available in the line, in excess of the required pressure head necessary to deliver the water to Potrerillos. This excess head and hydraulic drop occurs between the points where the line is tunneled through the mountain range which intervenes between La Ola Pampa and Potrerillos, and the head of Montandon Siphon; that is, in a distance of 4.40 miles beginning at Piñoncito Tunnel and ending at a point 525 ft. in elevation above the Potrerillos outlet of the line.

Piñoncito Tunnel was used as a forebay for the penstock and was provided with a spillway to care for any overflow of the line which might occur during

a shut-down of the plant. The penstock between Piñoncito Tunnel and the Montandon Power Plant consists of 1 mile of 36-in. riveted pipe, 2.4 miles of 26-in. riveted pipe, 0.6 mile of 26-in. Matheson joint lap-welded pipe, and 0.3 mile of 26-in. Van Stone joint lap-welded pipe. The latter is used on the lower end of the line. This penstock contains a total of twenty inverted siphons, the greatest of which is under a pressure head of 430 ft.

As the primary purpose of the pipe line was to furnish water to Potrerillos, the power station was a secondary consideration and therefore was designed to by-pass water when the plant was shut down. Provision for this feature was made by governing the plant with a deflector which cuts the jet at the nozzle, allowing the water to pass, but deflecting it from the wheel. The plant is also provided with a by-pass for diverting the total stream away from the wheel. The needle-valves of both wheel and by-pass nozzles are hand-operated and interconnected in a manner to close one valve while the other one opens.

The hydraulic equipment in the plant consists of a single-nozzle, overhung, impulse wheel, 53 in. in diameter, propelled by a 4¼-in. jet. This wheel is designed to operate under 1 132 ft. of head, at a speed of 514 rev. per min. It is mounted on the end of the generator shaft which runs on 9 by 15-in. bearings. The exciter unit is direct-connected to the other end of the generator shaft through a flexible coupling.

The tail-race and by-pass flumes discharge into a steel tank 25 ft. in diameter in which baffles have been constructed to check the velocity of the water. This tank is provided with a spillway and an outlet gate that discharges into the pipe line to Potrerillos. When closed, this gate diverts the flow over the spillway into a canyon which takes the water entirely away from the pipe line. The capacities of this plant for corresponding quantities of flow, are as follows:

Quantity of flow, in cubic feet per second.	Capacity, in kilowatts.	Quantity of flow, in cubic feet per second.	Capacity, in kilowatts.
21	1 575	26	1 950
22	1 650	27	2 025
23	1 725	28	2 100
24	1 800	29	2 175
25	1 875	30	2 250

Excavation for the foundations of this plant was started on March 15, 1926, and was completed by the end of that month. During April, all the substructure concrete was poured up to the level of the operating floor. During May, the work was delayed for lack of the superstructure steel and plant equipment, but during that month a permanent building for housing operators was constructed. By the end of June all the building steel had been received and erected, and the equipment had been transported to the site. On July 27, 1926, the plant was completed, and has been in continual operation since that date, except for minor interruptions.

The only difficult part of this work was the transportation of the plant equipment from the assembly plant yard to the power plant site. The heaviest piece to be transported was the generator shaft and rotor, which

weighed more than 10 tons and was of a very difficult shape for handling because the rotor was mounted on the shaft. The weight of the piece taxed the capacity of the incline railway and was of such shape that the crate had to be bridled ahead to the haulage cable of the car, in order to prevent it from tipping over backward. From the top of the incline the transformers and other heavy pieces were carried in a four-wheeled wagon drawn by eighteen mules.

Conclusion

The difficulties attending this construction make a story in themselves. The organization of the necessary supervising staff, the selection and training of a large number of men in all departments of the work, the effect of climatic conditions at high altitudes, the difficulties in getting together and organizing the transportation facilities, and last, but not least, the delay in the receipt of necessary material were the disturbing factors encountered on this work. To appreciate the task, it is necessary to see the country traversed. The last pipe was in place and all work completed on November 21, 1926. To date (June, 1930), the La Ola Pipe Line has demonstrated its fitness to serve the purpose for which it was built.

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PAPERS AND DISCUSSIONS

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TESTS OF BROAD-CRESTED WEIRS

By James G. Woodburn,* Assoc. M. Am. Soc. C. E.

Synopsis

This paper gives the results of 305 tests made during 1928-29 at the University of Michigan, Ann Arbor, Mich., on broad-crested weirs of various designs in a rectangular wooden flume 2 ft. wide. The crests varied in breadth from 10 to 15.5 ft., and in slopes and combinations of slopes from level to 0.085. The range of head was from 0.5 ft. to 1.5 ft. and of volume of flow from 2.0 to 11.0 cu. ft. per sec.

The purpose of the tests was to obtain experimental data in regard to certain advantages suggested in favor of the broad-crested weir over the sharp-crested weir as a device for measuring flowing water. One advantage established by the tests is that the tail-water can be backed up to an elevation only slightly lower than the water surface at the down-stream end of the weir without changing appreciably the relation between head-water elevation and discharge. Moreover, if the slope of the weir is such that a hydraulic jump can be made to occur on the crest, nearly all the initial head can be recovered, still without changing the head-water-discharge relation.

This ability to operate when submerged adapts the broad-crested weir for use under conditions where head is at a premium, as, for example, in the measurement of irrigation water. The broad-crested weir, moreover, even when operating under a considerable degree of submergence, requires the measurement of head at only one point and is thus preferable to the submerged sharp-crested weir and the Venturi flume, which require the measurements at two points.

Another advantage is the possibility of causing flow to occur at critical depth at some predetermined point on the crest at which the depth can be readily measured. The discharge is then a simple function of the depth, independent of the confusing effects of velocity of approach.

Previous discussions of this subject had indicated that critical depth might be expected to occur at the change in grade between slopes of channel one of

Note.—Written discussion on this paper will be closed in **January, 1931**, *Proceedings*.

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which would maintain flow above, and the other below, the critical depth. To investigate this theory tests were made of weirs formed of 10 ft. of level crest followed by 3½ ft. of apron with a small down-stream slope, the entrance to the weir being rounded.

These tests showed that the location of critical depth is affected by waves on the crest. The conformation of these waves is, in turn, affected by the volume of flow. With flows up to about 8 cu. ft. per sec., critical depth in a number of tests occurred at the change in grade, provided the slope of the apron was not too steep; but in other tests the waves caused the position of critical depth to move short distances up or down stream. When the flow was increased to more than 8 cu. ft. per sec. the position of critical depth moved from 1 to 3 ft. up stream from the change in grade.

Waves occurred at certain volumes of flow with every weir model tested in which part or all of the crest was level. If the crest was given a slope sufficient to maintain flow without waves, critical depth was found to occur, as a rule, only at some point of the initial drop at the entrance to the weir, at which the water surface sloped steeply and a small error in selection of the point of measurement would cause considerable error in the measured depth.

The tests showed that there was no single location of critical depth for all volumes of flow. Some interesting relations, however, were brought out between the design of the weir, the number of waves on the crest, and the location of critical depth.

Much information was obtained in regard to weir coefficients and to the factors affecting their variation. Coefficients as high as 3.08 were reached with a weir with rounded entrance followed by a down-stream slope of 0.026. This slope of crest permitted the formation of the hydraulic jump and the recovery of nearly all the initial head.

Introduction

Few previous tests have been made of broad-crested weirs of the designs described in this paper. Experiments have been made by Bazin, Blackwell, Fteley and Stearns, the U. S. Geological Survey, and others, on many designs of dams and weirs not sharp-crested, but only a small part of their investigations has dealt with weirs with broad level crests or with broad crests having a small down-stream slope.

However, recent discussions and tests on the subject of the critical depth and hydraulic jump* have suggested certain advantages which the broad-crested

* "The Hydraulic Jump in Open-Channel Flow at High Velocity," by K. R. Kennison, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 338; "Theory of Hydraulic Jump and Back-Water Curves," by S. M. Woodward, M. Am. Soc. C. E.; also "The Hydraulic Jump as a Means of Dissipating Energy," by R. M. Riegel and J. C. Beebe, The Miami Conservancy Dist., Tech. Repts., Pt. III, 1917; "The Hydraulic Jump and Critical Depth in the Design of Hydraulic Structures," by Julian Hinds, M. Am. Soc. C. E., *Engineering News-Record*, November 25, 1920; "The Improved Venturi Flume," by Ralph L. Parshall, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 89 (1926), particularly discussions by Julian Hinds and J. C. Stevens, Members, Am. Soc. C. E., p. 841 *et seq.*; "Berechnung der Wasserspiegellage," by P. Boess, *Karlsruhe Forschungsheft No. 284*, Verein Deutscher Ingenieure, Berlin, 1927; "The Hydraulic Design of Flume and Siphon Transitions," by Julian Hinds, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 1423.

weir may have over the sharp-crested weir as a device for measuring flowing water. These suggested advantages are:

1.—Its adaptability to the use of the hydraulic jump to reduce to a minimum the head lost in the weir; and

2.—The possibility of producing flow at critical depth with a resulting simple relation between depth and discharge which is independent of the confusing effects of velocity of approach.

Purpose and Scope of the Tests

The purpose of the tests was to obtain experimental data in regard to these two suggested advantages of the broad-crested weir. The breadth of crest of the weirs tested varied from 10 to 15.5 ft., with various down-stream slopes and combinations of slopes from level to 0.085. The range of heads was from 0.5 ft. to 1.5 ft. and of volume of flow from 2 to 11 cu. ft. per sec. These and other details of the tests will be given in the summaries of the experiments.

Coefficients of discharge of various weir models were determined both with free over-fall and with the weir submerged. The data thus obtained permitted a study of the effect of submergence, of the effect on the coefficient of various factors in the design of the weir, and of the variation of the coefficient with the head. Profiles of the water surface were made with different weir models to permit a study of the factors affecting the position of critical depth, with a view to suggesting if possible a design of weir which would produce flow at critical depth at the same point on the crest for all volumes of flow.

Notation

The following notation is used in the paper:

H = head on the weir, in feet.
H_a = total head, in feet, at any point in a flowing stream.
h_v = head due to velocity of approach, in feet.
D = depth, in feet.
D_a = depth in channel of approach, in feet.
D_c = critical depth, in feet.
V = velocity, in feet.
v_a = velocity of approach, in feet.
Q = discharge, in cubic feet per second.
Q_1 = discharge, in cubic feet per second, per foot of width.
C = weir coefficient.
g = acceleration due to gravity, = 32.16 cu. ft. per sec. per sec.
α = velocity-head coefficient, correcting for unequal distribution of velocities in cross-section; here considered = 1.0.

Review of Critical Depth Theory

The total head in any point of a flowing stream is:

$$H_a = D + \alpha \frac{V^2}{2g}$$

Considering $\alpha = 1.0$.

$$V = \sqrt{2g(H_a - D)}$$

Then,

$$Q_1 = D\sqrt{2g(H_a - D)}$$

Critical depth, by definition, is the depth at which, for any given total head, the discharge is a maximum. Differentiating Q_1 with respect to D and equating to zero,

$$H_a = \frac{3}{2} D$$

Substituting,

$$Q_1 = \sqrt{g}\ (D_c)^{\frac{3}{2}}$$

The discharge is, therefore, a simple function of the critical depth; the velocity of approach is not involved. Thus, if the critical depth can be made to occur at any known point where it can be measured accurately, a simple method of determining the discharge is available.

The effect of error in measurement of D_c may be shown by differentiating Q_1 with respect to D_c and dividing; that is,

$$d\ Q_1 = \frac{3}{2} \sqrt{g}\ (D_c)^{\frac{1}{2}}\ d\ D_c$$

$$\frac{d\ Q_1}{Q_1} = \frac{3}{2} \frac{d\ D_c}{D_c}$$

Thus, for any percentage, p, of error in D_c, the computed Q is in error 1.5 p per cent. This degree of accuracy is the same as that of the usual equations for rectangular weirs (aside from velocity of approach) and is greater than that of the equations for V-notch weirs, where an error of $p\%$ in measuring the head results in an error of $2.5p\%$ in the computed Q.

The Laboratory

Details of the hydraulic laboratory are shown in Figs. 1 and 3. The quantity of water flowing in the system was controlled by the gate valve at the upper end of the 15-in. pipe line.

The 90° V-notch weir by which the discharge was measured, was calibrated by means of the weighing tanks. The methods and results of this calibration are described under "Measurement of Discharge".

Weir.—At a point 32 ft. from the head end of the flume, the bottom was stepped up 1.75 ft. (Fig. 1), and carried for 10 ft. at this elevation. This formed a weir crest 10 ft. broad, with square-cornered entrance. Series *A* includes the tests of this weir. Rounded entrances of various curvatures were obtained by means of laminated wooden blocks set on bases of proper height and thickness against the up-stream end of the weir.

Variation in slope was obtained by a false bottom 12 ft. long. This bottom was given the desired slope by laying it on shims on the 10-ft. level section. Series *D D* and *D E* give the results obtained with two different slopes of crest.

Combinations of level and sloping crests were obtained by means of a platform 3½ ft. long, used with either the permanent level section or the sloping false bottom. With the former the platform was used as a down-stream apron with three different slopes (Series *DA*, *DB*, and *DC*) while with the sloping bottom the platform was placed ahead to form a level section of crest (Series *DF* and *DG*).

The flume and weir were built in August, 1928. Just prior to the tests which began in November, the crest of the permanent weir was planed to a true level surface, and the sides of the flume, as well as the weir crest, were scraped and sand-papered to give as smooth a surface as possible. The surfaces of the false bottom and short platform were also made smooth.

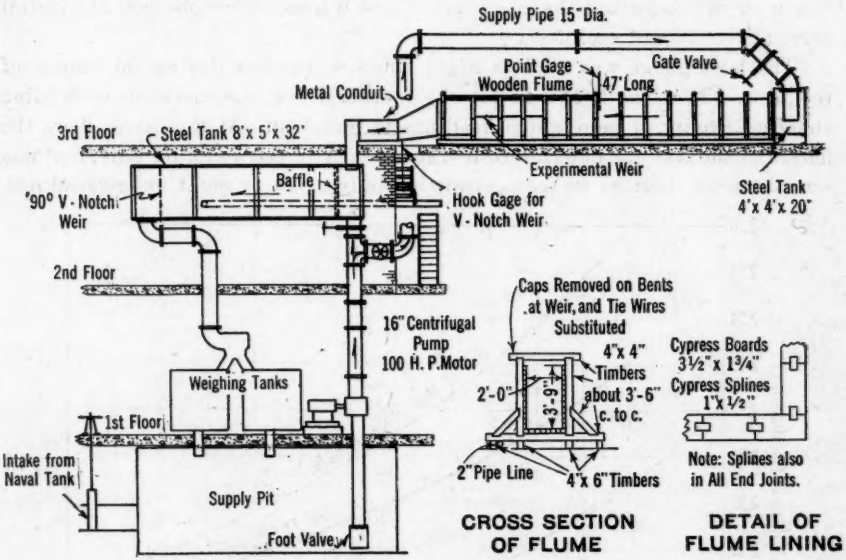


FIG. 1.—DETAILS OF EXPERIMENTAL FLUME.

Aeration.—Five 1-in. holes to provide aeration of the nappe were bored in the wooden bulkhead forming the down-stream end of the permanent weir section. These holes were in a horizontal line 5 in. below the weir crest. Wooden plugs were used to stop the holes when required. As a general rule, the plugs were removed and the nappe was aerated in tests in which the back-water from the fall did not rise high enough to cause leakage through the holes. The nappe was thus aerated for flows up to about 7 cu. ft. per sec.

Measuring Elevation of Water Surface.—The elevation of the head water was measured by a point-gauge mounted in the center of the flume on the cross-cap of one of the bents (Figs. 1 and 3). The distance from the up-stream edge of the weir to the gauge was 7 ft. When the weir was extended up stream by the addition of the 3½-ft. platform, the gauge was moved up to the next bent. Profiles of the water surface over the weir were taken with another point-gauge mounted on a light channel iron. This movable gauge was supported by parallel channels attached to the tops of the posts.

Measurement of Discharge.—The discharge over the broad-crested weir was measured by means of a 90° V-notch weir which was calibrated gravimetrically just previous to the tests. The methods and results of this calibration will be discussed here briefly.

The flying-start-and-stop method was used in measuring the water. With the flow steady and the water discharging through the weighing tank, the

weight on the scale-beam was set to a small initial reading and the outlet valves of the weighing tank were closed. At the instant the beam rose to the cross-bar the two watches were started. The weight was then set ahead to the desired final quantity, and the watches were stopped when the beam rose to the bar again. The average of the watch readings, corrected for initial error, was used in computing the coefficient. The watches were checked for initial error at the start of each test.

The hook-gauge was read as many times as possible during the course of the tests. The length of test for the smaller flows was made about 4 min., enabling twenty or more gauge readings to be taken. At the larger flows the length of the test was determined by the capacity of the weighing tank, and was sometimes as short as 50 sec., permitting only seven or eight gauge readings.

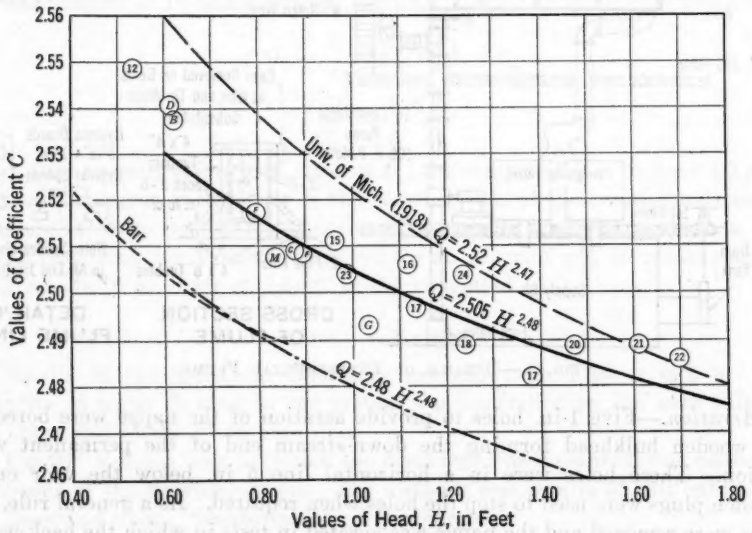


FIG. 2.—CALIBRATION TESTS OF 90° V-NOTCH WEIR: VARIATION OF C WITH H IN $Q = CH^{2.5}$ FOR THE EQUATIONS SHOWN.

The volume of flow, in cubic feet per second, was obtained by using 62.40 lb. as the weight of 1 cu. ft. of water. The computed volume of flow and the observed head on the weir corrected for zero reading of the gauge, were used to compute the coefficient, C, in the equation, $Q = CH^{\frac{5}{2}}$. Values thus obtained for C are plotted against values of H in Fig. 2, which shows the usual decrease in coefficient with increase in head. The equation, $Q = CH^{\frac{5}{2}}$, is awkward to use because it involves two variables. By making a slight change in the exponent of H, a suitable equation can be derived in which C is constant. The equation best fitting the points on the graph was adopted for use, namely, $Q = 2.505\,H^{2.48}$. The curve representing this equation (Fig. 2) was obtained by plotting values of C against H. C is computed from the equation, $C = \dfrac{Q}{H^{\frac{5}{2}}}$,

Q having been computed from the equation, $Q = 2.505\ H^{2.48}$. Thus, the curve and the test results are comparable graphically.

Curves of two other series of experiments with 90° V-notch weirs are also shown on Fig. 2. The equation, $Q = 2.52\ H^{2.47}$, was obtained as the result of experiments performed at the University of Michigan in 1918. In Scotland, some years previously, Barr made some experiments with results as also shown on Fig. 2. Barr's coefficients fit quite closely the equation, $Q = 2.48\ H^{2.48}$. The curve corresponding to this equation is drawn on the graph to serve as an extension of Barr's results to higher heads.

The writer's equation is seen to lie between the Barr experiments and the University of Michigan equation, but approaching more nearly the latter at the higher heads. The care taken in calibrating the V-notch weir and the agreement with former V-notch weir tests offer reasonable assurance that the subsequent measurement of discharge over the broad-crested weir was correct at least within 1 per cent.

Test Procedure

The usual method of making a test required four observers. One man read the hook-gauge which showed the head on the V-notch weir by which the discharge was measured. Another read the fixed point-gauge up stream from the broad-crested weir. These two gauges were read regularly at 30-sec. intervals. Twenty readings constituted the usual test.

A third observer operated the movable gauge to obtain the profile of the water surface (Fig. 3). Cross-sections were taken, proceeding down stream, at as many points as were required to permit drawing an accurate profile. The usual cross-section consisted of three readings, one at the center and one 3 in. from each side. The average of these readings was used in plotting the profile.

The fourth man kept notes for the second and third observers. The man reading the hook-gauge recorded his own notes.

A number of tests were made merely to determine coefficients of discharge without taking profiles of the water surface. These tests required an observer on the hook-gauge, one on the fixed point-gauge, and a recorder.

Method of Computing Weir Coefficients

The weir coefficient for each test was computed by the method outlined by Horton in his treatise on "Weir Experiments, Coefficients, and Formulas."* The steps in the computation will be given briefly.

The average reading of the hook-gauge, corrected for the zero reading of the gauge, was used to find the discharge, Q, from a table prepared by the writer, based on the formula already stated, $Q = 2.505\ H^{2.48}$.

The average reading of the fixed point-gauge, corrected for the zero reading, was used as the observed head, H, on the weir.

The depth of water in the channel of approach,

$$D_a = H + \text{the height of the weir}$$

* U. S. Geological Survey, *Water Supply and Irrigation Paper No. 200*, pp. 66–67.

Then,

$$v_a = \frac{Q}{2\,D_a}$$

$$h_v = \frac{{v_a}^2}{2\,g}$$

$$H_a = H + h_v$$

$$C = \frac{Q}{2\,{H_a}^{\frac{3}{2}}}$$

$$D_c = \left(\frac{Q_1}{\sqrt{g}}\right)^{\frac{2}{3}} = \left(\frac{Q}{2\sqrt{g}}\right)^{\frac{2}{3}} = 0.1982\ Q^{\frac{2}{3}}$$ (see page 1586).

D_c was found from a table prepared by the writer for a rectangular flume, 2 ft. wide, giving critical depths for values of Q from 0 to 12 cu. ft. per sec.

Outline of Tests

An outline of the 305 tests of broad-crested weirs, with the series designations, in the chronological order of their performance, is as follows:

Tests of weirs with level crest, 10 ft. broad, with varying curvature of entrance:

Square cornered entrance	37 tests,	Series *A*
Rounded entrance, 2-in. radius	13 tests,	Series *B*
Rounded entrance, 3-in. radius	11 tests,	Series *C*
Rounded entrance, 6-in. radius	26 tests,	Series *D*
Rounded entrance, 3 by 6-in. ellipse, long axis horizontal	10 tests,	Series *E*
Rounded entrance, 8-in. radius	12 tests,	Series *F*
Rounded entrance, 4 by 8-in. ellipse, long axis horizontal	12 tests,	Series *G*

Tests of weirs from 12 to 15.5 ft. broad, with various slopes and combinations of slopes. Entrance rounded on 6-in. radius:

10-ft. level, followed by 3.5 ft. of 0.015 slope	25 tests,	Series *DA*
10-ft. level, followed by 3.5 ft. of 0.004 slope	36 tests,	Series *DB*
10-ft. level, followed by 3.5 ft. of 0.085 slope	8 tests,	Series *DC*
12 ft. of 0.004 slope	33 tests,	Series *DD*
12 ft. of 0.026 slope	36 tests,	Series *DE*
3.5-ft. level, followed by 12 ft. of 0.026 slope	33 tests,	Series *DF*
3.5-ft. level, followed by 12 ft. of 0.004 slope	13 tests,	Series *DG*
Total tests	305	

Analysis of Results

The results will be discussed under three heads:

1.—Effect of submergence on the weir coefficient.

2.—Location of critical depth.

3.—Variation of coefficient with head for different weir models.

Effect of Submergence on Coefficient.—The results of fifty-four pairs of tests which show the effect of submergence on the weir coefficient, are sum-

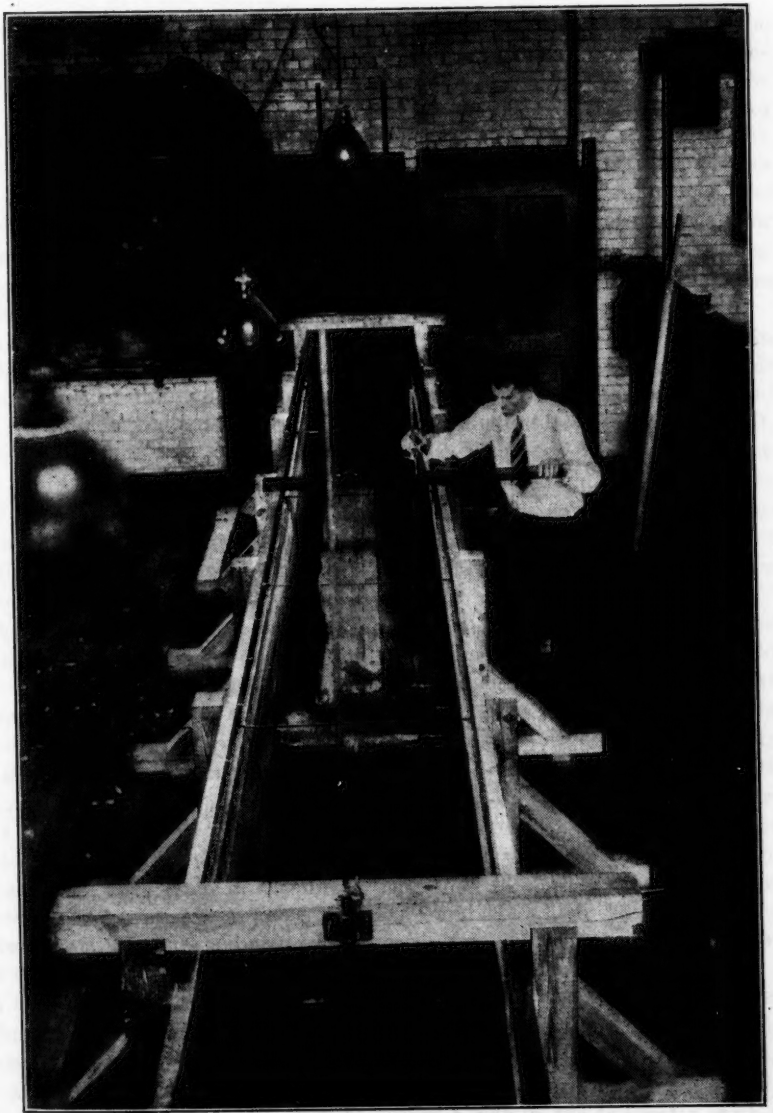
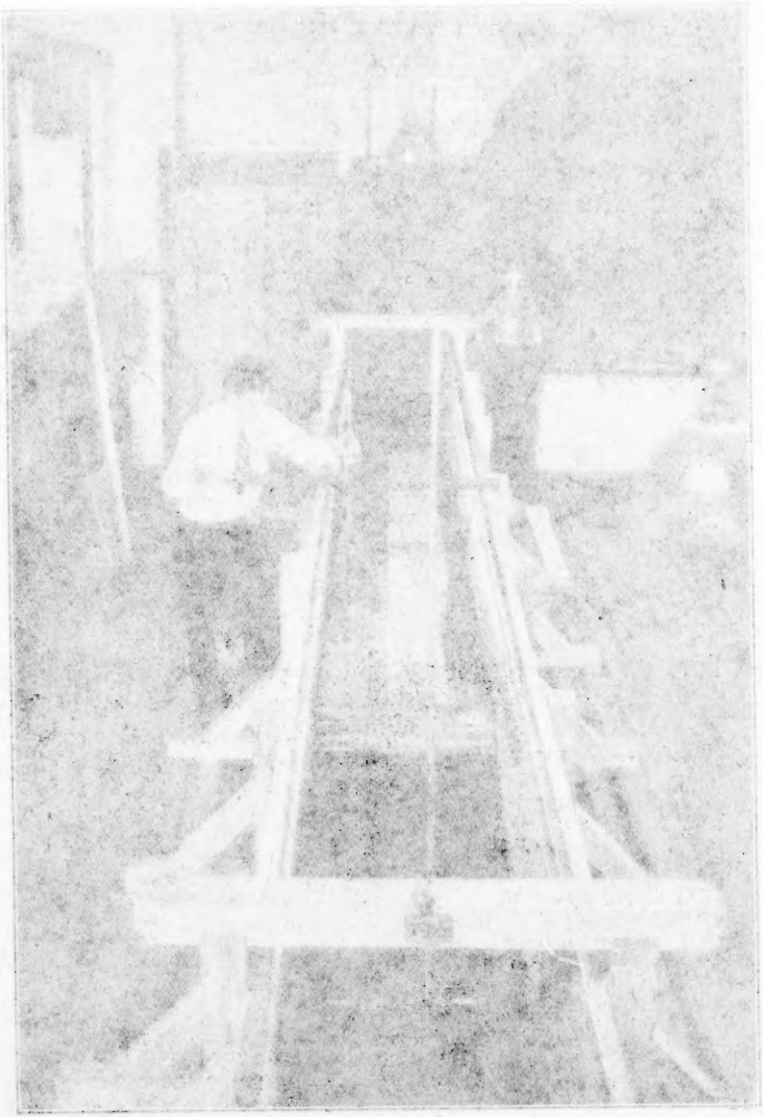


FIG. 3.—SET-UP AND DETAILS OF HYDRAULIC FLUME.



marized in Table 1. These tests were made on eight different weir models and covered the entire range of head from 0.5 ft. to 1.5 ft. Each pair, in nearly every case, was performed consecutively with no change between tests in the quantity of water flowing.

Either with or without the hydraulic jump, the average difference in the coefficients of the weirs submerged and with free over-fall was found to be inappreciable. In thirty-two pairs of tests in Table 1 (Part I), in which the jump could not be produced when the weir was submerged, the coefficient for the submerged weirs was, on the average, 0.219% less than for the weirs with free over-fall; while in twenty-two pairs of tests in Table 1 (Part II), in which the jump was formed on the crest when the weir was submerged, the coefficient for the submerged weirs was, on the average, 0.081% less than for the weirs with free over-fall. In the fifty-four pairs of tests in both parts, the average coefficient for the submerged weirs was 0.163% less than for the weirs with free over-fall.

The depths of submergence are shown in the profiles of water surface in Fig. 4. In the tests in which the slope of the crest was steep enough to permit the formation of the hydraulic jump (Series *DE* and *DF*), the loss of head over the weir was limited to about 0.2 ft. at the smallest flow and about 0.3 ft. at the largest flow. In the tests of weirs with slopes so flat that the jump could not be formed (Series *DA*, *DD*, and *DG*), the tail-water could be raised only to an elevation somewhat below the water surface at the down-stream end of the weir without noticeably affecting the conditions of flow over the weir, and the loss of head was from about 0.3 ft. for the smallest flow to about 0.6 ft. for the largest flow.

The attempt was made at the start of each test with the weir submerged to back up the tail-water to an elevation just under the maximum which would leave the weir coefficient unchanged. If at the start of a test, however, the tail-water was very close to the permissible elevation, slight fluctuations in volumes of flow might have caused this elevation to be exceeded momentarily with a resulting change in the conditions of flow over the weir and a decrease in the weir coefficient. The permissible elevation of the tail-water was purposely approached more closely in the tests in which the jump could not be formed than in those in which it could be formed, and as a result was probably exceeded more often in the former case than in the latter. This fact is reflected by the averages of Table 1 (Parts I and II) which show a difference in coefficients of weirs submerged and with free over-fall somewhat larger when the jump could not be formed than when it could be formed.

A typical example of the change in conformation of the water surface over a submerged weir without the jump when the tail-water rises through this permissible elevation is shown by Tests 4 and 5 of Series *DD* (Fig. 4). The volume of flow in the two tests was practically the same; but a small closure of the down-stream gate between tests caused a rise in the tail-water, with a marked change in profile of the water surface over the weir and a rise in the head-water. A change of this nature, although much smaller in extent and caused by a slight fluctuation in the volume of flow instead of by closure of the gate, doubtless occurred during those six tests of Table 1 (Part I) in

TABLE 1.—TESTS OF BROAD-CRESTED WEIRS: COMPARISON OF COEFFICIENTS OF SUBMERGED WEIRS AND WEIRS WITH FREE OVER-FALL.

(Length of weir, $L = 2$ ft.; entrance rounded on 6-in. radius.)

Test No.	Discharge, in cubic feet per second, Q.	Total head = observed head plus velocity head, H_a.	Coefficient in equation, $Q = C L H_a^{3/2}$, C.	Percentage by which C submerged is greater or less than with free over-fall.
PART I: WEIRS WITH SLOPE OF CREST SO SMALL THAT HYDRAULIC JUMP WILL NOT FORM.				
SERIES *DA*.				
1	2.603	0.6062	2.757	−0.62
2*	2.582	0.6054	2.740	
9	3.291	0.7056	2.776	−0.76
8*	3.278	0.7074	2.755	
10	4.135	0.8201	2.784	−0.22
11	4.118	0.8189	2.778	
12	5.649	1.0074	2.794	−0.21
13*	5.634	1.0070	2.788	
14	10.108	1.4758	2.817	−0.53
15*	10.040	1.4752	2.802	
16	8.170	1.2910	2.786	+0.32
17*	8.184	1.2892	2.795	
18	6.770	1.326	2.808	−0.46
19*	6.775	1.369	2.795	
20	4.922	0.9150	2.812	−0.71
21*	4.913	0.9183	2.792	
22	7.577	1.2241	2.798	+0.07
23*	7.483	1.2132	2.800	
24	9.189	1.3964	2.785	0.00
25*	9.212	1.3986	2.785	
Average for Series *DA*				−0.31
SERIES *DB*.				
1	2.628	0.6057	2.787	−1.04
2*	2.622	0.6092	2.758	
3	4.036	0.8046	2.796	−0.11
4*	4.055	0.8077	2.793	
5	5.654	1.0041	2.810	−0.18
6*	5.640	1.0035	2.805	
7	7.601	1.2230	2.810	−0.43
8*	7.585	1.2249	2.798	
9	9.478	1.4189	2.804	+0.18
10*	9.503	1.4196	2.809	
Average for Series *DB*				−0.32

* Submergence.

TABLE 1.—(*Continued.*)

Test No.	Discharge, in cubic feet per second, Q.	Total head = observed head plus velocity head, H_a.	Coefficient in equation, $Q = C\,L\,H_a^{3/2}$, C.	Percentage by which C submerged is greater or less than with free over-fall.
		SERIES *DD*.		
1	2.670	0.5903	2.943	−1.02
2*	2.671	0.5946	2.913	
3	4.005	0.7755	2.932	−0.27
4*	3.993	0.7753	2.924	
7	5.656	0.9826	2.904	−0.45
8*	5.621	0.9813	2.891	
9	7.429	1.1855	2.878	−0.07
10*	7.369	1.1797	2.876	
11	8.879	1.3377	2.869	+0.24
12*	8.864	1.3343	2.876	
13	2.061	0.4954	2.951	+0.37
14*	2.064	0.4951	2.962	
15	3.423	0.6980	2.935	−0.03
16*	3.415	0.6970	2.934	
17	4.892	0.8884	2.921	−0.14
18*	4.878	0.8900	2.917	
22	2.396	0.5483	2.951	−0.03
24*	2.410	0.5505	2.950	
25	4.396	0.8277	2.919	+0.03
26*	4.427	0.8314	2.920	
27	7.137	1.1529	2.883	−0.21
28*	7.120	1.1527	2.877	
29	8.097	1.2570	2.873	−0.03
30*	8.076	1.2552	2.872	
31	9.318	1.3830	2.865	−0.10
32*	9.321	1.3840	2.862	
Average for Series *DD*	.....	.....	.....	−0.13
		SERIES *DG*.		
4	2.758	0.6103	2.893	−0.21
5*	2.746	0.6094	2.887	
8	6.781	1.1208	2.857	−0.07
9*	6.810	1.1249	2.855	
12	9.711	1.4308	2.837	−0.28
13*	9.713	1.4336	2.829	
Average for Series *DG*	.....	.....	.....	−0.19

* Submergence.

TABLE 1.—(*Continued.*)

Test No.	Discharge, in cubic feet per second, Q.	Total head = observed head plus velocity head, H_a.	Coefficient in equation, $Q = C L H_a^{3/2}$, C.	Percentage by which C submerged is greater or less than with free over-fall.
		SERIES *D*.		
17	5.144	0.9412	2.817	−0.04
18*	5.258	0.9552	2.816	
Average for thirty-two pairs of tests in Part I				- 0.219
		PART II: WEIRS WITH THE HYDRAULIC JUMP FORMED ON THE CREST BY SUBMERGENCE.		
		SERIES *DE*.		
1	4.361	0.8027	3.082	−0 03
2*	4.361	0.8032	3.031	
3	5.835	0.9780	3.016	+0.03
4*	5.845	0.9791	3.017	
5	7.385	1.1473	3.005	−0.33
6*	7.378	1.1492	2.995	
7	8.860	1.3024	2.981	+0.17
8*	8.856	1.3005	2.986	
10	2.822	0.5983	3.049	−0.39
11*	2.792	0.5956	3.037	
16	2.513	0.5502	3 079	−0.19
17*	2.529	0.5533	3.073	
18	4.012	0.7555	3.055	−0.20
19*	3.998	0.7548	3.049	
20	6.605	1.0600	3.026	−0.36
21*	6.558	1.0577	3.015	
22	5.229	0.9054	3.035	+0.16
25*	5.225	0.9039	3.040	
23	5.233	0.9051	3.039	0.00
26*	5.224	0.9039	3.039	
24	5.228	0.9035	3.044	−0.23
27*	5.239	0.9061	3.037	
28	7.546	1.1582	3.027	−0.36
29*	7.528	1.1592	3.016	
30	8.548	1.2669	2.997	+0.13
31*	8.547	1.2658	3.001	
32	9.772	1.3830	3.004	−0.20
33*	9.747	1.3825	2.998	
35	3.122	0.6385	3.060	+0.26
36*	3.146	0.6406	3.068	
Average for Series *DE*				−0.10

* Submergence.

TABLE 1.—(*Continued.*)

Test No.	Discharge, in cubic feet per second, Q.	Total head = observed head plus velocity head, H_a.	Coefficient in equation, $Q = C L H_a^{3/2}$, C.	Percentage by which C submerged is greater or less than with free over-fall.
		SERIES *DF*.		
9	5.025	0.9059	2.914	−0.38
10*	5.007	0.9060	2.903	
11	6.995	1.1363	2.887	+0.10
12*	7.011	1.1375	2.890	
13	8.825	1.3274	2.885	+0.31
14*	8.833	1.3255	2.894	
16	7.642	1.2051	2.888	+0.10
17*	7.659	1.2060	2.891	
18	2.104	0.5050	2.931	−0 07
19*	2.092	0.5034	2.929	
20	3.416	0.6987	2.925	−0.10
21*	3.411	0.6984	2.922	
22	5.947	1.0192	2.890	−0.21
23*	5.956	1.0217	2.884	
Average for Series *DF*				−0.04
Average for twenty-two pairs of tests in Part II				−0.081
Grand average of fifty-four pairs of tests				−0.163

* Submergence.

which the coefficient submerged was more than 0.5% less than the coefficient with free over-fall. If these six pairs of tests are disregarded, the remaining twenty-six pairs give coefficients for the submerged weirs only 0.09% less on the average than for the weirs with free over-fall.

With the jump formed on the crest, the largest difference in coefficients submerged and with free over-fall was 0.39 per cent. Various positions of the jump were tried, but none closer to the entrance of the weir than about 5 ft., at which distance a slight fluctuation in volume of flow merely caused a small movement of the jump up or down stream with no other visible effect on the conditions of flow.

LOCATION OF CRITICAL DEPTH

Profiles of the water surface over the weir for each weir model tested, both submerged and with free over-fall, and with a range of head from 0.5 ft. to 1.5 ft., are shown in the series of graphs in Fig. 4; also the points at which the measured depth equals the computed critical depth for each flow.

The graphs show that none of the weir models tested produced flow at critical depth at any fixed location throughout the entire range of heads. At points where previous discussions had indicated that critical depth might be

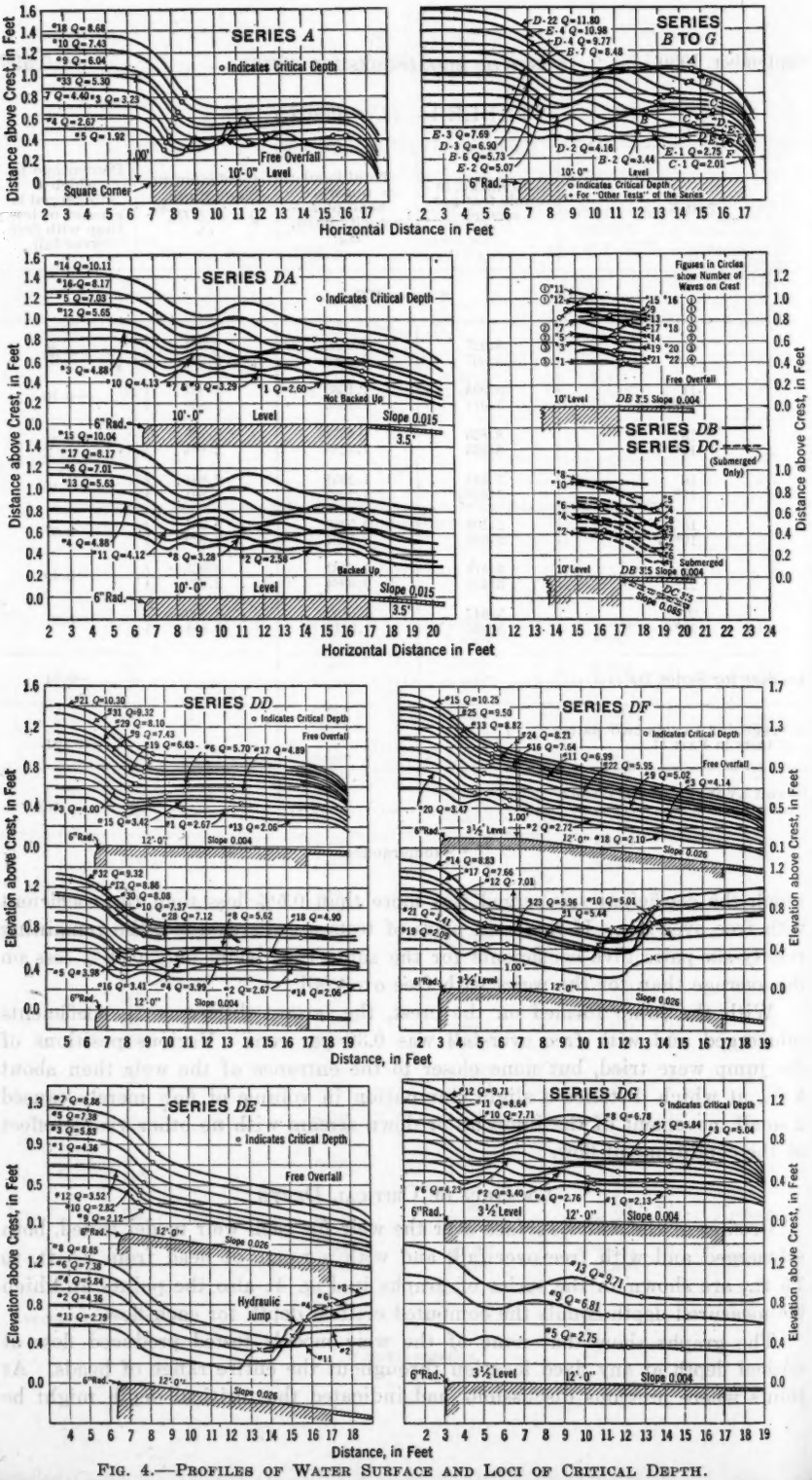


FIG. 4.—PROFILES OF WATER SURFACE AND LOCI OF CRITICAL DEPTH.

expected to occur, as, for instance, at the change from level crest to a small down-stream slope (Series *DA*, *DB*, *DF*, and *DG*), the depth of flow was found to be affected by waves on the crest of the weir. On the other hand, with crest slopes sufficient to maintain flow free of waves (Series *DE*), critical depth occurred, as a rule, only on the initial drop in water surface at the entrance to the weir. One series of tests in which the crest had a down-stream slope of 0.004 (Series *D D*) showed for the smaller flows a depth very near the critical through a distance of 3 or 4 ft. at about the center of the crest; but for flows larger than about 6 cu. ft. per sec., the depth was distinctly less than the critical at all points except on the initial entrance drop.

Although for the models tested no one location was indicated at which critical depth may be expected to occur at any flow, nevertheless, the profiles show some interesting facts in regard to the manner in which it varies with the number of waves on the crest; and also the manner in which the width and slope of the crest, the rounding of entrance, and the volume of flow affect the number and position of the waves.

Waves occurred at certain flows with every weir model tested in which part or all of the crest was level. The profiles of Series *B* to *G* show typical conditions of flow over a weir with a level crest, 10 ft. wide, with rounded entrance. The locus of the points of critical depth in the profiles of this group is a series of disconnected straight lines, apparently with about the same slope, each line containing the points of critical depth accompanying a particular number of waves on the crest. Thus, all profiles showing only one wave on the crest have their points of critical depth on Line *BB*; profiles with two waves have their points on Line *CC*; those with three waves, on Line *DD*; with four waves, on Line *EE*; and with five waves, on Line *FF*. Test No. 6 in Series *B* appears to be a border-line case between one wave and two waves, since its profile shows points of critical depth on both Line *BB* and Line *CC*. Line *AB*, containing critical depth points for flows of about 11 cu. ft. per sec. and greater, appears, from the few tests made at those flows, to slope up to the left instead of up to the right like the other loci.

The same arrangement of the points of critical depth is apparent also in the profiles in Series *DA* and *DB* for free over-all. A majority of the points of critical depth for the lower flows in these two series lie at, or very close to, the change in slope. However, even with these smaller flows, the effect of the waves is apparent in the variations of some of the points from the change in slope. At the larger flows, the points of critical depth all occur from 1 ft. to 3 ft. up stream from the change in slope, and their locus is similar to that of the points on the profiles with one wave in Series *B* to *G*.

The profiles show the variation of the number of waves on the crest with the volume of flow and with the breadth and slope of the crest. At the smallest flows tested (Series *B* to *G*, and *DA*), five distinct waves appear. With increasing flow, the number of waves decreases consecutively until at the largest flows the profiles present a smooth curve free from waves. Moreover, it is apparent that at least within the limits of these tests, the waves on a level crest continue to occur until smoothed out by sufficient increase in the slope

or by a fall at the down-stream end of the level section. Narrowing the weir would thus obviously result in fewer waves at the smaller flows and in a reduction of the volume of flow at which the last wave disappears.

The effect of an increase in slope is shown by the profiles of Series *DF* and *DG*. The weir in Series *DF* consisted of a level section 3½ ft. long, with rounded entrance, followed by 12 ft. of 0.026 slope. Waves occurred singly and only at the two smallest flows. The narrow section of level crest and the steepness of the down-stream slope combined to smooth out the flow quickly. The slope of crest in Series *DG*, however, was not sufficient to smooth out the flow quickly and the profiles show at least one wave for all heads tested and as many as three waves for the smallest flow.

The foregoing discussion of the position of critical depth pertains to the weirs in which the entrance was rounded. With a square-cornered entrance the conformation of the water surface over the weir was quite different, as is seen by comparing the profiles of Series *A* with those of Series *B* to *G*. The initial drop in the water surface was greater with the square-cornered entrance than with the rounded entrance preceding a level crest 10 ft. broad. With the square-cornered entrance the water surface dropped to a point below the critical depth and then commenced to rise gradually. This rise continued either until it approached the critical depth, when the surface became wavy and turbulent, or until it reached the vertical curve caused by the fall. The critical depth was reached and waves were formed only with flows of less than about 4 cu. ft. per sec. The waves formed with the rounded entrance were smooth in profile, and the water surface was practically level in cross-section at all points. With the square-cornered entrance the waves reached more of a peak, being considerably higher in the center of the stream than at the sides; and there was more surface turbulence.

Variation of Weir Coefficient with Head

The variation of the weir coefficient, C, with the total head, H_a, in the equation, $Q = CLH_a^{\frac{3}{2}}$, for thirteen different weir models tested, is shown by the series of graphs in Fig. 5. A longitudinal section of the weir model is shown with each graph. The points on the graphs include only tests of weirs with free over-fall.

To facilitate comparisons of the coefficients for the different weir models, all thirteen curves are shown on one graph in Fig. 6; also, two curves obtained by Bazin and three by the U. S. Geological Survey for weirs of the same general type as those reported in this paper.

One important comparison (Fig. 6) is that for the weirs with the broader part of the crest level, with rounded entrance, and either with or without a narrow sloping down-stream apron, the coefficient increases with the head up to values of H_a of about 1.10, after which it is approximately constant (Series *B* to *G*, *DA*, and *DB*); while for weirs with the broader part of the crest sloping down stream, with rounded entrance, and either with or without a narrow level platform preceding the sloping section, the coefficient decreases with the head and shows a tendency to become constant at values of H_a of about 1.30 (Series *DD*, *DE*, *DF*, and *DG*).

Series *B* to *G* (Fig. 6) also indicate that for the breadth of weirs tested, varying the curvature of the weir entrance between a 2-in. and an 8-in. radius has practically no effect on the weir coefficient. The graphs obtained with six different curvatures—four circular and two elliptical—all lie within 0.5% of one another at all points. The graph for a square-cornered entrance (Series *A*), however, is considerably lower, showing values of the coefficients from 6 to 8% less than for rounded entrances. Apparently, the entire advantage of rounding the entrance is obtained with the change from the square corner to the 2-in. radius, and further increase in the radius does not appreciably increase the coefficient.

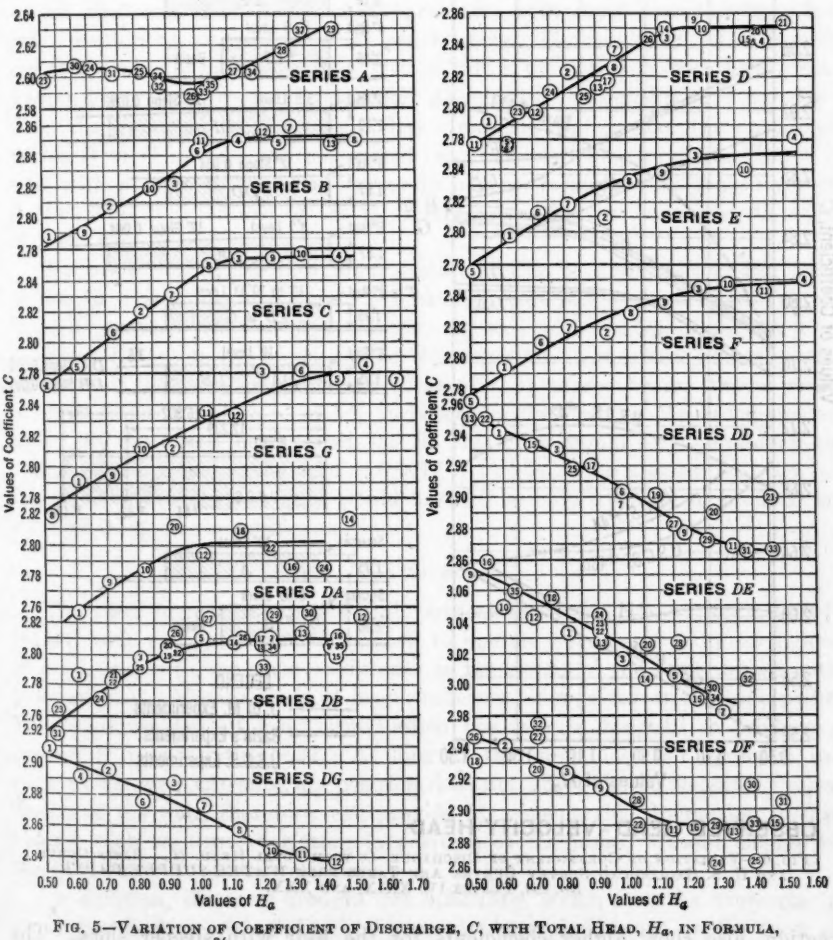


FIG. 5—VARIATION OF COEFFICIENT OF DISCHARGE, *C*, WITH TOTAL HEAD, H_a, IN FORMULA, $Q = C\,L\,H_a^{3/2}$. H_a = OBSERVED HEAD + VELOCITY HEAD; L = 2.0 FEET.

A definite increase in coefficient with increase in slope of the crest is also shown. A comparison of the curves of Series *DE* and *DD* for the same width

of crest, but with different slopes, shows that the coefficients for the weir with a crest slope of 0.026 are about 4% higher throughout the range of heads tested than the coefficients for the weir with a crest slope of only 0.004. Tests in Series *DF* and *DG*, in which a 3½-ft. level section preceded the sloping

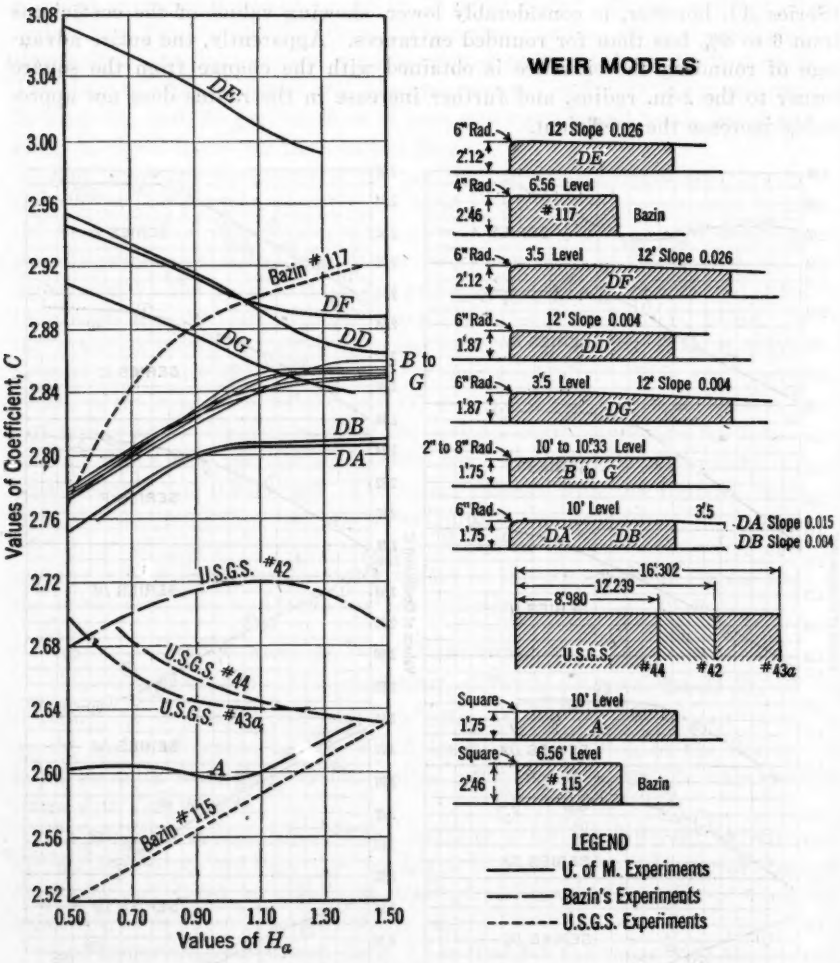


FIG. 6.—VARIATION OF COEFFICIENT OF DISCHARGE, *C*, WITH TOTAL HEAD, H_a. BAZIN CURVE AND U. S. GEOLOGICAL SURVEY CURVES ARE TAKEN FROM *WATER SUPPLY PAPER No. 200*, PLATES IV, XXIX. AND XXX.

section, also show higher coefficients for the weir with steeper slope. The difference is not so great, however, as in the narrower weirs of Series *DE* and *DD*, indicating that a greater width tends to reduce the effect of a change in slope.

The tests show a reduction in coefficient with an increase in the width of the weir. Comparison of Series *DA* and *DB* with Series *B* to *G* shows that a decrease of about 1% in the coefficient resulted when a 3½-ft. downstream apron with a slight slope was added to the weir with level crest. Moreover, comparison of Curves *DE*, *DF*, and *DD* shows that the coefficients were reduced about as much by the addition of 3½ ft. of level section ahead of the sloping section as by reducing the slope from 0.026 to 0.004. The height of these weirs varied from 1.75 to 2.12 ft., but there is little probability that the conclusions are appreciably in error on this account, especially since the variation in height was largely compensated for by including the correction for velocity of approach in the head used to compute the weir coefficients.

All the tests show fairly good agreement with those obtained by other experimenters on weirs of the same general type. The results of Series *B* to *G* are comparable with Bazin's Test No. 117 (Fig. 6) on a weir with level crest, 6.56 ft. broad, with rounded entrance. As might be expected with a narrower weir, the curve of Bazin's results lies somewhat higher than the curves of Series *B* to *G*. For a sharp-cornered entrance, the curve of Series *A* for a weir with a crest 10 ft. broad agrees in general location with the curves of Bazin's Test No. 115 and of the U. S. Geological Survey Tests Nos. 42, 43A, and 44 (Fig. 6).

An inspection of the individual graphs of Fig. 5 shows that in 201 tests of the 13 weirs, only one point, No. 21 in Series *DD*, is more than 1% off from its curve, while the large majority are well within 0.5% of the curve.

Since H_a enters the broad-crested weir equation to the three-halves power, an error of 0.5% in the computed value of C is caused by an inverse error of approximately one-third of 1% in observing the head on the broad-crested weir. At the lowest head tested, 0.5 ft., this error would be produced by an actual error in reading the head of 0.0017 ft.; while at the highest head, 1.5 ft., the error in reading the head would have to be only 0.005 ft. to produce an error of 0.5% in the coefficient.

Conclusions

The tests furnished experimental verification of the theory that a broad-crested weir can be properly designed to operate submerged with small loss of head, and with the same coefficients as for a weir of the same design with free over-fall. Measurement of head, moreover, is required at only one point. The minimum loss of head occurs when the weir is designed to produce a hydraulic jump on the crest. As long as the jump does not approach too close to the weir entrance, its position does not affect the coefficient. Proper design of the weir, therefore, can provide for considerable fluctuation in discharge, with resulting variation in position of the jump, without change in the head-water-discharge relation.

In addition, the tests brought out difficulties which must be overcome in producing flow at critical depth at some predetermined point on the crest of the weir. The chief difficulty is the formation of waves on the crest which affect the location of critical depth. Tests of other types of entrance with different alignment of the sides of the channel may lead to a design of weir

which will maintain the flow at the desired depths without these disturbing waves. The measurement of flowing water with a broad-crested weir of this type would require only a determination of the depth at the predetermined point and would not involve velocity of approach, which has been a continued source of uncertainty in measurements with sharp-crested weirs.

Personnel

The tests were performed by the writer under the leadership and guidance of H. W. King, M. Am. Soc. C. E., and with the assistance of C. O. Wisler, Assoc. M. Am. Soc. C. E., and Messrs. H. A. Seaman, J. G. Jobes, C. L. Palmer, A. J. Paddock, P. Singh, and C. Li, graduate students and seniors at the University of Michigan. Most of the drawings were by Mr. Seaman.

APPENDIX

SUPPLEMENTAL TESTS—WEIRS WITH APRONS INCLINED UP STREAM AND DOWN STREAM

By Alex. R. Webb,* M. Am. Soc. C. E.

Scope

This Appendix describes experiments conducted in continuation of Mr. Woodburn's work on broad-crested weirs. The object was to determine if possible a form of weir crest which would cause the locus of critical depths to lie in a vertical line. The general dimensions of the weir used are illustrated in Fig. 7. The apex of the two sloping aprons forms the highest section of the weir crest.

Results are given for 235 separate experiments. The down-stream apron was set at slopes of 1 in 15.4, 1 in 24.8, 1 in 25.0, 1 in 25.5, and 1 in 46.4. The slope of the up-stream apron was varied between 1 in 7.5 and 1 in 98.5. The heads ranged from 0.12 to 1.33 ft. and the discharges from 0.22 to 9.33 cu. ft. per sec.

The experiments show that for the flatter slopes waves formed on the up-stream apron and that the location of critical depth changed with the position of the wave. This condition was less marked for the smaller discharges. Except for these very flat slopes it was found that for each slope of the down-stream apron there was a corresponding slope of the up-stream apron, which caused critical depths for all discharges to lie in a vertical straight line near, but not necessarily above, the apex of the crest.

The effect of submergence on the relation of head to discharge was investigated by making parallel runs both with and without submergence. In all, fourteen pairs of comparative tests were made. The value of the coefficient, C, in the weir formula, $Q = C L H_a^{\frac{3}{2}}$, was also investigated. In addition, a series of experiments was conducted to check the formula for the V-notch weir which was used to measure discharges.

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Laboratory

With slight modification the equipment used is the same as that described in the paper. The horizontal broad-crested weir was modified by building a small bulkhead at the up-stream end and attaching the sloping aprons, shown in Fig. 7. The down-stream apron, the false bottom referred to by Mr. Woodburn, was fixed for each set of experiments. The up-stream apron was attached by hinges to the bulkhead so that the slope could be easily changed.

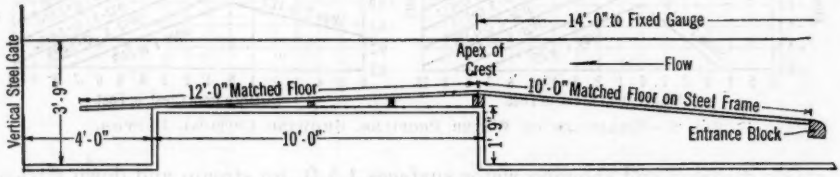

FIG. 7.—LONGITUDINAL SECTION OF WEIR.

During the first experiments, Series *WA* to *WH*, uplift pressure caused the floor to pull away from the joists, forming a slightly convex surface instead of a plane. Thereafter, matched flooring attached to a welded steel frame ensured a plane surface under all conditions.

For all experiments except Series *WA*, *WB*, and *WC*, a 3 by 6-in. elliptical laminated block was bolted to the up-stream apron to give a rounded entrance.

Test Procedure

An outline of conditions to which the experiments conform is given in Table 2. In making each run, the profile of water surface over the weir was determined from observations 2 ft. or less apart. At the time of taking readings along the center line of the channel, simultaneous readings were made for the fixed gauge and the V-notch-weir gauge. Six readings were taken across the apex of the weir while simultaneous readings were being taken at the fixed gauge. While these observations were being made, as many readings as possible of the V-notch-weir gauge were taken. (See Table 3.) One reader attended each of the three gauges, and one man acted as recorder for the fixed gauge and the movable gauge.

Analysis of Results

The results of the investigation will be discussed under several heads.

Location of Critical Depth.—A profile of water surface was plotted for each run. Usually 6 to 8 runs (constituting a series of experiments) were conducted for each setting of the aprons. In all there were 37 series. The point of critical depth was determined from the formula, $Q = \sqrt{g}\ L\ D_c^{\frac{3}{2}}$, or, for length of weir, $L = 2$, $D_c = 0.1981\ Q^{\frac{2}{3}}$. This depth was plotted on each profile.

Profiles for each series of experiments were plotted on a single sheet. Typical examples are shown in Fig. 8. Five sets of experiments containing, respectively, 8, 10, 9, 6, and 4 series were performed. In each set the slope of the down-stream apron was kept constant and that of the up-stream apron was varied. Profiles of each set of experiments plotted to a larger scale on a

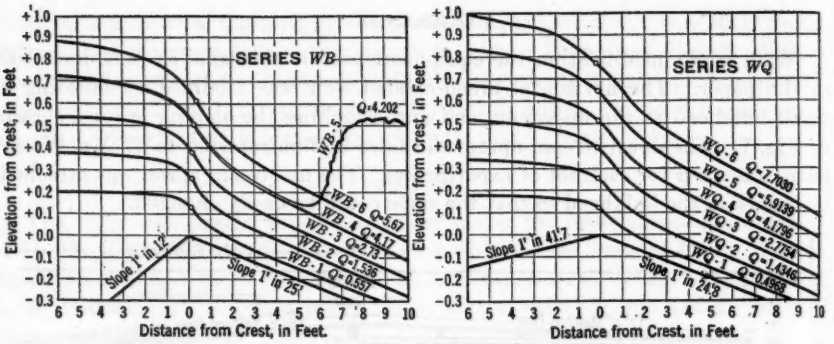

FIG. 8.—EXAMPLES OF WATER PROFILES, SHOWING CRITICAL DEPTHS.

single diagram and showing water surfaces 1.5 ft. up stream and down stream from the apex (Fig. 9), indicate that while there are individual variations, in general the points of critical depth lie on well-defined curves. These curves plotted to a greatly distorted scale are shown in Fig. 10.

TABLE 2.—OUTLINE OF EXPERIMENTS.

(Series *WA*, *WB*, and *WC* were made without the use of the block for rounded entrance. The wooden frame of the up-stream apron was changed to a welded steel frame after Series *WH*.)

Series.	Number of runs.	Up-stream slope.	Down-stream slope.
W A	6	1 in 7.4	1 in 25
W B	6	1 in 12	1 in 25
W C	3	1 in 46.8	1 in 25
W D	6	1 in 46.8	1 in 25
W E	13	1 in 31.5	1 in 25
W F	12	1 in 26.2	1 in 25
W G	3	1 in 22.2	1 in 25
W H	5	1 in 30 to 32	1 in 25
W I	15	1 in 25.6	1 in 24.8
W J	16	1 in 24.0	1 in 24.8
W K	7	1 in 24.45	1 in 24.8
W L	9	1 in 27.1	1 in 24.8
W M	5	1 in 30.3	1 in 24.8
W N	6	1 in 33	1 in 24.8
W O	5	1 in 34.8	1 in 24.8
W P	5	1 in 39.8	1 in 24.8
W Q	6	1 in 41.7	1 in 24.8
W R	6	1 in 50.2	1 in 24.8
W A A	8	1 in 40 3	1 in 25.5
W A B	5	1 in 37.6	1 in 25.5
W A C	6	1 in 42.3	1 in 25.5
W A D	6	1 in 47.1	1 in 25.5
W A E	6	1 in 34.1	1 in 25.5
W A F	6	1 in 29.8	1 in 25.5
W A G	6	1 in 40.0	1 in 25.5
W A H	6	1 in 41.3	1 in 25.5
W A I	6	1 in 40.5	1 in 25.5
B A	5	1 in 40 9	1 in 46.4
B B	6	1 in 50.0	1 in 46.4
B C	5	1 in 65.1	1 in 46.4
B D	4	1 in 80.5	1 in 46.4
B E	6	1 in 89.55	1 in 46.4
B F	6	1 in 98.5	1 in 46.4
C A	4	1 in 40.2	1 in 15.37
C B	8	1 in 30.15	1 in 15.37
C C	5	1 in 19.95	1 in 15.37
C D	4	1 in 24.99	1 in 15.37

TABLE 3.—SERIES *WP*-5: GAUGE READINGS AND HEADS FOR DETERMINING DISCHARGE, CRITICAL DEPTH, AND PROFILE OF WATER SURFACE.

V-Notch Weir.		Fixed Gauge.		Point on weir.	Movable Gauge.				Elevation above crest, in feet.
					Observed Readings, in Feet.				
Observed readings, in feet.	Head, in feet.	Observed readings, in feet.	Head, in feet.		Left.	Center.	Right.	Average.	
1.157	1.102	3.796	0.650	10	.....	0.762	.....	.....	—0.172
1.157	1.102	3.797	0.651	8	.....	0.856	.....	.....	—0.078
1.157	1.102	3.796	0.650	6	.....	0.956	.....	.....	+0.022
1.156	1.101	3.796	0.650	4	.....	1.041	.....	.....	0.107
1.158	1.103	3.796	0.650	3	.....	1.096	.....	.....	0.162
1.158	1.103	3.797	0.651	2	1.152	1.154	1.158	1.155	0.221
1.159	1.104	3.798	0.652	1.5	1.189	1.189	1.198	1.192	0.258
1.158	1.103	3.798	0.652	1.0	1.239	1.235	1.242	1.239	0.305
1.158	1.103	3.797	0.651	0.5	1.294	1.293	1.296	1.294	0.360
1.1581*	1.1036	3.7972	0.6517	{ 0.0	1.353	1.354	1.352 }	1.3527	0.4183
				{ 0 0	1.353	1.352	1.352 }		
1.158	1.103	3.796	0.650	0.5	1.393	1.395	1.394	1.394	0.460
1.158	1.103	3.797	0.651	1.0	1.416	1.417	1.421	1.418	0 484
1.156	1.101	3.795	0.649	1.5	1.435	1.435	1.440	1.437	0.503
1.156	1.101	3.795	0.649	2	1.448	1.447	1.450	1.448	0.514
1.155	1.100	3.796	0.650	3	.....	1.468	.....	.....	0.534
1.154	1.099	3.796	0.650	4	.....	1.482	.....	.....	0.548
1.155	1.100	3.795	0.649	5	.....	1.494	.....	.....	0.560
1.156	1.101	3.797	0.651	6	.....	1.505	.....	.....	0.571
Average..	1.1025	.....	0.6509	....	.....	.....	.....	.....	.....

*Average of 12 readings.

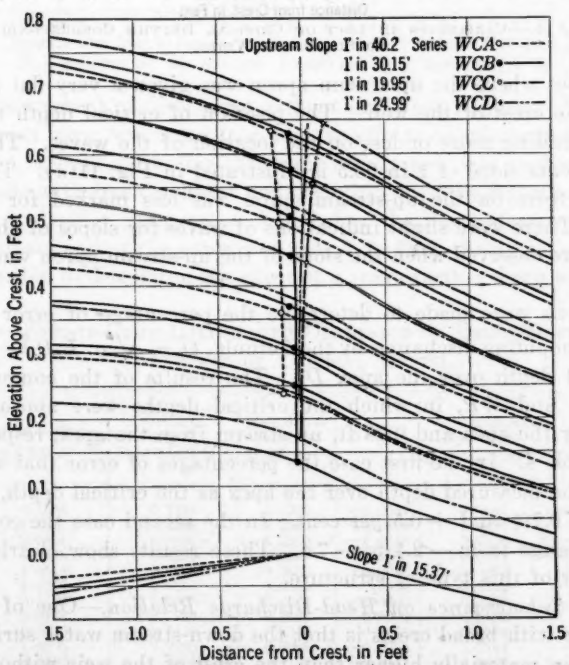

FIG. 9.—ENLARGED PROFILES, SHOWING CRITICAL DEPTHS.

In one series of experiments, where the slope of the down-stream apron was 1 in 25.5 and that of the up-stream apron, 1 in 40.3, the points of critical depth for all discharges fell on a vertical straight line over the apex. This condition was not duplicated for any other combination of slopes. With down-stream and up-stream slopes, respectively, 1 in 24.8 and 1 in 39.8, critical depths fell in a vertical line, 0.08 ft. up stream from the apex. Experiments on a much steeper down-stream slope (1 in 15.4) indicate that critical depths would lie in a vertical line about 0.04 ft. up stream from the apex when the up-stream slope was about 1 in 28.

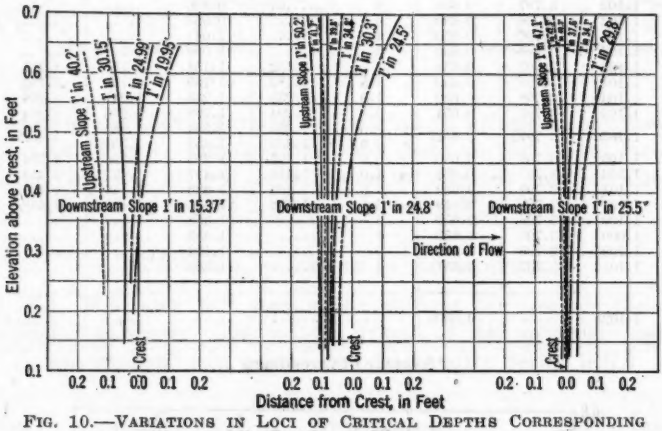

FIG. 10.—VARIATIONS IN LOCI OF CRITICAL DEPTHS CORRESPONDING TO SLOPES OF WEIRS.

In all cases where the up-stream apron was given a very flat slope, waves formed on the crest of the weir. The position of critical depth then became variable, depending more or less on the location of the waves. The condition for an up-stream slope of 1 in 98.5 is illustrated in Fig. 11(*a*). The tendency of waves to form on the up-stream apron was less marked for the smaller discharges. There were slight indications of waves for slopes of about 1 in 50. No waves were observed when the slope of the up-stream apron was 1 in 40, or less.

Calculations were made to determine the percentage of error that would result from computing discharges by the formula, $Q = \sqrt{g}\ L\ D_c^{\frac{3}{2}}$, substituting the measured depth over the apex D_c. The results of the computations for Series *WAA* and *WP*, in which the critical depths were approximately in verticals, over the apex and 0.08 ft. up stream from the apex, respectively, are shown in Table 4. In the first case the percentages of error that would result from using the measured depth over the apex as the critical depth, lie between the limits, — 0.7% and + 0.5 per cent. In the second case the corresponding percentages range from — 2.1 to — 7.4. These results show clearly the sensitive character of this type of structure.

Effect of Submergence on Head-Discharge Relation.—One of the advantages of weirs with broad crests is that the down-stream water surface may be at an elevation materially higher than the crest of the weir without affecting the relation between head and discharge. Weirs having this characteristic per-

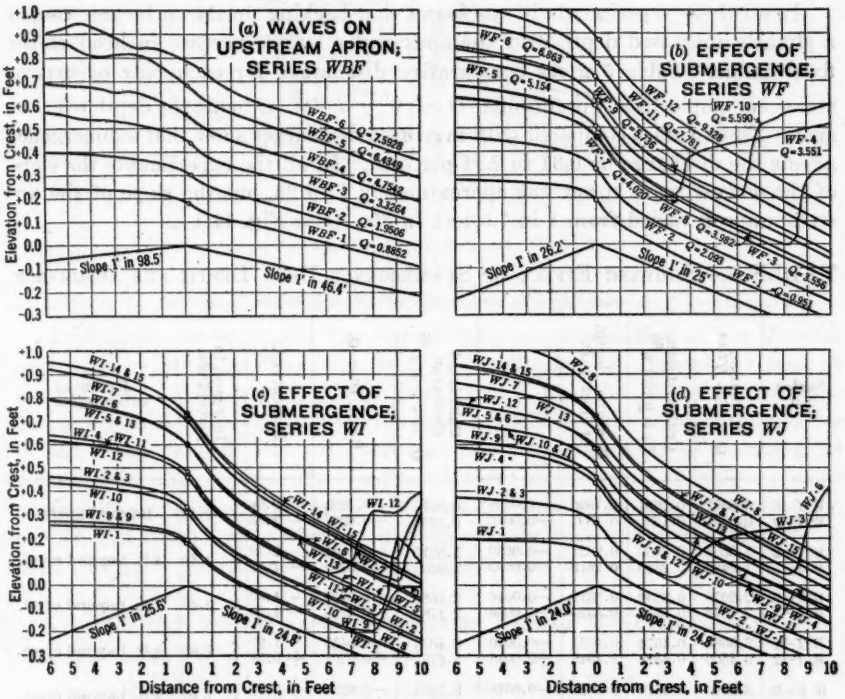
(a) WAVES ON UPSTREAM APRON; SERIES WBF
(b) EFFECT OF SUBMERGENCE; SERIES WF
(c) EFFECT OF SUBMERGENCE; SERIES WI
(d) EFFECT OF SUBMERGENCE; SERIES WJ
Elevation from Crest, in Feet
Distance from Crest, in Feet
Slope 1' in 98.5
Slope 1' in 46.4'
Slope 1' in 26.2'
Slope 1' in 25'
Slope 1' in 25.6'
Slope 1' in 24.8'
Slope 1' in 24.9'

FIG. 11.—EXAMPLES OF WATER PROFILES AND INFLUENCE OF WAVES.

mit the measurement of discharges without great loss of head. Fourteen pairs of experiments were conducted to study the effects of submergence. In so far as practicable the discharge for both runs of any pair was kept constant. One run was made with free discharge and the other with the water backed up so as to cause a hydraulic jump on or just below the down-stream apron. Since the down-stream end of the apron was only 2 ft. from the outlet of the flume, it was not possible to determine the effect of a jump farther down stream.

TABLE 4.—EFFECT UPON CALCULATED DISCHARGE OF USING DEPTH OVER APEX INSTEAD OF CRITICAL DEPTH.

Series No.	Discharge, $Q = 2.505\ H^{2.48}$.	Critical depth, $D_c = 0.1981\ Q^{2/3}$.	Measured depth at crest = D.	$D - D_c$.	Calculated discharge, $Q' = \sqrt{g}\ L\ D^{3/2}$.	Error, $Q' - Q$.	Percentage.
W A A-3	0.4984	0.1246	0.1250	+0.0004	0.5006	+0.0022	+0.44
W A A-4	1.4377	0.2523	0.2515	−0.0008	1.4300	−0.0077	−0.53
W A A-5	2.7604	0.3898	0.3880	−0.0018	2.7410	−0.0194	−0.70
W A A-1	3.1153	0.4226	0.4231	+0.0005	3.1211	+0.0065	+0.21
W A A-6	4.0544	0.5038	0.5055	+0.0017	4.0750	+0.0206	+0.51
W A A-7	5.8270	0.6416	0.6405	−0.0010	5.8129	−0.0141	−0.24
W A A-2	6.1185	0.6628	0.6626	−0.0002	6.1157	−0.0028	−0.05
W A A-8	8.0123	0.7933	0.7933	0.0000	8.0123	0.0000	0.00
W P-5	0.6040	0.1416	0.1346	−0.0070	0.5594	−0.0446	−7.40
W P-4	1.8595	0.2995	0.2906	−0.0089	1.7764	−0.0831	−4.47
W P-3	3.1982	0.4300	0.4183	−0.0117	3.0678	−0.1304	−4.09
W P-2	5.1040	0.5873	0.5784	−0.0089	4.9875	−0.1165	−2.28
W P-1	7.5400	0.7619	0.7509	−0.0110	7.3788	−0.1612	−2.14

In all these experiments it was found that backing up the tail-water caused a slightly increased depth over the apex and a slight increase in head at the fixed gauge. Table 5 gives comparative discharges for each pair of experiments computed from the formula, $Q = \sqrt{g}\ L\ D^{\frac{3}{2}}$, D being the depth over the apex. These values compared with measured discharges show that submergence affects the discharge by 0.31 to 2.81 per cent. In all the experiments the slope of the down-stream apron was approximately 1 in 25, but the slope of the up-stream apron varied from 1 in 7.5 to 1 in 30. (See Fig. 11.)

TABLE 5.—Showing Effect of Submergence Upon Depth and Discharge.

Series No.	Discharge, $Q = 2.505\ H^{2.48}$.	Critical depth, $D_c = 0.1961\ Q^{\frac{2}{3}}$.	Measured depth, at crest $= D$.	$D - D_c$.	Calculated discharge, $Q' = \sqrt{g}\ L\ D^{\frac{3}{2}}$.	Error, $Q' - Q$.	Percentage.	Increase, in percentage.	Extent of submergence.
W I- 8	0.9657	0.1934	0.1896	−0.0038	0.9372	−0.0285	− 2.96	−0.12	Beyond apron.
W I- 9	0.9633	0.1931	0.1891	−0.0040	0.9336	−0.0297	− 3.08		
W J- 2	1.5558	0.2660	0.2621	−0.0039	1.5217	−0.0341	− 2.19	0.47	4 ft. beyond crest.
W J- 3	1.5465	0.2649	0.2619	−0.0030	1.5200	−0.0265	− 1.72		
W I- 2	2.2078	0.3359	0.3319	−0.0040	2.1682	−0.0396	− 1.80	0.45	7 ft. beyond crest.
W I- 3	2.2078	0.3359	0.3329	−0.0030	2.1780	−0.0298	− 1.35		
W I-11	3.5088	0.4574	0.4534	−0.0040	3.4633	−0.0455	− 1.29	1.87	8 ft. beyond crest.
W I-12	3.4566	0.4528	0.4546	+0.0018	3.4767	+0.0201	+ 0.58		
W F- 3	3.5556	0.4615	0.4597	−0.0018	3.5335	−0.0221	− 0.62	1.86	3 ft. beyond crest.
W F- 4	3.5508	0.4611	0.4649	+0.0038	3.5944	+0.0436	+ 1.24		
W F- 7	4.0195	0.5009	0.5014	+0.0005	4.0244	+0.0049	+ 0.12	1.85	8 ft. beyond crest.
W F- 8	3.9820	0.4978	0.5035	+0.0057	4.0511	+0.0691	+ 1.73		
W A- 4	3.9266	0.4931	0.5480	+0.0549	4.6000	+0.6734	+17.14	1.58	4 ft. beyond crest.
W A- 5	4.0102	0.5001	0.5607	+0.0606	4.7613	+0.7513	+18.72		
W B- 4	4.1699	0.5134	0.5422	+0.0288	4.5263	+0.3564	+ 8.55	1.28	7 ft. beyond crest.
W B- 5	4.2021	0.5160	0.5492	+0.0332	4.6150	+0.4129	+ 9.83		
W J-10	4.2434	0.5194	0.5121	−0.0073	4.1550	−0.0884	− 2.08	0.99	Beyond apron.
W J-11	4.2224	0.5177	0.5139	−0.0048	4.1763	−0.0461	− 1.09		
W J- 5	5.0350	0.5821	0.5771	−0.0050	4.9713	−0.0637	− 1.27	1.54	9 ft. beyond crest.
W J- 6	5.0340	0.5820	0.5831	+0.0011	5.0475	+0.0135	+ 0.27		
W I- 5	5.2280	0.5967	0.5961	−0.0006	5.2200	−0.0080	− 0.15	0.62	10 ft. beyond crest.
W I- 6	5.2690	0.5999	0.6018	+0.0019	5.2938	+0.0248	+ 0.47		
W F- 9	5.7360	0.6349	0.6330	−0.0019	5.7113	−0.0247	− 0.43	2.81	6 ft. beyond crest.
W F-10	5.5900	0.6241	0.6339	+0.0091	5.7229	+0.1329	+ 2.38		
W J-14	6.9138	0.7190	0.7156	−0.0034	6.8643	−0.0495	− 0.72	0.31	Beyond apron.
W J-15	6.9842	0.7238	0.7219	−0.0019	6.9557	−0.0285	− 0.41		
W I-14	7.0235	0.7266	0.7204	−0.0062	6.9343	−0.0892	− 1.27	1.48	Beyond apron.
W I-15	7.0168	0.7261	0.7271	+0.0010	7.0314	+0.0146	+ 0.21		

The Variation of Weir Coefficient with Head and Slope

Values of the coefficient, C, in the weir formula, $Q = C\ L\ H_a^{\frac{3}{2}}$, show a wide variation for comparatively small changes in discharges and in slopes of aprons. This variation is indicated quite clearly in Fig. 12. In determining Q and H, gauge readings for a complete run were averaged.

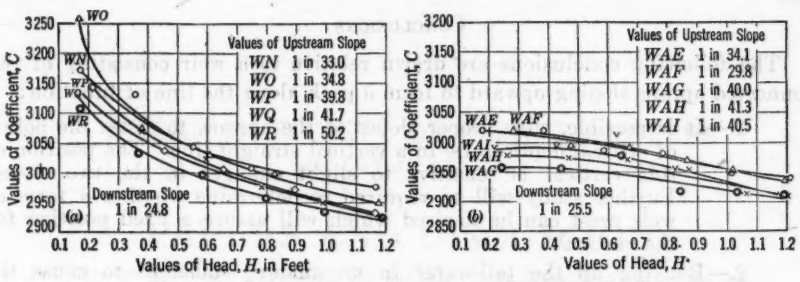

FIG. 12.—VARIATION IN VALUES OF COEFFICIENT, C.

CHECKING V-NOTCH WEIR FORMULA

Preliminary to beginning the experiments, an examination of the V-notch weir used by Mr. Woodburn for determining discharges, showed the face of the weir to be quite heavily coated with mineral deposits. A series of experiments was made with the weir in this coated condition and another series after it had been cleaned. The quantity of flow was obtained by weighing as described in the paper. The results of these experiments (Table 6) show average discrepancies with Mr. Woodburn's formula before and after cleaning, of 0.44 and 0.33%, respectively. In view of this close agreement Mr. Woodburn's formula was accepted as being substantially correct.

TABLE 6.—EFFECT OF DEPOSITS ON FACE OF V-NOTCH WEIR.

Head on V-notch weir, in feet.	Discharge obtained by weighing.	Discharge computed by formula, $Q = 2.505\ H^{2.48}$.	Percentage in error.
	BEFORE CLEANING.		
0.83156	1.5664	1.5862	+1.26
0.93995	2.1256	2.1488	+1.10
1.05436	2.8309	2.8570	+0.92
1.15504	3.5636	3.5808	+0.48
1.15995	3.6118	3.6197	+0.22
1.16639	3.6546	3.6693	+0.40
1.26225	4.4590	4.4634	+0.10
1.39642	5.7543	5.7342	−0.35
1.39914	5.7743	5.7617	−0.21
Average.....	.....	.....	+0.44
	AFTER CLEANING.		
1.15604	3.5688	3.5893	+0.54
1.16309	3.6197	3.6426	+0.64
1.23025	4.1533	4.1892	+0.87
1.22749	4.1545	4.1657	+0.15
1.29310	4.7165	4.7390	+0.48
1.29335	4.7234	4.7411	+0.38
1.39090	5.6864	5.6779	−0.15
1.39220	5.7041	5.6910	−0.24
Average.....	.....	.....	+0.33

Conclusions

The following conclusions are drawn relative to a weir consisting of two connected aprons sloping upward to form a peak along the line of junction:

1.—It is possible, with proper slopes of the aprons, to cause the points of critical depth to lie in a vertical straight line. The position of this vertical is sensitive to slight changes in the two slopes. Further study will be required to determine whether a form of weir crest can be devised which will assure a fixed position for the vertical.

2.—Backing up the tail-water in an amount sufficient to cause the hydraulic jump to occur upon the apron increases the depth of water on the weir.

3.—Values of the coefficient, C, in the weir formula, $Q = C L H_a^{\frac{3}{2}}$, are very sensitive to small changes in the slopes of the aprons.

Acknowledgments

The experiments were performed under the direction of H. W. King, M. Am. Soc. C. E. Harry Larson, Assoc. M. Am. Soc. C. E., collaborated in conducting the experiments and in computations. The following students in the College of Engineering, University of Michigan, assisted in the laboratory, Messrs. E. V. Schulhauser, V. K. Ipe, M. Mortenson, D. W. Richardson, I. Curtis, F. F. Smalla, and C. O. Bloom.

During the past year (1929-30) the Society's Special Committee on Irrigation Hydraulics has co-operated in the experiments. In particular, D. C. Henny, and J. C. Stevens, Members, Am. Soc. C. E., Chairman and Secretary, respectively, of the Committee have made valuable suggestions.

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STRESSES IN GRAVITY DAMS BY PRINCIPLE OF LEAST WORK

By B. F. Jakobsen,* M. Am. Soc. C. E.

Synopsis

In order to determine the stresses in gravity dams, it has been customary to assume a linear stress distribution on horizontal planes. An examination of English tests on model dams shows a non-linear stress distribution. A theoretical investigation, in which the Principle of Least Work was substituted for the usual assumption of a linear stress distribution, leads to a stress curve similar to that found in the English tests.

The bending stresses, that is, the vertical normal stresses due to the water load, found by the usual method, are equal at the up-stream and down-stream faces, while the proposed theory gives a considerably higher bending stress at the up-stream face and a correspondingly lower stress at the down-stream face.

The maximum stress is found to be less than by the usual method, but this is of no particular advantage, since the compressive stress in the up-stream face with reservoir full should exceed a definite minimum, and this will generally be the determining factor in the design.

The writer believes that the stress distribution based on the Principle of Least Work will more nearly approach the true load stresses than the results obtained by the usual method.

I.—Introduction

The section of the straight gravity dam to be considered is triangular with a vertical up-stream face. It is assumed, that the concrete is homogeneous and isotropic in every respect, that is, that each unit of volume weighs exactly as much as any other; that its temperature, moisture content, and general conditions do not vary from point to point; and that the modulus of elasticity is the same for compression as for tension and is the same at all points of the dam. It is also assumed that there is no uplift and no residual stress. The

Note.—Written discussion on this paper will be closed in **January, 1931**, *Proceedings*.

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stresses to be considered are those due to the load imposed by the weight of the structure itself and by the horizontal thrust of the water acting on the up-stream face. It is further assumed, that the deformations are elastic and infinitesimal, so that the shape of the structure is not altered when the load is applied. These are the usual assumptions made when dealing with engineering structures.

Evidently none of these assumptions is rigorously true. Some of them, as for example, those regarding uplift and residual stress, may be so far wrong that for purposes of engineering design they must be separately considered in order to obtain the actual stresses. Föppl has shown,* that the load stresses are independent of the residual stresses, when the usual assumptions are made. Regarding plastic deformations (time effect), the tests made by R. E. Davis, M. Am. Soc. C. E., indicate† that these increase linearly with the stress and therefore are not likely to influence the stress distribution. They may be taken into account by assuming a lower modulus of elasticity.

II.—Usual Theory—Stresses

Referring to Fig. 1, let:

n_y and n_x = vertical and horizontal normal stresses at any point;
t = shear or tangential stress in the same point;
w = weight of concrete, in pounds per cubic foot;
g = weight of 1 cu. ft. of water, in pounds;
M = moment, in foot-pounds, about a point on the Y-axis, y ft. below the crest;
W = weight, in pounds, of a 1-ft. thick slice of dam, y ft. high;
b = base width, in feet, y ft. below the crest;
ϕ = angle at crest between up-stream and down-stream faces; and,

$$k = \tan \phi = \frac{b}{y}.$$

Then,

$$W = 0.5\, w\, b\, y = 0.5\, w\, k\, y^2 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and,

$$M = 0.5\, k\, y^2 \frac{b}{3} + \frac{g}{6} y^3 = (g + w\, k^2) \frac{y^3}{6} \ldots\ldots\ldots\ldots (2)$$

Referring to Fig. 2, take tension as positive and the sign for shear such that on the plane on which the positive direction of a normal stress is in the positive direction of the axis to which it is parallel, the positive direction of shear will also be in the positive direction of the axis to which it is parallel; thus, the horizontal shear in a gravity dam will have the negative sign.

It is assumed that there are no stresses acting at right angles to the section, that is, normal to the plane of the paper in Fig. 1. The only stresses which will be considered are n_y, n_x, and t. This implies, that the vertical limiting planes of the section are free to deform in accordance with these stresses. (See Appendix III.)

* "Drang und Zwang," Vol. 2, Article 91 (1920 Edition).

† *Proceedings*, Am. Soc. C. E., May, 1928, Pt. 3, Table 28, p. 213. When plotted, these tests give a straight line between deformation and stress.

The statical conditions require that,

$$\int_0^b n_y \, dx = -W = -0.5 \, w \, k \, y^2 \quad \ldots\ldots (3)$$

$$\int_0^b n_y \, x \, dx = -M = -(g + w \, k^2) \frac{y^3}{6} \quad \ldots\ldots (4)$$

and, also, that,

$$\int_0^b t \, dx = -S = -0.5 \, g \, y^2 \quad \ldots\ldots (5)$$

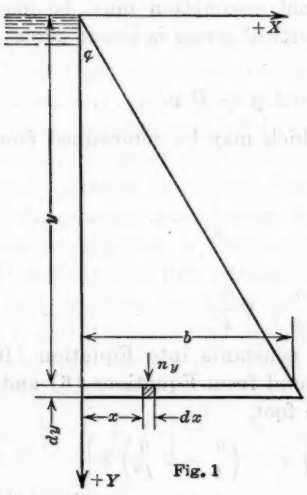

Fig. 1

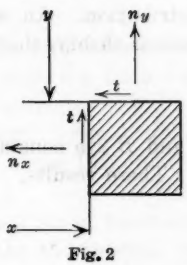

Fig. 2

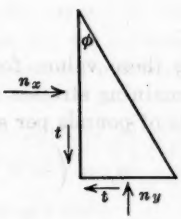

Fig. 3

Equation (5) is not an independent condition, since for transverse loading, $S = \frac{d\,M}{d\,y}$, which checks, inasmuch as the influence of w on the bending moment must be excluded. The bending produced by vertical forces gives rise to horizontal shear stresses, which, however, must total zero across the entire section. The equilibrium equations for any small particle, $d\,y$, $d\,x$, $d\,z$, in a point (y, x), are:

$$\frac{d\,n_y}{d\,y} + \frac{d\,t}{d\,x} + w = 0 \quad \ldots\ldots (6)$$

$$\frac{d\,n_x}{d\,x} + \frac{d\,t}{d\,y} = 0 \quad \ldots\ldots (7)$$

in which, $\frac{d\,n_y}{d\,y}$, etc., are the partial differential coefficients, usually written $\frac{\partial\,n_y}{\partial\,y}$. The boundary conditions are, at the up-stream face, $t = 0$ and $n_x = -g\,y$. At the down-stream face (Fig. 3), it must be,

$$t = n_y \tan \phi \quad \ldots\ldots (8)$$

and,

so that,

$$\left.\begin{aligned} n_x &= t \tan \phi = n_y \tan^2 \phi \\ n_y\, n_x &= t^2 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Equations (8) and (9) follow from the fact that the forces acting on the small triangle must be in equilibrium. The weight of the triangle is not involved, since this is infinitely small compared to the forces.

These conditions, that is, Equations (3) to (7), inclusive, and the stress relations at the up-stream and down-stream faces, do not suffice to determine the stress distribution. An additional assumption must be made; and this is, in the classical theory, that the vertical stress is linear across a horizontal joint, that is,

$$n_y = A\, y + B\, x \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

in which, A and B are constants, which may be determined from Equations (3) and (4). There results,

$$A = -w + \frac{g}{k^2} \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

and,

$$B = \frac{w}{k} - 2\frac{g}{k^3} \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Introducing these values for the constants into Equation (10) gives n_y; and the two remaining stresses are found from Equations (6) and (7). There results, in units of pounds per square foot,

$$\left.\begin{aligned} n_y &= \left(-w + \frac{g}{k^2}\right) y + \left(\frac{w}{k} - 2\frac{g}{k^3}\right) x \\ t &= -\frac{g}{k^2}\, x \text{ and } n_x = -g\, y \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

These are the usual linear equations found when the classical assumption, that is, Equation (10), is made. In textbooks the vertical stress is generally given only for the up-stream and down-stream faces, that is, for $x = 0$ and $x = b$, these being the two most important values.* For the purpose in hand, the stresses are expressed as functions of the co-ordinates, y and x.

III.—Usual Theory—Deformations

The usual assumption, that n_y is a linear function of x, is equivalent to stating that horizontal planes remain plane, provided the modulus of elasticity is constant and that the shear deformation is neglected. Fredrik Vogt, Assoc. M. Am. Soc. C. E., has shown,† that the bending moment produces an **S**-shaped curving of the foundation; therefore, the classical assumption is incorrect, at least for sections near the foundation. The tests made at the Stevenson Creek Test Dam show a remarkably close agreement between the actual displacements of the foundation and those computed from Dr. Vogt's formulas.‡

* See "Notes on Arched Gravity Dams," *Transactions*, Am. Soc. C. E., Vol. 92 (1928), Equation (14), p. 771. Note that for a straight dam, $R_x = \infty$.

† "Ueber die Berechnung der Fundamentdeformation," Det Norske Videnskaps Akademi, Oslo, 1925.

‡ *Proceedings*, Am. Soc. C. E., May, 1928, Pt. 3, p. 127, and Fig. 82.

That the usual assumption was probably incorrect near the foundation has been accepted for many years by authorities on dam design.*

IV.—Stress Distribution by the Principle of Least Work

This principle was first applied to engineering structures by Castigliano in his work† published in 1879. It was introduced into American technical literature by William Cain, M. Am. Soc. C. E., in a paper entitled "Determination of the Stresses in Elastic Systems by the Method of Least Work"‡ and it was applied by him to obtain a correct and elegant solution of the problem of stresses in arches of dams in his well-known paper, "The Circular Arch under Normal Loads."§

The writer has shown‖ that the arch stresses found from the tests on the Stevenson Creek Dam agree remarkably well with the Cain formulas, when the deformations and displacements of the abutments are included in the calculations in the manner suggested by Dr. Vogt. (See Article III.) These tests are, therefore, an experimental verification of the Principle of Least Work as applied to the stresses in an arch dam.

In lieu of the classical assumption, that n_y is a linear function of x, assume that the stress distribution is such as will make the internal work the least possible.¶ It is clear that this assumption is less restrictive than the classical assumption and includes the latter as a specific case. If, for example, the shear stress, t, and the horizontal normal stress, n_x, are assumed as negligible, the principle of least work requires that the stress, n_y, must be linear.

The internal work is,**

$$L = \frac{1}{2\,G} \int\int\int \left[\frac{{n_y}^2 + {n_x}^2}{2} - \frac{(n_y + n_x)^2}{2\,(m+1)} + t^2 \right] d\,y\,d\,x\,d\,z \ldots . (14)$$

in which, G is the shear modulus and $\frac{1}{m}$ is Poisson's ratio.

The shear modulus is defined by,

$$G = \frac{m\,E}{2\,(m+1)}$$

in which, E is Young's modulus of elasticity. Equation (14) implies that the modulus of elasticity is constant and the same for tension as for compression, which is believed to be true for ordinary working stresses.††

Instead of the simple Equation (10) for n_y, the condition that Equation (14) must be a minimum is now introduced. This is a problem in variational calculus, but instead of treating it as such, which would be very complicated,

* Professor Cain calls attention to this in the opening sentence of his paper, "Stresses in Masonry Dams," *Transactions*, Am. Soc. C. E., Vol. LXIV (1909), p. 208; and Professor Résal, in his paper, "Formes et Dimensions des grands barrages en maçonnerie" in *Annales* des Ponts et Chaussées, II, 1919, deals at some length with this subject.

† "Théorie de l'equilibre des systèmes élastiques"; this book is out of print, but there is an English translation available.

‡ *Transactions*, Am. Soc. C. E., Vol. XXIV (1891), p. 265.

§ *Loc. cit.*, Vol. LXXXV (1922), p. 233.

‖ *Proceedings*, Am. Soc. C. E., May, 1929, Papers and Discussions, p. 1241.

¶ See Föppl, "Drang und Zwang," Vol. I, 1924, Paragraphs 13, 14, and 55.

** *Loc. cit.*, p. 336.

†† *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 887.

a method of approximation may be resorted to, by assuming n_y as a function of y and x. This method, due to Ritz,* transforms a problem in variational calculus into a simple problem of differentiation. The procedure is as follows: The stress, n_y, is assumed with more constants than are needed to satisfy Equations (3) and (4); the equilibrium equations, (6) and (7), serve to determine n_x and t as functions of y and x and of the constants in the equation for n_y. The condition that Equation (14) must be a minimum then serves to determine the free constants in the equation for n_y. Assume,

$$n_y = A\,y + B\,x + C\,\frac{x^2}{y} + F\,\frac{x^3}{y^2} \ldots\ldots\ldots\ldots\ldots (15)$$

in which, A, B, C, and F are four constants, any two of which, for example, A and B, may be expressed as functions of C and F by means of Equations (3) and (4), by carrying out the simple integrations indicated. There results,

$$A = -w + \frac{g}{k^2} + C\,\frac{k^2}{6} + F\,\frac{k^3}{5} \ldots\ldots\ldots\ldots\ldots (16)$$

and,

$$B = \frac{w}{k} - 2\,\frac{g}{k^3} - C\,k - 0.9\,F\,k^2 \ldots\ldots\ldots\ldots\ldots (17)$$

The two independent constants, C and F, must now be determined so that the internal work is a minimum, or,

$$\frac{d\,L}{d\,C} = 0;\ \text{and}\ \frac{d\,L}{d\,F} = 0 \ldots\ldots\ldots\ldots\ldots (18)\dagger$$

It will simplify the calculations to differentiate under the integral sign, instead of first integrating and then differentiating. There results,

$$\frac{1}{2\,G}\int_0^1 d\,z \int_0^H d\,y \int_0^b d\,x \left[\frac{m}{m+1}\left(n_y\frac{d\,n_y}{d\,C} + n_x\frac{d\,n_x}{d\,C}\right) - \frac{1}{m+1}\left(n_y\frac{d\,n_x}{d\,C} + n_x\frac{d\,n_y}{d\,C}\right) + 2\,t\,\frac{d\,t}{d\,C}\right] = 0 \ldots\ldots\ldots (19)$$

in which, H is the total height of the section considered.

Likewise,

$$\frac{1}{2\,G}\int_0^1 d\,z \int_0^H d\,y \int_0^b d\,x \left[\frac{m}{m+1}\left(n_y\frac{d\,n_y}{d\,F} + n_x\frac{d\,n_x}{d\,F}\right) - \frac{1}{m+1}\left(n_y\frac{d\,n_x}{d\,F} + n_x\frac{d\,n_y}{d\,F}\right) + 2\,t\,\frac{d\,t}{d\,F}\right] = 0 \ldots\ldots\ldots (20)$$

It is now necessary to express t and n_x as functions of the co-ordinates, y and x, and of the constants, A, B, C, and F. Differentiating Equation (15) toward y and introducing the result into Equation (6),

$$A - C\,\frac{x^2}{y^2} - 2\,F\,\frac{x^3}{y^3} + \frac{d\,t}{d\,x} + w = 0$$

* *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 325. The writer used this method to determine the stresses in the buttress of a multiple-arch dam.

† $\frac{d\,L}{d\,C}$ and $\frac{d\,L}{d\,F}$ indicate the partial differential coefficients usually written $\frac{\delta\,L}{\delta\,C}$ and $\frac{\delta\,L}{\delta\,F}$. In this paper, d has been used for δ as there is no danger of misunderstanding.

Integrating and bearing in mind that, for $x = 0$ (that is, at the up-stream face), $t = 0$,

$$t = -A\,x + \frac{C}{3}\frac{x^3}{y^2} + \frac{F}{2}\frac{x^4}{y^3} - w\,x \ldots\ldots\ldots\ldots (21)$$

Differentiating Equation (21) toward y and introducing this into Equation (7),

$$\frac{d\,n_x}{d\,x} = -\left(-\frac{2}{3}\,C\,\frac{x^3}{y^3} - \frac{3}{2}\,F\,\frac{x^4}{y^4}\right)$$

Integrating toward x, remembering that the constant of integration, $f(y)$, is a function of y, but not of x, and that for $x = 0$, $n_x = -g\,y$, or $f(y) = -g\,y$, there results,

$$n_x = \frac{C}{6}\,\frac{x^4}{y^3} + 0.3\,F\,\frac{x^5}{y^4} - g\,y \ldots\ldots\ldots\ldots\ldots (22)$$

The equations for t and n_x must also satisfy Equations (8) and (9) at the down-stream face; that is, for $x = b = k\,y$, $t = k\,n_y$, and $n_x = k\,t$.

By introducing the values of n_y, t, and n_x, from Equations (15), (21), and (22), into Equations (8) and (9), and eliminating the two constants, A and B, by means of Equations (16) and (17), the desired check is obtained. Equations (15), (21), and (22) for the stresses, n_y, t, and n_x, therefore, satisfy the statical requirements (Equations (3) and (4)), the equilibrium equations, (6) and (7), the boundary conditions at the up-stream face (that is, $t = 0$ and $n_x = -g\,y$), and also the boundary conditions at the down-stream face (Equations (8) and (9)). This means that Equations (15), (21), and (22) represent a possible stress distribution, no matter what arbitrary finite values, including zero, are assigned to the two constants, C and F.

The differentials required for Equations (19) and (20) may now be formed, bearing in mind that the constants, A and B, are functions of C and F as given by Equations (16) and (17), so that,

$$\frac{d\,A}{d\,C} = \frac{k^2}{6};\ \frac{d\,B}{d\,C} = -k;\ \frac{d\,A}{d\,F} = \frac{k^3}{5};\ \frac{d\,B}{d\,F} = -0.9\,k^2$$

Then,

$$\frac{d\,n_y}{d\,C} = \frac{k^2}{6}\,y - k\,x + \frac{x^2}{y}$$

$$\frac{d\,n_x}{d\,C} = \frac{x^4}{6\,y^3}$$

$$\frac{d\,t}{d\,C} = -k^2\,\frac{x}{6} + \frac{x^3}{3\,y^2} \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

Likewise,

$$\frac{d\,n_y}{d\,F} = \frac{k^3}{5}\,y - 0.9\,k^2\,x + \frac{x^3}{y^2}$$

$$\frac{d\,n_x}{d\,F} = 0.3\,\frac{x^5}{y^4}$$

$$\frac{d\,t}{d\,F} = -\frac{k^3}{5}\,x + 0.5\,\frac{x^4}{y^3} \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

Substituting these values in Equations (19) and (20), integrating from $x = 0$ to $x = b = k\,y$, and arranging, the final Equations (25) and (26) are obtained. The calculations are somewhat extensive, but involve nothing but multiplications and integrations of simple algebraic expressions. Equation (25) has been divided by k^5 and Equation (26) by k^6. The final results are,

$$\frac{1}{2\,G}\int_0^1 d\,z\int_0^H y^3\,d\,y\left[\frac{m}{m+1}\left(\frac{C}{180}+\frac{F}{120}\,k+\frac{C}{324}\,k^4+\frac{F}{200}\,k^5-\frac{g}{30}\right)-\frac{1}{m+1}\left(\frac{A}{30}+\frac{B}{36}\,k+\frac{8}{315}\,C\,k^2+\frac{F}{42}\,k^3\right)-\frac{A}{45}+\frac{C}{105}\,k^2+\frac{F}{72}\,k^3-\frac{w}{45}\right]=0 \quad \ldots\ldots (25)$$

$$\frac{1}{2\,G}\int_0^1 d\,z\int_0^H y^3\,d\,y\left[\frac{m}{m+1}\left(\frac{C}{120}+\frac{9}{700}\,F\,k+\frac{C}{200}\,k^4+\frac{9\,k^5}{1\,100}\,F-\frac{g}{20}\right)-\frac{1}{m+1}\left(\frac{A}{20}+\frac{3}{70}\,B\,k+\frac{C}{25}\,k^2+\frac{4}{105}\,F\,k^3\right)-\frac{A}{30}+\frac{3}{200}\,C\,k^2+\frac{F}{45}\,k^3-\frac{w}{30}\right]=0 \quad \ldots\ldots (26)$$

These two equations, combined with Equations (16) and (17), suffice to determine the four constants, A, B, C, and F. It is apparent that the values of the constants are independent of the height, H, of the dam, since in Equations (25) and (26) the expressions in the square brackets must be zero.

An examination of these equations shows, that if the shear stress, t, and the horizontal stress, n_x, are both equal to zero, Equations (25) and (26) reduce to,

$$\frac{C}{180}+\frac{F}{120}\,k=0;\ \text{and}\ \frac{C}{120}+\frac{9}{700}\,F\,k=0$$

Both these equations cannot be correct, except when $C = F = 0$. Then A and B take on the values given in Equations (11) and (12) and all the stresses are linear.

V.—Stress Distribution—Numerical Example

In order to investigate the stress distribution found by the Principle of Least Work and to compare it with the usual theory, assume that $k = \tan\phi = \frac{b}{y} = 0.645$, $g = 62.5$, and $m = 8$. Let the weight of the concrete be chosen so that the stress in the up-stream face is zero for reservoir full, when the ordinary theory is used. This requires that the coefficient for y in Equation (10) must be zero, or from Equation (11), that $w = \frac{g}{k^2}$, or $w = 150.23$ lb. per cu. ft.*

When these values are introduced into Equations (16) and (17), there results,

$$A = 0.069337\ C + 0.05367\ F \quad \ldots\ldots (16a)$$

$$B = -232.92 - 0.645\ C - 0.37442\ F \quad \ldots\ldots (17a)$$

* All computations have actually been carried out several decimal places beyond the recorded results.

From Equations (25) and (26), when multiplied by 1 000,

$$8.2013\ C + 8.2909\ F - 25.926\ A - 1.9907\ B = 5\,190.3 \ldots\ldots (25a)$$

and,

$$12.568\ C + 13.011\ F = 38.889\ A - 3.0714\ B = 7\,785.5 \ldots\ldots (26a)$$

The solution of these four equations gives the values of the constants, when the stresses are expressed in pounds per square foot. Dividing these values by 144, there results, $A' = 0.3765$; $B' = -5.777$; $C' = 9.50$; and $F' = -5.263$. The stresses are in pounds per square inch,

$$\left.\begin{aligned} n_y &= 0.3765\ y - 5.777\ x + 9.504\frac{x^2}{y} - 5.263\frac{x^3}{y^2} \\ t &= -1.420\ x + 3.168\frac{x^3}{y^2} - 2.632\frac{x^4}{y^3} \\ n_x &= -0.4340\ y + 1.584\frac{x^4}{y^3} - 1.579\frac{x^5}{y^4} \end{aligned}\right\} \ldots\ldots\ldots (27)$$

TABLE 1.—STRESSES IN NO-UPLIFT SECTION ($k = 0.645$), 100 FEET BELOW CREST ($y = 100$), WHEN RESERVOIR IS FULL.

(Stresses, in pounds per square inch.)

x.	n_y.	t.	n_x.
0	+37.650	0.000	−43.403
3	+21.161	.....	.....
6	+ 5.274	.....	.....
10	−11.140	−13.908	−43.389
20	−44.082	−26.238	−43.200
30	−64.332	−36.172	−42.503
40	−75.048	−43.254	−40.962
50	−79.389	−47.839	−38.438
60	−80.512	−50.868	−35.201
64.5	−80.809	−52.119	−33.616

For a horizontal section, 100 ft. below the crest ($y = 100$ ft.), and with reservoir full, the stresses are as given in Table 1. At the down-stream face, $n_y\ n_x = t^2$; this acts as a check on the calculations.

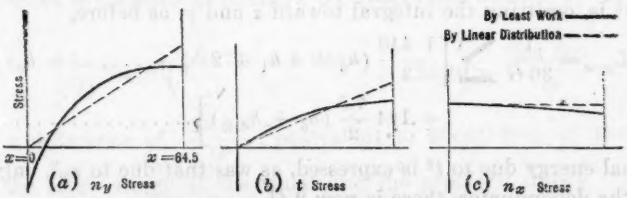

FIG. 4.—COMPARATIVE STRESS DISTRIBUTIONS. UP-STREAM FACE TO THE LEFT.

The stresses are plotted to scale in Fig. 4. On the assumption of a linear stress distribution, Equations (11) and (12) give, when divided by 144, $A'' = 0$ and $B'' = -1.6175$ and the stresses are Equation (13), $n_y = -1.6175\ x$; $t = -1.0433\ x$; and $n_x = -0.43403\ y$. These stresses are also plotted in Fig. 4, for $y = 100$ ft. For the down-stream face, $x = 64.5$ ft., and, $n_y = -104.33$; $t = -67.291$; and $n_x = -43.403$.

A comparison of the vertical normal stresses shows, that according to the new stress distribution, there is considerable tension in the up-stream face and the angle between the n_y-stress and the horizontal decreases as x increases, while the usual method gives zero stress at the up-stream face and a constant angle. Furthermore, the tension in the up-stream face increases directly with y.

VI.—Calculation of Internal Work—Numerical Example

As a check on the calculations, from Equation (3),

$$\int_0^b n_y \, dx = A\, y\, b + B\,\frac{b^2}{2} + \frac{C}{3}\,\frac{b^3}{y} + \frac{F}{4}\,\frac{b^4}{y^2} \quad \ldots\ldots\ldots (28)$$

in which, $b = 64.5$ and $y = 100$. This gives 484 791 lb., while $0.5\, w\, k\, y^2$ gives 484 497 lb., an error of about 6 in 10 000. Equations (4) and (5) were likewise found to check closely. The stresses at the down-stream face are related, as shown by Equations (8) and (9). Since $n_y = -80.809$, t should be -52.122, and n_x should be -33.619, whereas in Table 1, $t = -52.119$ and $n_x = -33.616$. Also, from Table 1, $n_y\, n_x = 2\,716$ and $t^2 = 2\,716$. That these figures check, proves merely that the calculations are correct as far as Equations (16) and (17) and Equations (6) and (7) are concerned; it proves nothing with respect to the correct values of the two free constants, C and F.

When the stresses are expressed in pounds per square inch, the internal work, in foot-pounds, is, from Equation (14),

$$L = \frac{144}{2\,G}\int_0^1 dz \int_0^H dy \int_0^b dx \left[\frac{n_y^2 + n_x^2}{2} - \frac{(n_y + n_x)^2}{18} + t^2\right] \ldots (29)$$

In order to obtain the internal work, Simpson's rule was used. The internal work due to n_y^2 is,

$$L_y = \frac{1}{4\,G}\sum\left[\frac{1}{3}\,\frac{440}{}(h_0 + 4h_1 + 2h_2 + \ldots\ldots + h_6) + 144\,\frac{4.5}{2}(h_6 + h_{6.45})\right] \ldots (30)$$

h_0, h_1, h_2, etc., represent the ordinates, n_y^2, for $x = 0$, 10, 20, etc., as given in Table 2. The same formula applies to n_x^2. The internal work due to $(n_y + n_x)^2$ is, omitting the integral toward z and y, as before,

$$L_{y+x} = \frac{1}{36\,G}\sum\left[\frac{1}{3}\,\frac{440}{}(h_0 + 4\,h_1 + 2\,h_2 + \ldots\ldots + h_6) + 144\,\frac{4.5}{2}(h_6 + h_{6.45})\right] \ldots\ldots\ldots\ldots\ldots\ldots (31)$$

The internal energy due to t^2 is expressed, as was that due to n_y^2, only, instead of $4\,G$ in the denominator, there is now $2\,G$.

The internal work, when the usual assumption is maintained, was found by integration. It was $n_y = -1.6175\,x$; $n_x = -0.43403\,y$; and $t = -1.0433\,x$, so that,

$$L' = \frac{144}{2\,G}\int_0^1 dz \int_0^H dy \int_0^b dx \left[\frac{1.6175^2}{2}\,x^2 + \frac{0.043403^2}{2}\,y^2 - \frac{(1.6175\,x + 0.43403\,y)^2}{18} + 1.0433^2\,x^2\right] \ldots\ldots\ldots\ldots (32)$$

TABLE 2.—VALUES OF STRESSES FOR CALCULATION OF INTERNAL WORK.
(Stresses, in pounds per square inch.)

x.	n_y.	n_y^2.	n_x.	n_x^2.	t.	t^2.	$(n_y + n_x)$.	$(n_y + n_x)^2$.
0	+37.65	1 417.5	−43.40	1 883.6	0.00	0.0	− 5.75	33.1
10	−11.14	124.1	−43.39	1 882.7	−13.91	193.5	− 54.79	3 002.
20	−44.08	1 943.0	−43.20	1 886.2	−26.28	690.6	− 87.28	7 618.
30	−64.33	4 138.3	−42.50	1 806.3	−36.17	1 308.3	−106.83	11 413.
40	−75.05	5 632.5	−40.96	1 677.7	−43.25	1 870.6	−116.01	13 458.
50	−79.39	6 302.8	−38.44	1 477.6	−47.84	2 288.7	−117.83	13 884.
60	−80.51	6 481.9	−35.20	1 239.0	−50.87	2 587.8	−115.71	13 389.
64.5	−80.81	6 530.3	−33.62	1 130.3	−52.12	2 716.5	−114.43	13 094.

When the calculations are carried out, as indicated, and the integrations toward z and y, are omitted, the results given in Table 3 are obtained. The difference between the two theories amounts to 564 480, or about 3.26 per cent.

TABLE 3.—COMPARISON OF INTERNAL WORK BY USUAL AND PROPOSED THEORY.
INTERNAL WORK $\times$ G FOR A STRIP, b, $d\,z$, $d\,y$.

Due to.	By usual theory.	By proposed theory.
(n_y^2)	8 424 000	8 891 000
(n_x^2)	4 374 000	3 902 000
$(n_y + n_x)^2$	−2 590 000	−2 489 000
t^2	7 010 000	6 349 000
Total energy	17 218 000	16 653 000

There was some doubt in the writer's mind regarding the accuracy of the term depending upon $(n_y)^2$ because this becomes zero between $x = 0$ and $x = 10$. Applying Simpson's rule to the first three values of n_y, that is, for $x = 0$, 10, and 20, the internal work is,

$$\frac{1}{4\,G}\,144\,\frac{10}{3}\,(h_0 + 4\,h_1 + h_2) = \frac{462\,800}{G}$$

If the n_y curve is assumed as straight, the energy represented by the first three values is,

$$\frac{1}{4\,G}\,1\,440\left(\frac{1\,417.5 + 1\,943.0}{2} + 124.1\right) = \frac{604\,900}{G}$$

There is a difference of $\frac{142\,100}{G}$, equivalent to about 40% of the difference between the total energy of the usual and the proposed theory. The correct value, therefore, was determined by integration for $y = 100$,

$$\frac{144}{4\,G}\int_0^b d\,x\left(0.3765\,y - 5.777\,x + 9.504\,\frac{x^2}{y} - 5.263\,\frac{x^3}{y^2}\right)^2 \quad \ldots\ldots(33)$$

This gives 8 875 000, which is 16 000, or 0.18%, less than is found by Simpson's rule. The internal work by the new theory is, therefore, slightly less than that given in Table 3. It is interesting to note that if t and n_x are neglected (see

Table 3) the usual theory gives the smaller value for the internal work. The terms inside the square bracket in Equations (14) and (29) then reduce to $\frac{m}{2\,(m+1)}\,n_y^2 = \frac{4}{9}\,n_y^2$.

VII.—Reservoir Empty ($g = 0$)

When, in Equations (16), (17), (25), and (26), g is put equal to zero, there results, in pounds per square inch, after dividing by 144,

$$\left.\begin{aligned} n_y &= -1.017\,y + 1.444\,x - 0.06558\,\frac{x^2}{y} + 0.5752\,\frac{x^3}{y^2} \\ \text{and,} \\ t &= -0.02632\,x - 0.02186\,\frac{x^3}{y^2} + 0.2876\,\frac{x^4}{y^3} \end{aligned}\right\} \quad \ldots\ldots(34)$$

TABLE 4.—Stresses in No-Uplift Section ($k = 0.645$), 100 Feet Below Crest ($y = 100$), When Reservoir is Empty.
(Stresses, in pounds per square inch.)

x.	n_y.	t.
0	−101.695	0.0000
10	− 87.259	−0 2635
20	− 72.609	−0.4979
30	− 57.400	−0.6157
40	− 41.287	−0.4564
50	− 23.924	+0.2083
60	− 4.967	+1 6760
64.5	+ 4.176	+2.6930
55		+0.8203

For $y = 100$ and $b = 64.5$ the values are given in Table 4 and the shear stress is plotted in Fig. 5. The value of n_y checks Equation (3) and t checks Equation (5); the total horizontal shear is zero. The n_y stress distribution is practically a straight line, the same as that found by the usual method. This was to be expected, because the shear stresses are small. There is a slight tension at the toe, while the usual method gives $n_y = 0$ at the toe. Equations (11) and (12) give for $y = 100$ ft., $A = -w$ and $B = \frac{w}{k}$, so that, $n_y = -w\,y + \frac{w}{k}\,x$, which, for $x = 0$ and $x = 64.5$, gives $n_y = -104.327$ and $n_y = 0$. The shear is zero throughout as is also n_x.

VIII.—Bending Stresses ($g = 62.5$ and $w = 0$).

With the usual assumption, the stresses for a full reservoir may be obtained by superimposing the stresses produced by the water on those obtained when the reservoir is empty. The stresses for $g = 0$ and $w = 150.23$ have been determined in Article VII. If, now, $w = 0$ and $g = 62.5$ are introduced into the equations determining the constants, A, B, C, and F, there is obtained, when the stresses are expressed in pounds per square inch,

$$n_y' = 1.4344\,y - 7.6016\,x + 10.159\,\frac{x^2}{y} - 5.8380\,\frac{x^3}{y^2} \quad \ldots\ldots(35)$$

When this is added to the stress found for $g = 0$ in Article VII, there results,

$$n_y'' = 0.41741\, y - 6.1572\, x + 10.093\, \frac{x^2}{y} - 5.2628\, \frac{x^3}{y^2} \ldots\ldots (36)$$

This is nearly the same, as was found directly in Article V (see Equation (27)). The writer has checked his calculations carefully and found no error. Since E is assumed constant, the two results should be equal. The reason for the discrepancy is no doubt to be sought in the circumstance that Equation (15) is only an approximation, and that it contains only a few terms of a possibly infinite series. This explanation was suggested by Dr. Vogt.

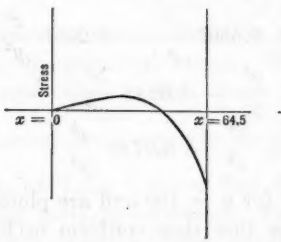

FIG. 5.—VARIATION IN SHEAR STRESS, $g = 0$

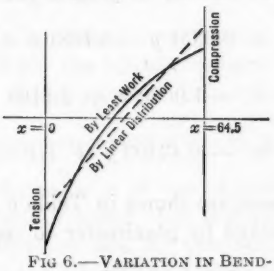

FIG. 6.—VARIATION IN BENDING STRESS, $w = 0$.

It may be noted that the bending stress found by the usual method is uniform and gives the same value at the up-stream face as at the down-stream face. In this manner the bending stresses for a triangular section are the same as for a rectangular one. This does not seem plausible, and the resulting principal stresses are not alike. By the usual method, the bending stress is, for $y = 100$,

$$n_y' = \frac{62.5}{0.41602}\, \frac{100}{144} = 104.33 \text{ lb. per sq. in.}$$

This may also be obtained by the usual formula,*

$$n_y' = \frac{M}{I}\, y = 6\, \frac{M}{b^2} = \frac{1\,000\,000 \times 0.43403}{4\,160} = 104.33$$

TABLE 5.—BENDING STRESSES IN NO-UPLIFT SECTION ($k = 0.645$), 100 FEET BELOW CREST ($y = 100$); RESERVOIR FULL AND WEIGHT OF DAM NEGLECTED.

(Stresses, in pounds per square inch.)

x.	n_y.	x.	n_y.
0	+143 41	40	— 35.45
10	76.99	50	— 55.65
20	27.37	60	— 73.04
30	— 8.94	64.5	— 80 88

The bending stress found by the Principle of Least Work is listed in Table 5 and both curves are shown in Fig. 6. The distribution of the bending stress could readily be tested, since it does not involve w.

* "Notes on Arched Gravity Dams," *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 766.

IX.—The Section of Maurice Lévy

Lévy laid down the rule that when the normal vertical stresses are calculated by the usual method, the compression in the up-stream face with reservoir full should be equal to, or greater than, the hydro-static pressure of the water.* Assume $g = 62.5$ and $k = 0.844$, then from Equation (13), for $x = 0$,

$$n_y = -62.5\, y = \left(-w + \frac{62.5}{0.844^2}\right) y$$

or, $w = 150.24$ lb. per cu. ft. Carrying out the calculations for the constants, as before, there results, in pounds per square inch,

$$\left.\begin{aligned} n_y &= 0.0351\, y - 3.7381\, x + 6.6507\, \frac{x^2}{y} - 3.2487\, \frac{x^3}{y^2} \\ t &= -1.0083\, x + 2.2169\, \frac{x^3}{y^2} - 1.6243\, \frac{x^4}{y^3} \\ n_x &= -0.43403\, y + 1.1085\, \frac{x^4}{y^3} - 0.9746\, \frac{x^5}{y^4} \end{aligned}\right\} \quad \ldots\ldots\ldots (37)$$

These stresses are shown in Table 6 for $y = 100$ and are plotted in Fig. 7; they were checked by planimeter to see that they conform to Equations (3) and (5).

TABLE 6.—Stresses in Full Uplift Section ($k = 0.844$), 100 Feet Below Crest ($y = 100$); Reservoir Full.

(Stresses, in pounds per square inch.)

x.	n_y.	t.	n_x.
0	− 3.507	0.000	−43.403
10	−34.562	− 9.877	−43.393
20	−54.265	−18.652	−43.257
30	−64.566	−25.578	−42.850
40	−67.412	−30.301	−41.563
50	−64.754	−32.854	−39.521
60	−58.540	−33.662	−36.616
70	−50.721	−33.539	−33.169
80	−43.244	−33.688	−29.937
84.4	−40.546	−34.237	−28.896

When the usual theory is employed, the stresses at the up-stream face, for $y = 100$ ft., are $n_y = -43.403$ and $t = 0$, and at the down-stream face, $n_y = -60.930$ and $t = -51.425$. The proposed theory gives compression

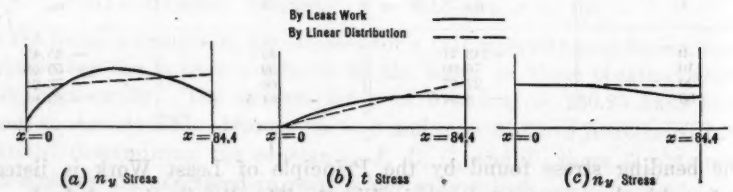

Fig. 7.—Comparative Stress Distribution for Full Uplift Section.

* "Quelques considérations sur la construction des grands barrages," *Comptes Rendus*, Vol. 121 (1895).

in the up-stream face, which is materially smaller than that found by the usual theory.

X.—Maximum Stress

The two principal stresses* are,

$$\left.\begin{aligned} s_1 &= \frac{n_y + n_x}{2} - \frac{1}{2}\sqrt{(n_y - n_x)^2 + 4\,t^2} \\ s_2 &= \frac{n_y + n_x}{2} + \sqrt{(n_y - n_x)^2 + 4\,t^2} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (38)$$

The maximum stress occurring in a horizontal cross-section is the greatest value of the principal stress. With the usual assumption, all three stresses increase with x and reach their greatest value for $x = b$, so that the maximum stress occurs at the down-stream face and is, $s_{\max.} = \frac{n_y}{\cos^2 \phi}$. An inspection of Table 1 shows that the same is at least approximately true for the section there investigated. Table 6 shows that it does not hold for the Maurice Lévy section. The principal stresses for this section are given in Table 7. The maximum stress is 87.4, while on the usual assumption it is $\frac{60.93}{0.584} = 104.33$ lb. per sq. in. As a check, $n_y + n_x$ should equal $s_1 + s_2$.

TABLE 7.—Principal Stresses in Full-Uplift Section ($k = 0.844$), 100 Feet Below Crest ($y = 100$); Reservoir Full.

(Stresses, in pounds per square inch.)

x.	s_1.	s_2.
0	−43.4	− 3.5
10	−49.8	−28.2
20	−68.2	−29.3
30	−81.5	−25.9
40	−87.4	−21.6
50	−87.3	−16.9
60	−83.0	−12.2
70	−76.6	− 7.3
80	−70.9	− 2.2
84.4	−69.4	0.0

At the up-stream face, s_1 is horizontal and at the down-stream face, s_1 is parallel to the face of the dam. The angle between the vertical and s_1 should be,

$$\tan (2\,\phi) = \frac{2\,t}{n_y - n_x} \quad \ldots\ldots\ldots\ldots\ldots\ldots (39)$$

This gives $\phi = 40° 10'$, for $x = 84.4$ ft., which checks $\tan \phi = k = 0.844$, as it should.

In engineering literature too much emphasis has been placed on the necessity of limiting the maximum stress and too little on the importance of not

* *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 294. A more comprehensive explanation is given by Professor Cain in his paper, "Stresses in Masonry Dams," *Transactions*, Am. Soc. C. E., Vol. LXIV (1909), Appendix, p. 220. In Fig. 5 of that paper, the direction of the greatest principal stress is shown for a number of points of a horizontal section.

designing with too low a value of the least principal stress at the up-stream face, or rather that stress which is parallel to the up-stream face and which is generally the least principal stress. The writer has called attention to this before,* but it needs to be insisted upon, quite irrespective of whether or not the proposed stress distribution is the correct one. The stress in the up-stream face with reservoir full, is obtained by subtracting one stress from another, and the result will always be subject to considerable uncertainty, especially when the residual stresses are also considered, as they should be. Professor Cain has also called attention to this point.†

The writer has never heard of the failure of a triangular-shaped, gravity dam due to excessive maximum stress, but a number have failed because they were designed with too little compression in the up-stream face, which, by uplift or other action, resulted in tension.‡

XI.—The Influence of Poisson's Ratio, $\frac{1}{m}$

There is considerable variation between the values of m for concrete given by various investigators, and it is admittedly most difficult to obtain reliable values. It may also be that m varies, as does the modulus of elasticity, with the water-cement ratio, the aggregates, the age of the concrete, and the length of time during which the load is applied.

Consider again the section, $k = 0.645$, for which the stresses were determined in Article V on the assumption that $m = 8$, and calculate the constants, A, B, C, and F, on the assumptions that $m = \infty$ and $m = 2$. The latter value is applicable to rubber; the English tests, to which a more detailed reference will be given, were made on rubber having a value of m approximately equal to 2. In each case, the calculations are exactly the same as before, and the results are given in Tables 8 and 9.

TABLE 8.—Value of Constants for Various Values of m.
($k = 0.645$; $g = 62.5$; and $w = 150.2$.)

	$m = 8$.	$m = \infty$.	$m = 2$.
$A' =$	0.3765	0.3832	0.3581
$B' =$	−5.7768	−5.8061	−5.7047
$C' =$	9.5037	9.3941	9.8553
$F' =$	−5.2633	−4.9958	−6.0611

There is no great difference between the three sets, the tension is smallest for $m = 2$. When $m = \infty$, C and F are independent of w, the weight of the concrete, and the constant, B, does not enter into Equations (25) and (26). The value of w determines the slope, B, and also affects the value of A, which is the stress at the up-stream face.

* *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 763.

† *Loc. cit.*, Vol. 92 (1928), p. 796.

‡ Report of Committee of the Board of Direction on Essential Facts Concerning the Failure of the St. Francis Dam, *Proceedings*, Am. Soc. C. E., January, 1930, p. 163, and April, 1930, p. 827.

TABLE 9.—VALUES OF n_y FOR VARIOUS VALUES OF m.
($y = 100$ ft.; $k = 0.645$; stresses, in pounds per square inch.)

x.	$m = 8$.	$m = \infty$.	$m = 2$.
0	+37.650	+38.325	+35.806
10	−11.140	−10.842	−11.992
20	−44.082	−44.217	−43.716
30	−64.332	−64.800	−62.003
40	−75.048	−75.587	−73.488
50	−79.389	−79.576	−78.812
60	−80.512	−79.765	−82.607
64.5	−80.809	−79.408	−84.785

XII.—DISCUSSION OF THE ENGLISH TESTS

Some tests on gravity dam models were made by Mr. L. W. Atcherly and Professor Karl Pearson in 1904.* These models were built up of thin strips of wood, arranged horizontally in one set of tests and vertically in another. It was claimed that these tests indicated the presence of considerable tension across vertical planes near the down-stream toe. In order to throw additional light on this important subject, some additional tests were made.†

One set of tests was made with a model of plasticine, a kind of modeling clay. An inspection shows that the results near the up-stream face are too erratic to permit of any conclusions being drawn. The curving of the n_y-stress seems established and also the unlikelihood of tension near the toe. The model was constructed so that the resultant fell in the down-stream middle-third point when the reservoir was full. Another set of tests was made on rubber models. The weight of the dam was increased by attaching weights acting vertically downward, and the water loading was imitated by weights acting horizontally. It is not quite clear what the corresponding water load was, but it seems to have been intended that the resultant should fall at the down-stream middle-third point.

These tests have been generally taken to demonstrate the correctness of the ordinary assumption of linear stress, although, as stated in the conclusions,‡ tensile stresses were observed in every case at the up-stream heel, that is, where the concrete meets the foundation. Moreover, all the tests show the curving of the n_y-stress curves, as found by the Principle of Least Work. Fig. 8 (which is Fig. 11 in the original paper) shows the strains as measured, and Fig. 9 (which is Fig. 19 in the original paper) shows plainly the curving of the n_y-stress for the nearly triangular section, C. It shows no tension, except at the heel. The curves at Elevations 3-3 and 4-4 show the curving in the opposite direction to those at Elevations 5-5 and 6-6.

More details than given are necessary to estimate the reliability of the results. The tests are not wholly satisfactory on account of the possibility

* "On Some Disregarded Points in the Stability of Masonry Dams," *Drapers' Company Research Memoir*, Lond., 1904; abstract in *Minutes of Proceedings*, Inst. C. E., Vol. CLXII, p. 456.

† "Experimental Investigations of the Stresses in Masonry Dams Subjected to Water Pressure," by Sir John W. Ottley and A. W. Brightmore and by J. S. Wilson and William Gore, excerpt from *Minutes of Proceedings*, Inst. C. E., Vol. CLXXII, Session 1907–08, Pt. II, Lond., 1908.

‡ *Loc. cit.*, p. 40.

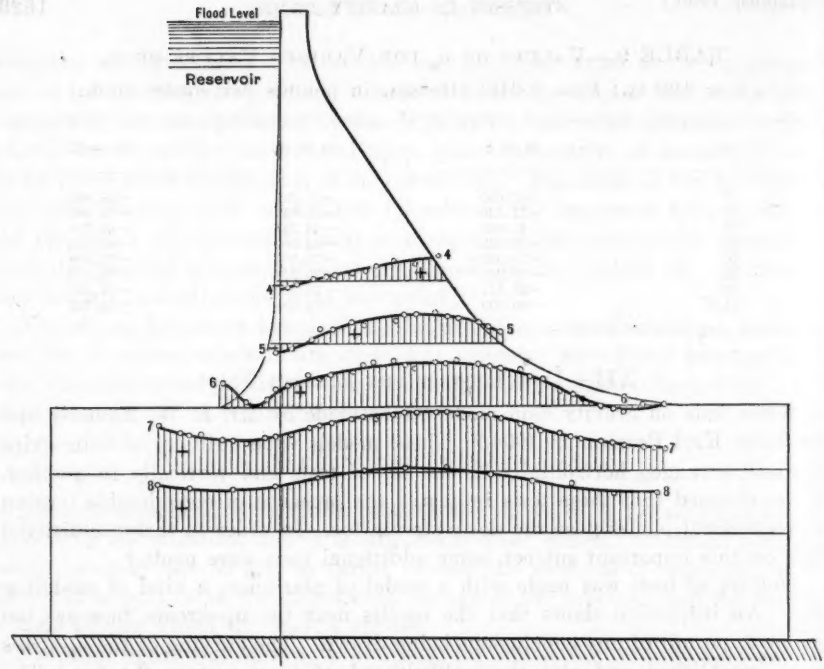

FIG. 8.—ENGLISH TESTS: VERTICAL STRAINS AS MEASURED. ENDS OF FOUNDATIONS NOT SUPPORTED.

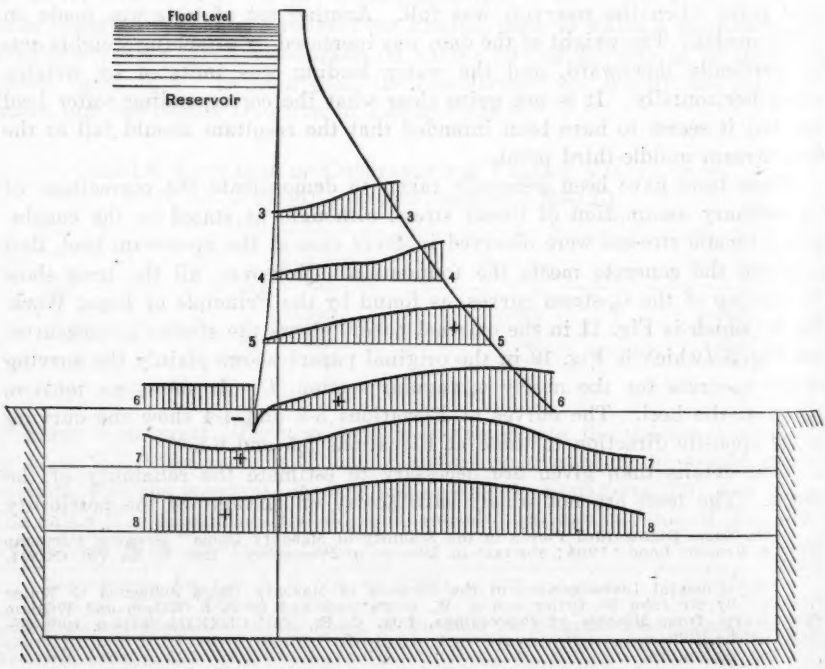

FIG. 9.—ENGLISH TESTS: VERTICAL PRESSURES ON HORIZONTAL PLANES; ENDS OF FOUNDATIONS SUPPORTED.

of considerable local strains, where the weights are attached, and the uncertainty as to the elastic behavior of the rubber, the time effect, etc. There is need of carefully made tests on models, which would represent a high gravity dam. The foundation should be sufficiently deep, and the method of attaching the dam to it might present a problem.

XIII.—General Remarks

For some years the writer has doubted the correctness of the assumption of a linear stress distribution and has suggested that some caution might be justified.* He has called attention to the fact that horizontal sections are not normal sections, as the theory of bending requires, and that the assumption of a linear stress distribution on horizontal sections is not a conservative assumption when dealing with multiple-arch dams.† This question was also discussed by Professor Cain‡ and had been discussed by Résal in his paper to which reference has already been given. The writer found, as did Résal, that for gravity dams the usual horizontal sections gave practically the same result as normal sections, when the usual assumption of linear stress distribution was made.§

The writer has used the Principle of Virtual Velocities to determine the stress distribution in thick arches of dams‖ and Professor Cain showed that the same result was obtainable by the Principle of Least Work.¶ After Dr. Vogt's work in connection with the deformation of the foundations** had become known to the writer, he made an attempt to determine the stress distribution at the abutments of the Pacoima Dam by the Principle of Least Work. Due to the fact that the abutments were not radial, it was evident, that the usual formulas were not exact. Meantime, further studies had convinced him, that the assumption of linear stress distribution in gravity dams was probably incorrect, entirely aside from the effect of internal stresses. It was believed,†† that if the assumption was in error, this would be likely to be aggravated in very high dams. The result of the investigation set forth in this paper seems to justify this apprehension.

If the proposed theory had given values for the stresses, which were less conservative than those found by the usual theory, the new theory should be disregarded until it could be firmly established by carefully made tests. As it is, the new theory gives stresses which are less advantageous, and dams designed by the usual method may not be safe when tested by the proposed theory. It will also be recognized, that the Principle of Least Work is more fundamental than the arbitrary assumption that n_y is linear. Tests only can determine which theory is the correct one.

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* *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 253.
† *Loc. cit.*, Vol. LXXXVII (1924), p. 300, Table 1.
‡ *Loc. cit.*, p. 315.
§ *Loc. cit.*, p. 294.
‖ *Loc. cit.*, Vol. 90 (June, 1927), p. 475.
¶ *Loc. cit.*, p. 522.
** *Loc. cit.*, p. 554.
†† Rept. by the writer to the Colorado River Comm. of Arizona, sent to the Colorado River Board of Engrs. (care, U. S. Reclamation Service), November, 1928.

The proposed solution is only an approximation, limited by the equation assumed for n_y, which has two free constants. If a more complicated equation is assumed for n_y, one having more free constants, a greater degree of approximation should be attained. This would involve much more work.

By putting $F = 0$ in Equation (15), a solution is obtained with only one free constant. There results, $A' = 0.296$; $B' = -4.37$; and $C' = 4.27$; and, for $y = 100$ ft., the corresponding values of n_y are shown in Table 10. These values agree quite closely with those found in Table 1, indicating that the true stress distribution is probably not far from that obtained when using two free constants. (See, remarks relative to Equation (36) in Article VIII.)

TABLE 10.—ONE FREE CONSTANT ONLY.
($y = 100$; $k = 0.645$; and $m = 8$.)

x.	n_y.	n_y from Table 1.
0	+29.6 (tension)	+37.7
10	− 9.87	−11.1
20	−40.8	−44.1
30	−63.2	−64.3
40	−77.0	−75.0
50	−82.4	−79.4
60	−79.2	−80.5
64.5	−75.0	−80.8

The writer has made an attempt to apply the theory to arched gravity dams. The mathematical complications were found to be considerable, although only one free constant was used in the expression for n_y. Equations (3) and (4) become more complicated in that W contains both y^2 and y^3 and M contains y^3 and y^4. Also, the equilibrium equations, (6) and (7), are more complicated. The solution found thus far agrees with the formulas presented in this paper when the radius becomes infinitely large, but it involves calculations with infinite series, which greatly augment the labor. This difficulty may be inherent in the problem, but it may also be that an expression can be selected for n_y which will avoid these difficulties. If the theory proves valid by tests on gravity dams the question arises as to how the stress distribution in multiple-arch dams is affected, when the principle is substituted for the assumption of linear stress across normal sections.

A few historical remarks may be of interest. In 1747, Maupertuis, President of the Berlin Academy under Frederick the Great, enunciated a principle, which he called "le principe de la moindre quantité d'action".* His paper was in three parts, the first being, "an examination of the proofs of the existence of God", which according to the second part must be sought in the general laws of motion. He declared the principle to be one which eminently accorded with the wisdom of the Creator and applied it to the equilibrium conditions of the lever and to the motion of light. Maupertuis was not a skilled mathematician and his exposition was confused and obscure.†

* "The Principle of Least Action," by Philip E. Jourdain, The Open Court Pub. Co., Chicago, Ill.

† "The Science of Mechanics," by Dr. Ernst Mach, late Professor at the University of Vienna, Open Court Pub. Co. (1919), pp. 364 and 454.

Mach suggests* that the principle would soon have been forgotten, had not Euler taken up the suggestion, magnanimously leaving Maupertuis the glory of the invention, and converted it into something serviceable. The theological point of view was retained by Euler who declared that since the universe is the handiwork of an All-Wise Maker, nothing can be met with in which some maximal or minimal property is not displayed. These were the studies which culminated in Euler's work in the calculus of variations and with the isoperimetrical problems; and the equation for the solution of these is still known as Euler's equation.†

That Nature performs in such a manner as to involve the least expenditure of energy, is an assumption which appeals strongly to the intuition; but, whether or not it applies in a specific case, or how it is to be interpreted, only actual tests can determine. For, as Mach says, it cannot be proved by mathematics that Nature must be exactly what it is.

Acknowledgment

The first draft of this paper was submitted for criticism to Professor Cain and to Dr. Vogt, to whom the writer's sincere thanks are due for calling attention to some errors of calculation and encouraging the writer to continue his studies of this subject.

APPENDIX I

From the tests on the plasticine model (having a base of 26 in.), referred to in Article XII, Sir John W. Ottley and Mr. A. W. Brightmore derived a stress distribution‡ which at the heel is zero (the same as found by the linear stress distribution) and at the toe is about 80% of the linear stress. Calling the stress at the toe for linear distribution, 100%, the stress distribution is as given in Table 11 and as plotted in Fig. 10.

TABLE 11.—Stress Distribution Found from Tests on Plasticine Model, in Terms of Percentage of Stress at Toe.

x, in inches.	Stress.	n_y.
(Heel) 0	0	0
2	7.7	5.2
4	15.4	11.8
6	23.1	18.8
8	30.8	27.4
10	38.5	36.8
12	46.1	48.4
14	53.9	60.8
16	61.5	72.6
18	69.2	80.6
20	77.0	83.6
22	84.6	83.6
24	92.4	82.2
(Toe) 26	100.0	80.0

* "The Science of Mechanics," by Dr. Ernst Mach, late Professor at the University of Vienna, Open Court Pub. Co. (1919), pp. 364 and 454.

† "Advanced Calculus," William F. Osgood, Macmillan Co. (1925), Chapter XVII.

‡ *Minutes of Proceedings*, Inst. C. E., Vol. CLXXII (Session 1907–08), Pt. II, Lond., 1908, Fig. 4, Pl. 3.

The question now arises whether Equation (15) can represent a curve similar to Fig. 10, so that in case this is the correct stress distribution, the constants could be determined so as to give a curve similar to Fig. 10.

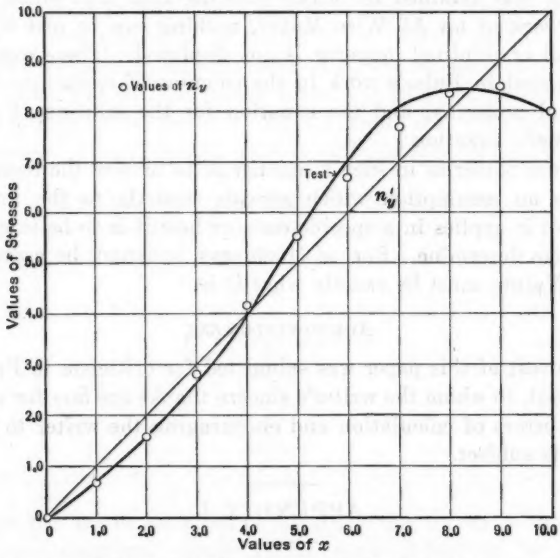

FIG. 10.—COMPARISON OF TEST AND EQUATION (15a), FOR $F = -0.02$.

Let $n_y' = a\,x$ be the linear stress distribution. From Equation (15), since y is a constant, $n_y = A + B\,x + C\,x^2 + F\,x^3$. Since, for $x = 0$, n_y is zero, it follows that $A = 0$.

From Equations (3) and (4),

$$\int_0^b n_y\,d\,x = \frac{B}{2}\,b^2 + \frac{C}{3}\,b^3 + \frac{F}{4}\,b^4 = \int_0^b n_y'\,d\,x = \frac{a}{2}\,b^2$$

and,

$$\int_0^b n_y\,x\,d\,x = \frac{B}{3}\,b^3 + \frac{C}{4}\,b^4 + \frac{F}{5}\,b^5 = \int_0^b n_y'\,x\,d\,x = \frac{a}{3}\,b^3$$

For the down-stream toe, $b = 10$ (Fig. 10), there results, $C = -12\,F$ and $B = 30\,F + a$; so that for $a = 1$, $n_y' = x$, and,

$$n_y = (30\,x - 12\,x^2 + x^3)\,F + x \ldots\ldots\ldots\ldots\ldots (15a)$$

For $x = 0$, $n_y = n_y' = 0$. If $F = 0$, then $C = 0$ and $n_y = n_y' = x$. If $F \neq 0$, the points in which n_y and n'_y are equal are found from, $30 - 12\,x + x^2 = 0$, or $x = 3.55$ and 8.45, no matter what F is. If F is less than zero, the resulting curve is similar to the one found in the English tests. If F is greater than zero, the curve is reversed with respect to the straight line, n_y', that is, the part nearest to the up-stream face lies above n_y', etc.

In Equation (15a), for n_y, F could be determined by the method of least squares, so as to fit the test curve in the best possible manner, but a quite close fit is obtained by assuming that $F = -0.02$, as may be seen by comparing

Tables 11 and 12, and from Fig. 10, in which the small circles are values of n_y determined from Equation (15a), when $A = 0$ and $F = -0.02$.

TABLE 12.—VALUES OF n_y FOR $A = 0$ AND $F = -0.02$.

x.	n_y.	x.	n_y.
0	0	60	6.72
10	0.62	70	7.70
20	1.60	80	8.32
30	2.82	90	8.46
40	4.16	100	8.00
50	5.50		

APPENDIX II

An Airy's stress function, F, may be determined to satisfy the linear stresses given by Equation (13). This stress function is defined by:

$$n_y = \frac{d^2 F}{d x^2}; n_x = \frac{d^2 F}{d y^2}; t = -\frac{d^2 F}{d x \, d y} - w x \ldots\ldots\ldots\ldots (40)$$

in which, n_y, n_x, and t are given by Equation (13). This gives,

$$F = \left(-w + \frac{g}{k^2}\right)\frac{y x^2}{2} + \left(\frac{w}{k} - 2\frac{g}{k^3}\right)\frac{x^3}{6} - g\,\frac{y^3}{6} \ldots\ldots\ldots (41)$$

Equation (41) satisfies Equation (13) and, therefore, satisfies the equilibrium equations, (6) and (7), the statical conditions, Equations (3) and (4), and the boundary conditions, which lead to Equation (13) for the stresses. According to Föppl, the compatibility equations, which guarantee the geometrical continuity of the structure, are satisfied when:*

$$\frac{d^4 F}{d x^4} + 2\frac{d^4 F}{d x^2 \, d y^2} + \frac{d^4 F}{d y^4} = 0 \ldots\ldots\ldots\ldots\ldots (42)$$

Equation (41) satisfies this condition. Of course, a function, F, may also be determined from Equation (27), and Equation (42) may be written,

$$\frac{d^2 (n_y + n_x)}{d y^2} + \frac{d^2 (n_y + n_x)}{d x^2} = 0 \ldots\ldots\ldots\ldots\ldots (43)$$

Equation (27) does not satisfy Equation (43); and, according to Föppl, it should not.†

APPENDIX III

The plane stress distribution considered in this investigation, assumes that $n_z = t_{zy} = t_{zx} = 0$ (Fig. 11). The quantity, n_z, is the normal stress acting parallel to the Z-axis and t_{zy} and t_{zx} are the shear stresses acting in the X-Y

* "Drang und Zwang," Vol. I, p. 248 (1924 Edition).

† *Loc. cit.*, Article 13, p. 97.

plane. When $n_z = 0$, the displacement, u_z, in the direction of the Z-axis, is determined by:

$$\frac{d\,u_z}{d\,z} = \frac{1}{2\,(m+1)\,G}\,(n_y + n_x) \qquad (44)$$

Since $n_y + n_x \neq 0$ (Table 2), it follows that the plane limiting surface, ABC, in the X-Y plane of Fig. 11, is transformed into a warped surface by the load stresses, n_y, n_x, and t. Consequently, t_{zy} and t_{zx} are zero only in the boundary surfaces, such as ABC and in the middle plane (on account of symmetry); and n_z, in general at least, is zero only in the boundary surfaces, ABC and $A'B'C'$, of Fig. 11. A plane stress condition, therefore, in general,

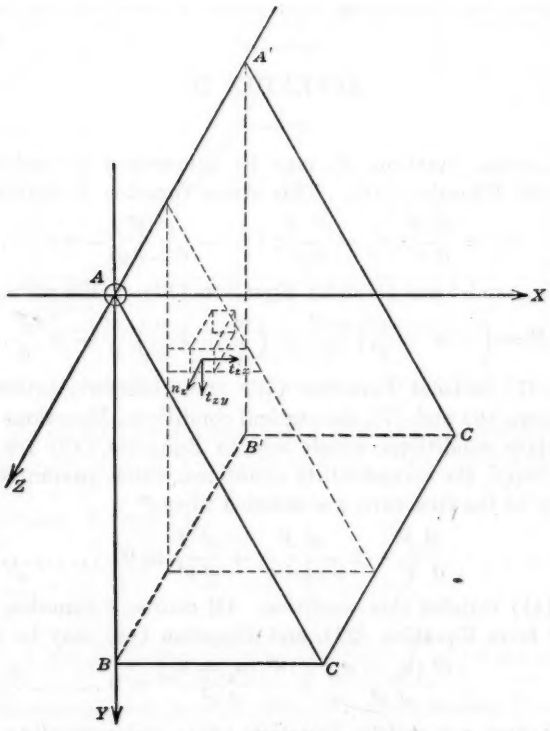

FIG. 11.

must lead to contradictions, if both the equilibrium and compatibility equations are used.* None the less Föppl advances the idea that if the exact stress function is not known and if F_a is an approximate function, containing free constants, these may be determined from:

$$\int \left(\frac{d^4\,F_a}{d\,y^4} + 2\,\frac{d^4\,F_a}{d\,y^2\,d\,x^2} + \frac{d^4\,F_a}{d\,x^4}\right)^2 d\,A = \text{Minimum} \qquad (45)$$

in which, dA is an element of area and the constants in F_a are to be deter-

* Teknisk Elasticitetslaere," by A. Ostenfeld, Article 20, p. 105 (1916 Edition); see, also, Föppl, "Drang und Zwang," Vol. I (1924), p. 129.

mined by the method of least squares.* This solution, according to Föppl, has the same right to be considered a valid approximation as the result obtained by the Principle of Least Work. Equation (45), however, must lead to Equation (41), which is equivalent to Equation (13), and represents a linear stress distribution, which does not agree with the result obtained by the Principle of Least Work.

It is clear that Equation (14) for the internal work is inaccurate, in that it assumes $n_z = t_{zy} = t_{zx} = 0$, for the entire volume under consideration. If $n_z = \frac{1}{m}(n_y + n_x)$, that is, if n_z is just sufficient to prevent displacements in the direction of the Z-axis, then t_{zy} and t_{zx} are both zero, and while there is no displacement in the direction of n_z, this latter, since it varies with the load stresses, influences the displacements in the direction of n_y and n_x. In general, nothing definite is known about the value of n_z in gravity dams. It is affected by shrinkage, temperature changes, etc., and no doubt varies from point to point and from time to time.

When $m = \infty$ it may be assumed, without error, that $n_z = t_{zy} = t_{zx} = 0$; all the compatibility equations are satisfied, and Equation (14) is exact. It has been shown in Article XI that there is no great difference between the stresses obtained for $m = 8$ and for $m = \infty$ and the assumption that $m = \infty$ leads to considerable simplification of the calculations. It is admitted that the assumption, $n_z = t_{zy} = t_{zx} = 0$, is not exact, except when $m = \infty$, but it is believed that this inaccuracy will not materially influence the stress distribution obtained by the Principle of Least Work, while the use of the compatibility equations in connection with the equilibrium equations may lead to contradictions due to the incorrect assumption.

The present investigation does not take account of the irregularities introduced by the foundation, where the section is suddenly changed. Whether or not the stress distribution furnished by the Principle of Least Work applies at the foundation more nearly than the ordinary assumption, only tests can decide. The ordinary assumption is incorrect at the foundation, since as Dr. Vogt has shown, the foundation does not remain plane when the stresses are linear. This is also shown by the English tests (Article XII), which also indicate that the stress distribution obtained by the Principle of Least Work may at least approximate the actual stress distribution at the foundation.

* "Drang und Zwang," Vol. I, p. 336 (1924 Edition).

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PAPERS AND DISCUSSIONS

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STRESSES IN GRAVITY DAMS BY PRINCIPLE OF LEAST WORK

Discussion*

By William Cain, M. Am. Soc. C. E.

William Cain,† M. Am. Soc. C. E. (by letter).‡—In designing straight gravity dams it has been customary to limit the position of the resultant of the water pressure and the weight of the dam to the middle third of any horizontal section. This rule held whether the uplift due to water penetrating the sections was considered, or otherwise. Furthermore, for the given resultant, the vertical unit stresses on a horizontal plane were supposed to be computed as if the vertical section of the dam was rectangular, on the hypothesis that plane sections remained plane sections after stress. This led to the so-called "trapezoid law" of variation of stress, or a uniformly increasing stress from the water face to the down-stream face. It followed that, if the resultant cut the corresponding horizontal section at one-third its width from the outer toe, the unit stress at the inner toe was zero and the stress increased uniformly from the inner to the outer toe where it was a maximum. There was consequently no tension exerted anywhere on the horizontal section.

It has long been the belief of some engineers that such conclusions or hypotheses were untenable; but it remained for Mr. Jakobsen to prove it theoretically§ and for English engineers by experiments on plasticene and India-rubber models of dams‖ to establish it experimentally. In the case of the plasticene model dam, water was applied to the face of the model through a thin rectangular India-rubber bag made to fit the frame. The results were only qualitative, but were sufficient to indicate the variation of stress on horizontal sections and to realize the fear that tension might be exerted at

* This discussion (of the paper by B. F. Jakobsen, M. Am. Soc. C. E., published in this number of *Proceedings*, but not presented at any meeting of the Society), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Prof. Emeritus, Univ. of North Carolina, Chapel Hill, N. C.

‡ Received by the Secretary, July 7, 1930.

§ *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 276.

‖ Excerpt, *Minutes of Proceedings*, Inst. C. E., Vol. CLXXII, Session 1907–08 Pt. II.

the inner toe and that cracks might be formed at the water face even when the resultant cut the base at one-third its width from the outer toe.

In the case of the India-rubber model water was not applied, but its exact equivalent was substituted in the form of weights, acting through strings over pulleys, the strings being given directions at right angles to the water surface where they were attached to small plates bearing, through thin rubber pads, directly on the model. It was found necessary virtually to increase the density of the rubber by attaching other weights to small pins passed through holes in the rubber cut through centers of gravity of certain imaginary sections. Such weights were adjusted to correspond with the volume of rubber they were made to represent. It was not found that the small holes appreciably affected the physical constants of the rubber. Wooden wedges were placed at the upper and lower ends of the large foundation portion of the dam to give it full support. The entire apparatus was most ingenious, as was likewise the method of measuring strains.

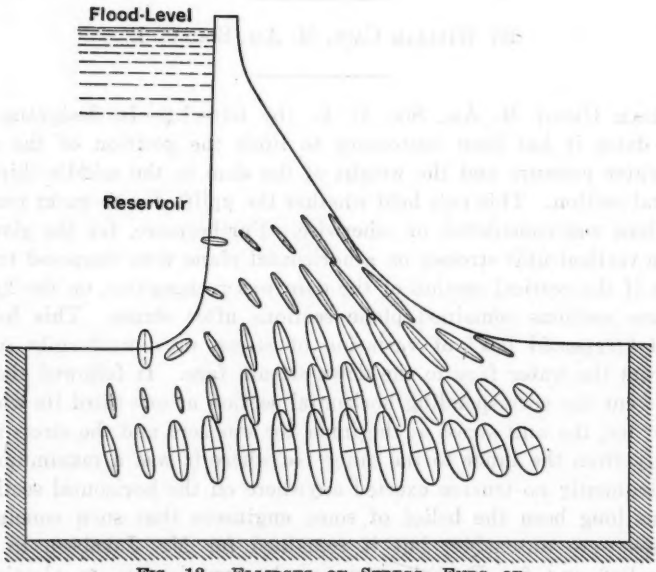

FIG. 12.—ELLIPSES OF STRESS, ENDS OF FOUNDATION SUPPORTED.

The unit stresses at numerous points in the dam and foundation were determined from the displacements by usual formulas and are represented by the ellipses of stress shown in Fig. 12. The principal stresses, or those acting normally to their planes, are represented in amount and direction by the semi-axes of the ellipse. These are observed to act parallel to the inclined down-stream face near that face, but at the up-stream face where the dam is subjected to water pressure the direction of the principal stress is nearly normal to that face to counteract the large normal pressure of the water. The variation of stress along horizontal planes above the foundation is of especial interest in relation to the author's main problem, and such variations for the

vertical pressure on horizontal planes, the horizontal pressure on vertical planes, and the shear on horizontal planes have all been computed and diagrams made to show the variation.

The author has endeavored to find this stress distribution theoretically by the method of least work and he has succeeded remarkably well in effecting it. To this end, after writing certain fundamental relations, he assumes that the vertical unit stress on a horizontal plane at (x, y) is given by an expression of the form of Equation (15),*

$$n_y = A\,y + B\,x + C\,\frac{x^2}{y} + F\,\frac{x^3}{y^2}$$

in which, A, B, C, and F, are constants; and in which, by antecedent equations, A and B can be expressed as functions of C and F, as given by Equations (16)* and (17).* The elastic internal work, L, is expressed by Equation (14),† as given by Föppl, which equation is likewise given in another form, easily reduced to this, by Molitor.‡

It will be convenient now to refer to Castigliano's "Systèmes Elastiques"§ for a fundamental principle (given in Chapter 1, Article 11, for an articulated system of bars, and in Chapter 2, Article 4, for a solid body), that if the elastic work of deformation is expressed as a function of the exterior forces (including the reactions as exterior forces), then its partial derivative with respect to any one of these forces will give the relative displacement of its point of application in the direction of the force. Similarly, it is proved (in Chapter 1, Article 12, for articulated structures and in Chapter 2, Article 5, for solids), that the derivative of the elastic work of deformation of the structure or body with respect to the moment at a certain section expresses the rotation of this section.‖

It is assumed, in developing these relations, that a linear relation exists between a force and its corresponding displacement, and, consequently, that the elastic work of deformation (or the potential energy) can be represented as a homogeneous function of the second order of the external forces, and that the forces are independent of each other. The author's Equation (14) affords an illustration. In fact, it is a very exceptional structure where the relations noted do not hold.¶

The "method of least work" is easily deduced by aid of these two principles. Thus, consider an arch "fixed at the ends", by which is meant that there is no displacement at the abutment sections and no rotation there, so that the sections remain plane after stress; then, if at the left abutment (say), the tangential component of the reaction is P_1, the radial or shearing component

* See p. 1618.

† See p. 1617.

‡ "Kinetic Theory of Structures," p. 37.

§ An English translation, with the title, "Elastic Stresses in Structures," is pub. by Scott Greenwood and Son, 8 Broadway, Ludgate, E. C. 4, London, England.

‖ All the fundamental principles of Castigliano's theory are given by the writer, by aid of the theory of "virtual velocities," in "Determination of the Stresses in Elastic Systems by the Method of Least Work," *Transactions*, Am. Soc. C. E., Vol. XXIV (April, 1891), p. 265.

¶ This subject is very clearly put in Timoshenko and Lessells' "Applied Elasticity," pp. 117–126.

is S_1 and the moment is M_1; and if L = elastic work of the entire arch, then, by the principles just given,

$$\frac{d\,L}{d\,P_1} = \text{displacement of } P_1 = 0$$

$$\frac{d\,L}{d\,S_1} = \text{displacement of } S_1 = 0$$

and,

$$\frac{d\,L}{d\,M_1} = \text{rotation of abutment section} = 0$$

These partial derivatives being equal to zero express the fact that the elastic forces after deformation are such as to render the elastic work a minimum. Thus is proved for this case (and similarly for other structures) the "Method of Least Work".

Castigliano's treatment of the change of the unknown quantities* is very informing. It proves that, if the work of deformation can be expressed in terms of an equal number of new unknowns, the latter can be found directly by placing the partial derivative of the elastic work with respect to each of them equal to zero.† This treatment suggests the following, referring to Mr. Jakobsen's main problem. Note that from Equations (16) and (17), it is seen that A and B are functions of C and F. For convenience, the values of n_y, n_x, and t, are grouped together. Thus, in Equation (15),

$$n_y = A\,y + B\,x + C\,\frac{x^2}{y} + F\,\frac{x^3}{y^2}$$

In Equation (22),‡

$$n_x = \frac{C}{6}\,\frac{x^4}{y^3} + \frac{3}{10}\,F\,\frac{x^5}{y^4} - g\,y$$

and in Equation (21),‡

$$t = -A\,x + \frac{C}{3}\,\frac{x^3}{y^2} + \frac{F}{2}\,\frac{x^4}{y^3} - w\,x$$

The partial derivatives as to C are:

$$\frac{d\,n_y}{d\,C} = \frac{x^2}{y};\quad \frac{d\,n_x}{d\,C} = \frac{x^4}{6\,y_3};\quad \frac{d\,t}{d\,C} = \frac{x^3}{3\,y^2}$$

If L is the total work of deformation for a 1-ft. length of dam, as given by the author's Equation (14), then L is a direct function of n_y, n_x, t; and as these are functions of C and F, by the known formula of the differential calculus (the derivatives with respect to C, being partial derivatives),

$$\frac{d\,L}{d\,C} = \frac{d\,L}{d\,n_y}\,\frac{d\,n_y}{d\,C} + \frac{d\,L}{d\,n_x}\,\frac{d\,n_x}{d\,C} + \frac{d\,L}{d\,t}\,\frac{d\,t}{d\,C} = \frac{d\,L}{d\,n_y}\,\frac{x^2}{y^2} + \frac{d\,L}{d\,n_x}\,\frac{x^4}{6\,y^3} + \frac{d\,L}{d\,t}\,\frac{x^3}{3\,y^2}$$

Now by Castigliano's theorems stated on the derivatives of work for any point (x, y) of a horizontal section of the dam,

$$\frac{d\,L}{d\,n_y} = \text{vertical displacement}$$

$$\frac{d\,L}{d\,n_x} = \text{horizontal displacement}$$

* "Systèmes Elastiques," Chapter 10, Article 2.

† This treatment of the change of unknown quantities is likewise given in full in the paper cited, *Transactions*, Am. Soc. C. E., Vol. XXIV (April, 1891), p. 292.

‡ See p. 1619.

and,

$$\frac{d\,L}{d\,t} = \text{displacement due to shear}$$

If at the base, each of these displacements is zero, the equation reduces to,

$$\frac{d\,L}{d\,C} = 0$$

Similarly, it can be found that,

$$\frac{d\,L}{d\,F} = 0$$

These two equations show that the internal work is a minimum, as stated by the author.* This demonstration assumes the base of the dam unyielding.

Fredrik Vogt, Assoc. M. Am. Soc. C. E., in a very interesting discussion† regards the base, whether the foundation of the dam or the abutment of an arch, as yielding for a certain small distance, below which it is absolutely unyielding and plane sections remain plane. If this hypothesis is adopted, it is perhaps only necessary to assume approximately the base of the dam removed to a parallel position, presumed to be unyielding, for which the supposed plane section remains plane and the method of least work is seen to apply to the revised position or to a slightly increased size of dam. On this supposition the original foundation will no longer remain plane, but will become a slightly warped surface.

Recurring now to the original problem, where the base of the dam (or foundation) is supposed to be unyielding, the writer is glad to testify to the complete scientific accuracy of the author's analysis, of which he has checked Equations (1) to (25), inclusive. For the stress distribution, for the numerical illustration of Article V,‡ the values of A', B', C', F', are readily found, and the formulas for the stresses at (x, y) are given by Equation (27),§ and their numerical values at the base, $y = 100$ ft., are recorded in Table 1.

The author has noted the important fact that the constants, A, B, C, F, are independent of the height, H, of the dam, "since in Equations (25) and (26) the expressions in the square brackets must be zero." It is thus only necessary in Equation (27) to substitute for (x, y), their numerical values to compute the stresses at any point (x, y) in the dam. This dam is designed so that the stress in the up-stream face is zero when the usual theory of (linear) stress distribution is used, straight lines limiting the unit stresses as shown in Fig. 4.§ Likewise, Fig. 4 gives the stress distribution as determined by the method of least work and it is seen that by this method there is appreciable tension at the up-stream face, and, further, that the angle between the curve of the n_y stress and the horizontal decreases as x increases. The important fact is the occurrence of tension at the up-stream face; and it is evident that the only way to avoid it is to increase the base or the value of

* See p. 1622.

† Of "Stresses in Thick Arches of Dams," by B. F. Jakobsen, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 90 (June, 1927), p. 554.

‡ See p. 1620.

§ See p. 1621.

$k = \frac{b}{y}$. This would alter slightly Equations (16a),* (17a),* (25a),† and (26a),† and lead to new values of A, B, C, F, and new formulas for n_y, t, n_x.

In place of assuming k and computing w, assume $w = 145$ (say), for the weight of 1 cu. ft. of concrete and compute, $k^2 = \frac{g}{w} = \frac{62.5}{145} = 0.431$; whence $k = \frac{b}{y} = 0.656$; or for $y = 100$ ft., $b = 65.6$ ft., thus giving the base for reservoir full for stress zero at the up-stream face when the usual linear stress distribution is assumed. Now b must be increased to avoid tension at $x = 0$ when the method of least work is used. As a first trial, suppose b increased by 5 ft., so that $b = 70.6$, whence for the same $w = 145$, $k = 0.706$, and the new values of A and B in terms of C and F are derived from Equations (16) and (17).

The solution proceeds by the use of Equations (25) and (26) exactly as the author indicates, so that eventually the formulas for n_y, t, n_x, can be written. If n_y at $x = 0$ should turn out to be either zero or a small negative value (compression), the solution may suffice. Otherwise, another small change in b will have to be assumed, and a new formula for n_y must be derived and tested as before. It should not require more than three trials before a satisfactory formula for n_y can be written.

As all the formulas refer to a triangular dam, they are approximate near the top when the usual additions to the section are made there; but for a high dam, they are sufficiently accurate for the lower sections. They furnish practical formulas for distribution of stress for most straight dams.

Hitherto, uplift (from water penetrating the base) has been neglected. If the uplift at the heel is assumed as the greatest possible (= 62.5 y lb. per sq. ft.), and as decreasing linearly to zero at the toe, then the total uplift is,

$$\tfrac{1}{2}\, 62.5\, y\, b = \tfrac{1}{2}\, 62.5\, k\, y^2$$

which is equivalent to the weight of a triangular mass of water acting upward on the base.

For 1 ft. in length of the triangular dam, its weight less the uplift is $\frac{1}{2}\, k\, y^2\, (w - 62.5)$.

It can now be easily verified that, for the triangular dam considered, all the general Equations (1) to (26) are valid when uplift is considered, by simply replacing w by $w' = (w - 62.5)$, which simplification is entirely due to the assumption of a triangular dam.

In Article VII,‡ Mr. Jakobsen has derived the results for reservoir empty (when, of course, there is no uplift to consider) by the method of least work, and finds a small amount of tension at the toe. This would probably disappear when the base is slightly increased as previously suggested.

Partly as a check, the author has computed the internal work by the usual and the proposed theories and has subsequently computed the bending stresses

* See p. 1620.
† See p. 1621.
‡ See p. 1624.

for the same cases, and illustrated the results in Fig. 6.* To render the solution more complete, the principal stresses have been computed and the influence of Poisson's ratio examined.

The labor involved in developing all the formulas and computing numerical results has been very great, and the author deserves the thanks of the profession for his excellent and convincing paper. The English experiments† first proved the ordinary "inner-third rule" inadequate, and now Mr. Jakobsen confirms it by a close theoretical analysis. It will not suffice that the resultant of the water pressure and the weight of the dam should be required to cut the base simply at one-third its width from the outer toe; it must be at a greater distance from this toe. In other words, to avoid tension at the heel, the resultant should be confined to narrower limits than the middle-third. It is a difficult problem to find such narrower limits, but, at least, it has been shown that the method of least work can materially aid in establishing them.

* See p. 1625.
† See p. 1629.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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REPORT OF THE COMMITTEE OF THE IRRIGATION DIVISION ON "A NATIONAL RECLAMATION POLICY"

Discussion*

By John E. Field, M. Am. Soc. C. E.

John E. Field,† M. Am. Soc. C. E. (by letter).‡—As a member of the Committee, the writer considered the question referred to in Section (9),§ more from the viewpoint of its effect on the settler than on the market for farm products, the competition with existing farms, or the needs of the Nation for farm products. The question of first importance is: Can the settler succeed? If he succeeds in his enterprise and if, in consequence, some other farmer finds it necessary to give up farming, that is competition, the "survival of the fittest," or the evolution of the industry.

The increasing cost of transportation and distribution has tended to revive or sustain agriculture in the more densely populated areas, and such revival from a National viewpoint is more desirable than the reclamation of new and isolated areas, and will tend to supply any future deficiency in area cultivated.

The evolution in menus, the rapidly increasing area of crops grown under glass and canvas, the artificial stimuli of fertilizers and light rays, the development of better seeds and better methods, all indicate that, even without additional areas being brought under cultivation, there is no prospect of a food shortage for a long time to come.

This condition, supplemented by the tendency toward urban employment and habitation and the lack of demand for land for farming, justifies the principle stated in Section (9).

The movement of people in the United States has been westward since the founding of the Government; but in the decade, 1920-1930, there has been a

* Discussion of the report of the Committee of the Irrigation Division on A National Reclamation Policy, continued from April, 1930, *Proceedings*.

† Cons. and Superv. Engr., Denver, Colo.

‡ Received by the Secretary, May 10, 1930.

§ *Proceedings*, Am. Soc. C. E., September, 1928, Papers and Discussions, p. 2099.

general trend toward the South. With this movement southward of large numbers of people, although predominantly commercial and industrial, development will create a demand for land in that region. This local demand will be met with increased cultivable areas or more intensive cultivation.

The South is in strenuous competition with the West for new citizens, more capital, and more enterprises. One section of the West is the rival of another section of the West. The same is true of the South and the Pacific Slope. Thus, reclamation of land is primarily a local, not a National, need; it should, therefore, be promoted and financed locally. The Federal Government, in so far as the Nation is benefited directly or indirectly, should aid these enterprises, but any other body, section, or region benefited should contribute in proportion to the benefits to be received. This principle is reflected in Sections (3)* and (4)* of the report.

"What's the hurry?" Many Governments have found it necessary to enter upon programs of large public undertakings involving the seeking of new territory and the founding of colonies, on account of the pressure of population, the necessity for more room, more work, and more revenue. Often, without such relief, the durations of government and even nations themselves have been threatened. Such conditions do not exist in the United States; nor will they for many years to come. It is all but certain, however, that, in time, this country will have like conditions to face and similar questions to answer. When that time comes it can be easily imagined that the people will deplore the too rapid growth of their country; they may wonder why the people of the present day were so desperately anxious to populate the country, that aliens and undesirables were imported; that Americans had preferred to hand over their inheritance to strangers and to deprive their own descendants of opportunity. Coming back to the present, one may well ask, "What's the hurry?"

Every section has the right to grow and expand, and to do its utmost to develop its resources. This is a commendable characteristic, but the problem is local and not National, and it must be judged by its effect on posterity as well as on prosperity.

The explanatory statement† by J. B. Lippincott, M. Am. Soc. C. E., is from the National rather than from the local viewpoint. As the membership of the Society is nation-wide and as the subject is a "National" reclamation policy, the Committee strove to view the question nationally, although of necessity the illustrations, discussions, and citations given in Committee meetings were local. Nevertheless, they covered quite one-half the area of the forty-eight States.

Throughout the report the principle is maintained that the States or local enterprise should be permitted to develop the natural resources and that the Government should do only those things the States cannot do—such as the gathering of data over wide areas and long periods of time—but the report also holds that where the Government acts it should be in co-operation with the State—with the State's consent, and at its request. It is also stated in the report that projects not now feasible should not be undertaken at this time.

* *Proceedings*, Am. Soc. C. E., September, 1928, Papers and Discussions, p. 2099.
† *Loc. cit.*, May, 1929, Papers and Discussions, p. 1193.

With reference to Section (1)* of the report, it was suggested by some members of the Committee that the proposed program for the expenditure of $100 000 000 should be held in abeyance and subjected to the same rules of feasibility as new enterprises; this is expressed in Sections (10)† and (11)† of the report. It was agreed that the Government should limit the expenditures of this fund and "fulfill its assumed obligations" only.

In proposing a reclamation policy the desire to prevent foolish, excessive, or premature expenditures of money—whether Federal, State, or private—should be expressed. Some of the proposals for the expenditure of the $100 000 000 Reclamation Fund were foolish; others excessive in cost; and many were premature. One project likely to be built has all these three faults. If the States in which these projects are located had been required to share the financial responsibility, and the localities had been required to guarantee repayment, some, at least, of the projects never would have been suggested. In the future, localities and States should be required to guarantee, in part at least, the interest and repayment of principal, in accordance with the terms of a contract and schedule of repayment, before any project is undertaken.

The writer approves of applying a test question, "Can the project be made to pay?" It should be applied in such a way as to discourage inopportune construction; to deter those who could not or would not pay a reasonable interest on the advance made to them; to discourage speculation; to banish the thought that Government funds were free; and to impress on the settlers a sense of entire responsibility.

In analyzing the differences in attitude of the Government and of private enterprise as applied to the ownership of water, the writer believes that attachment of the water to the land is a mistake. In Colorado titles to land and water are separate, and water may be transferred and its use extended to cover greater areas. This flexibility of use of water accounts in large measure for Colorado's marked progress in irrigation and her position as second of the arid States in the area of land irrigated. Similarly, the limitation of the area which one person could own on a Federal reclamation project was not entirely wise as it assumed that all settlers were equal in ability, enterprise, and resources; it assumed even that the number of children would be the same. A man of real executive and managerial ability would scarcely be content with an 80-acre tract in Colorado, Wyoming, or Nebraska, where only a living was in prospect, and where ultimate independence and the hope of providing advantages for his children were absent. The lack of experience and sufficient funds among the settlers was due, in part at least, to this limitation of the activities and prospects of the more experienced and able farmers.

State vs. Federal Control.—In Colorado the control and the responsibility for the distribution of water rests in appropriate State officials. Dual or joint control of the water by State and Nation would lead to confusion. The Enabling Acts of the States of the West give them the same standing as the original thirteen States. The public lands are held by the Government in

* *Proceedings*, Am. Soc. C. E., September, 1928, Papers and Discussions, p. 2098.
† *Loc. cit.*, p. 2099.

trust for those who desire to use them. Except for this condition of trustee, all the land within its boundaries is owned by the State and has the same status as lands in private ownership. It is not possible, therefore, for the Government to assume the original ownership and control of the natural resource of water. That the Government has called State ownership into question is true; but the Western States are on equal footing with the original thirteen States. Can the Government, therefore, claim control and ownership of the water of New York, Connecticut, and Pennsylvania? This is a form of bureaucratic aggression which the original thirteen States may consider with profit to themselves.

The writer does not advocate nationalization of the rivers. He believes the province of the Government should be limited to investigation and research, to the gathering of data, and to aiding in the preparation of comprehensive plans for the ultimate maximum utilization of available water. He hesitates to advocate even the construction of large reservoirs unless it is distinctly provided that the control and distribution of the stored water remains in the State water officials.

The writer does not subscribe to Mr. Lippincott's explanatory statement* that control of reservoirs should remain in the Federal Government if that means control of the water distribution. In Colorado there is an efficient organization for the distribution of the waters of the State. This control should, and of necessity will, remain in the State and under the State Engineer. A change from State to Federal or joint control can lead only to confusion because all canals and reservoirs now built are operated under State laws. Even the two Government projects (Uncompahgre and Grand Junction) receive their water (in accordance with the Reclamation Act) through the State water officials. Rather than submit to Federal distribution of water it would be better to forego Federal aid.

Considerable money has been spent in Colorado by the Government on these two projects. They are, or at least may yet be, advantageous to the State, but when one remembers the restrictions imposed on enterprises desiring to build irrigation works on the Rio Grande and North Platte Rivers, he is impressed with the fact that Colorado has gained nothing through Government aid; on the other hand, a question has been raised as to her right to administer and control the waters within her own boundaries. Interstate questions have been and can be settled by Court procedure or by compact. The writer consented to the recommendation in Section (2)† that "the conservation of the water in the rivers and lakes of the country should be under public control," only because "public control" means generally State control. It means Federal control only where the States cannot function alone or by compact. This thought is stated in Section (3), as follows: "utilization of the waters should be undertaken by the States," or, as an alternative, "jointly by the United States and the States."

The thought lying back of Sections (1) and (10), as the writer sees it, is that it is better to fix definitely and irrevocably what the settler is to pay and

* *Proceedings*, Am. Soc. C. E., May, 1929, Papers and Discussions, p. 1199.
† *Loc. cit.*, September, 1928, Papers and Discussions, p. 2099.

when and how he is to pay it. Indefiniteness in this regard has led in the past to many of the evils now recognized. If the obligations are fixed and irrevocable before construction begins the unwise projects would be dropped; speculation largely eliminated; the incompetent deterred; and the experienced farmer encouraged to settle. It is a species of fraud not to tell a settler the cost of his land and water, and especially is it fraud to tell him what it will cost at the time he enters the contract and then increase it later without his consent. Similarly, it is a deception to obtain public funds under promise of repayment and then to pay no interest and to ask for the cancellation of part of the indebtedness. The contract, once made, should be carried out to the letter, just as contracts are carried out in the business world; some losses are inevitable in all business and some will be suffered in Federal reclamation, resulting in losses for the Government and bankruptcy for the individual. The one will charge it off to poor judgment in building the project and the other will charge it to his own foolishness, inexperience, or "hard luck." In addition, it is fraud to construct a project and induce settlement where the settler cannot achieve independence in his old age. It is a fraud to treat public money as "easy money."

Generally speaking, the cost of reclamation is in excess of what the settler can pay. Recognizing this condition the Government, the State, and the locality desiring to build a project should make up the difference under the assumption that either the direct or indirect benefits, immediate or remote, justify the necessary aid. The probable benefits to Nation, State, and locality can be determined in advance, and the settler, knowing that the amounts due from him must be paid, will put his mind and labor on his farm work rather than in devising ways and means to obtain remissions from the Government. Also, in the knowledge that he must pay for the operation, maintenance, and improvements, he will weigh the necessity before demanding expenditures.

Even if it were true that original capital costs the Government nothing, this would not excuse the undertaking of projects that were not feasible or for which the settler would require heavy subsidy in the form of no interest payments or the remission of the original cost.

The only basis for any other policy is the theory that every locality is equally entitled to its share in the contents of the "pork barrel." The fact that the Reclamation Fund is derived from the sale of lands in certain States is scarcely sufficient reason for undertaking projects that will not pay the costs; nor does the fact that the money comes from sales of lands or revenues from a certain State justify the theory that, "the money comes from our State, we want it spent here, it will give our people employment; we do not care whether the project is feasible or not."

To some this may be surprising, but as early as 1903 and as late as 1929, the writer has heard this sentiment expressed by eminent "native sons" advocating a project in their States.

A five-year period would be sufficient to develop a reclamation project, when the improvements are opportune and when a demand for land exists. However, where improvements are premature or where postponed payments encourage the holding of land without cultivation, twenty-five years for

development would probably be required; in that time, however, prematureness would probably have disappeared.

The Colorado River Project with its Boulder Canyon Dam has been specifically mentioned as an enterprise which the Government alone can handle. This probably is true at present, but one can imagine either the seven Basin States, or the three lower ones, entering a partnership and compact to develop the basin. To declare (as some have done) that projects could not have been built without Government financing and Government contributions, is in the writer's judgment, an over-statement. There are projects which could not be built without Government aid. International questions might be such that the Government alone could function but, except that time is an essential element, the writer finds it difficult to imagine a project which a State, such as California, for instance, could not build. California is certainly equipped with engineers and scientists capable of undertaking the very largest projects. However, as long as Government financing will furnish the funds with the least expenditure of effort on the part of the States; as long as it will assume the risk and do the work, and will do these things in the shortest space of time, the line of least resistance will be followed. It is very easy to acquire the habit of being waited upon and relieved of work, worry, and responsibility. If hope of Government aid were not present one can imagine the States or even local enterprise and private capital undertaking the work—not at this time, perhaps, but at some future time.

It appears to the writer that the megalomania that is so characteristic of the United States is the real moving force back of most promotions, rather than any National or even State necessity. If there is a State necessity, then the State should assume the responsibility, and the part of the financial burdens needed to supplement the amounts that the settler can afford to pay. If such a rule prevailed the "log-rolling, pork-barrel" manifestations would abate, and loud shouts of "necessity" would sink to mere whispers.

Although the writer recognizes that extraordinary or supplemental aid is desirable from the settlers' point of view, and is often necessary on projects unwisely undertaken, he cannot subscribe to the universality of the need. In 1902, the Government examined the South Platte Project in Northeastern Colorado; it built the Interstate North Platte Project less than 100 miles north. These projects were similar in elevation, soil, climate, markets, crops, and potential settlers. The supply of water in the South Platte was questionable, that of the North Platte, unquestionable. Several years later portions of the land in Colorado were provided with canals and storage reservoirs through the irrigation district plan. The districts have been successful. The cost of construction, including expenses of promotion, discount on bonds, interest during construction, and costs of investigations, was not greatly different from the costs on the North Platte Project, exclusive of drainage and costs of maintenance and operation. The difference is even less if the character and the permanence of the irrigation works built by the two agencies are taken into consideration. Nearly all the bonds of the districts have now (1930) been paid off. In the Colorado projects, the settlement was less rapid at first than on the Federal project, but the general character of the settlers was better in

that they were more experienced and that the projects met local demands for land. Time, place, and cost corresponded with desire, demand, and necessity. A part of the indebtedness of the largest district was written off, in amount about equal to the promotion cost, and since then, in the knowledge of the futility of demanding further reductions or concessions, such demands have not been made and the district's credit is "A1." This illustrates, also, that land settlement can be taken care of locally. Had this been a Federal project less competent settlers would have rushed in; speculation based on the hope of avoiding repayment of costs would have been present; large maintenance, operation, and improvement costs would have been incurred on money advanced by the Government; and, probably, a demand for houses and leveling would have been made. As stated in Section (14)* of the report, "Land settlement * * * or any other kind of assistance * * * should be treated as a local matter." If the locality and the State are unwilling to assume this duty and responsibility, construction should be postponed until demand and feasibility induce it to do so.

Referring to Sections (1) and (11), the contributions by the Federal Government should be based upon the estimated benefits to the country as a whole and nothing more. As a maximum, the contribution should not reduce the settlers' obligations to less than what he can reasonably be expected to pay. The interest-payment rate should approximate the interest rate paid by the Government on its indebtedness. Such limitations and rates would undoubtedly leave obligations greater than the amount the settler could pay, and, in such cases, the project would be classed as premature and its construction postponed.

The proper desire of the U. S. Bureau of Reclamation has been to build projects so as to include all the lands possible. Often it was found that the earlier private enterprises took in only the choice areas, leaving the less desirable ones in such a condition that they could not be reclaimed feasibly. This suggests the popular expression, "they took the heart from the melon and left the rest." Carrying the simile further, the service in some cases undertook to "save the rind as well as the seed zone," and the results were unsatisfactory in that considerable areas were put under canal at great expense, but were later abandoned. A portion of the amounts written off by the Government were chargeable to this condition. Thus, reclamation went from the one extreme of private enterprise to the other by National expenditure. It is now proposed to construct a project in which only a very small percentage of the land proposed to be irrigated can support a settler, even if both land and water, leveling, and fencing were given him at one-fourth the cost. The advisability of imposing the conditions of Section (10) is under such conditions apparent.

In Sections (8)† and (9), the Committee's report indicates a preference for completing and bringing to perfection existing projects, and that great caution be exercised in attempting new projects. On existing projects much necessary expense has been met; facilities, such as roads, schools, and tele-

* *Proceedings*, Am. Soc. C. E., September, 1928, Papers and Discussions, p. 2100.
† *Loc. cit.*, p. 2099.

phones, have been provided; small business centers have been built; and markets have been established. On a new project these facilities will have to be provided. It is obvious that the bringing of existing projects to a higher state of efficiency is the more economical and promises quicker results than undertaking new projects.

One of the primary purposes of the irrigation congresses, and of individuals proposing Federal aid, before the passage of the National Reclamation Act, was to furnish supplemental water supplies to existing projects and the extension of those projects. About the time the Reclamation Act was passed, the scope was enlarged to include construction of main canals. The writer does not remember that the construction of laterals, administrations of systems, building of drains, colonization, or aids to settlers, were proposed until after the passage of the Act, and some of these activities were not suggested for some years thereafter. It was when the activities undertaken were broadened in scope that needs for new contracts with increased cost over original estimates appeared. In the "pre-Act period," it was assumed that the settlers would be drawn from experienced communities and that they would be capable of self-government, would have initiative and ability, and would be able to cope with all the problems except that of the supply of water. The present tendency is to revert to the original conceptions of the irrigation congresses, and this thought is expressed in Section (8) of the report.

The writer believes that criticisms and distrust of Federal reclamation activities are decreasing, due, first, to the withdrawal by former Secretary Work, of the Department of the Interior, of restrictions of the rights of private enterprise to build reservoirs in Colorado on the Rio Grande and North Platte Rivers. While this reversal of previous rulings came too late to be of great benefit to that State, because the time when funds could be secured had passed, this saner and more equitable attitude of the Department of the Interior is appreciated. Second, the general conduct of the engineers of the Reclamation Service is winning ever greater popular approval. The present-day opposition and criticism is due to the attitude of the Government's legal advisers who claim that the title to water is in the hands of the Government and thus deny any weight to the article of the Enabling Act to the effect that all States are equal and are admitted on the same terms as the original thirteen States. Criticism is now more outspoken than formerly against the growth of bureaucracy, of which President Hoover, President Coolidge, and others have warned the Nation.

In the writer's experience the management of projects by the water users who pay the bills (Section 15),* is more economical than that by either Government or corporations. In time, through experience, the water users become as efficient as and more conservative and less technical and theoretical than either.

Elwood Mead, M. Am. Soc. C. E., states† it to be his conviction that "one of two things should be done: (1) Either provide aid in carrying out the second stage of reclamation; or (2) quit building canals to irrigate unim-

* *Proceedings*, Am. Soc. C. E., September, 1928, Papers and Discussions, p. 2100.
† *Loc. cit.*, May, 1929, Papers and Discussions, p. 1210.

proved land." The writer agrees with this statement, but chooses the second alternative, because he believes that—except for local ambition, a megalomania, and the desire for local expenditures—there is no demand for such construction; and that, as indicated in Section (8) of the report, the construction of storage reservoirs for supplemental supplies and flood protection for existing projects meets the present need and also that for a very long time in the future. This need is aptly illustrated under the head of "Achievements of Federal Reclamation in Completing Private Development," in Mr. Mead's paper.* The Salt River, the Weber Valley, and the Snake River are mentioned; the Uncompahgre Project, where a supplemental supply for existing canals was secured through a tunnel, and the North Platte Project, where many existing canals were supplied with a supplemental supply from Pathfinder Reservoir, are other examples. In the latter case a community of interest was created and conflicts of water rights were eliminated. On that project priority of right has now little meaning; but the matter of control and distribution remains a source of conflict and litigation in a three-cornered fight—in which Wyoming, Nebraska, and the Federal Government are concerned. Allocation of the water between the States and joint control by compact would solve the problem.

The explanatory statement by Mr. Mead† is particularly worthy of study as it presents many facts pertinent to the question of reclamation by irrigation in general, and not to Federal reclamation alone. While the Committee's report was prepared by engineers, several of them were raised on irrigated farms; some now (1930) own and operate such farms; and all, probably, have had to deal with irrigation economics and with settlers.

It is probably impossible (outside the group of irrigation engineers) to secure a committee more familiar with all three phases that Mr. Mead mentions, namely, engineering, economics, and the settlement problem. Such being the case, it is vital that the subject be thoroughly discussed by engineers other than those of the Committee, and the fact that so many have entered the discussion and that ample time has been given for the discussions, is a matter for congratulations. While the discussions have been local in view application, taken in their entirety they have a National viewpoint. Out of them a correct conclusion will doubtless be reached that will be to the credit of the Engineering Profession in that it will demonstrate that engineers do not have as narrow an outlook as they are sometimes accused of having.

The subject discussed herein may be summarized as follows:

(*a*) There is no hurry; the Government should beware of camouflaged frauds in considering requests for Federal aid.

(*b*) Reclamation is at present a local and not a National question.

(*c*) Under present (1930) National conditions of market and crops the settler is likely not to succeed even with local, State, and Federal aid.

(*d*) To complete and perfect existing projects is desirable from a local viewpoint, although it is not necessary nationally.

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* *Proceedings*, Am. Soc. C. E., May, 1929, Papers and Discussions, p. 1211.

† *Loc. cit.*, p. 1207.

(*e*) Willingness on the part of the State and locality to assume responsibility and a part of the cost should be the test of necessity and feasibility.

(*f*) Most projects that are feasible, desirable, and opportune can be built without Federal aid.

(*g*) The control and distribution of water should remain in the State.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

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THE BEHAVIOR OF A REINFORCED CONCRETE ARCH DURING CONSTRUCTION

Discussion*

By C. M. Strahan, M. Am. Soc. C. E.

C. M. Strahan,† M. Am. Soc. C. E. (by letter).‡—In seeking to answer inquiries for the guidance of the bridge designer, Mr. Slack has interested the entire fraternity of reinforced concrete engineers by his invasion into a difficult and important research area. This involves the study of green concrete as it passes into the form of an elastic or semi-elastic solid progressively capable of direct stress within itself and of inducing stress in its associated steel reinforcement. The author directs attention to initial stresses acquired at high temperature during set, and has followed these and other stress phenomena during the four-month program of his observations.

His paper brings together a large body of original data valuable in itself, pioneer in character, and highly suggestive in relation to further basic research into stress behavior and stress distribution in structural concrete.

The tests described follow upon and supplement the elaborate series of high-load stress experiments executed at the Yadkin River Bridge, in North Carolina, in 1928, using the sensitive electric telemeter to measure the attendant deformations.§ Mr. Slack, likewise, has used the telemeter as his main oracle in the present tests. The paper makes clear the careful planning with which the test installation was conceived and handled under field conditions. The methods pursued and the observed data will command confidence and interest.

As far as the writer is informed, the electric telemeter devised and perfected in the U. S. Bureau of Standards has been applied chiefly in deformation measurements on large scale field tests of concrete bridges, as witnessed

* Discussion of the paper by Searcy B. Slack, M. Am. Soc. C. E., continued from May, 1930, *Proceedings*.

† Prof., Civ. Eng., Univ. of Georgia, Athens, Ga.

‡ Received by the Secretary, May 15, 1930.

§ *Proceedings*, Am. Soc. C. E., March, 1929, Papers and Discussions, p. 608.

by Mr. Slack's paper, by the 1928 Yadkin River tests, and by other important current research.

It is not out of place to call attention to its possibilities in laboratory research as well, where the control of the conditions of the experiment can permit the elimination of certain variables and where, in consequence, this highly sensitive detector of linear deformation can have its widest range of usefulness in the study of concrete. Without attempting detailed description, the telemeter is sensitive in the order of 0.00001 in., and is read on a Wheatstone bridge in terms of the varying electrical resistance in the carbon stack of the instrument, its direct readings being converted into the corresponding deformations by proper calibration curves. It measures total deformation from the initial reading, however it may be caused.

Applied to concrete under load, the main factors causing deformation are elastic stress itself, temperature changes (requiring the auxiliary use of an electric temperature coil), movements due to moisture changes, and, during long and continued loading, those movements which need further study connected with the theory of plastic flow in concrete and which seem to involve relatively large deformations gradually acquired without the existence of the elastic stress or the occurrence of the fractures normally to be expected. For these several variables, with the use of the auxiliary temperature coil, it would seem that the telemeter can be made of much confirmatory and research value. The fact that the instrument can be embedded in green concrete permits the study of early set phenomena, which, as Mr. Slack points out,* involves the existence of initial stresses of considerable magnitude between concrete and steel. This also may embrace areas of marked local stress in mass concrete of whose existence and magnitude nothing is known, and whose persistence or later relief by internal molecular flow is equally obscure. It lends itself also to the determination of various constants, especially the modulus of elasticity, which, at present, is determined by deformation in the outer shell of the test cylinder *versus* its determination by means of the embedded telemeter from the internal deformation in the center of the test piece.

In the field of plastic flow studies, it seems to offer great possibilities for a long-time record and variable loadings, care being taken to master any hysteresis behavior of the instrument when stresses are relieved and re-applied. Without further elaboration, it is hoped that this suggestion of the telemeter as a delicate supplementary tool in the study of basic concepts regarding concrete may awaken interest among laboratories and other agencies interested in perfecting scientific knowledge of the nature and behavior of concrete.

* *Proceedings*, Am. Soc. C. E., November, 1929, Papers and Discussions, p. 2289.

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HIGH DAMS

A SYMPOSIUM

Discussion*

BY MESSRS. A. FLORIS, JOSEPH WRIGHT, A. J. WILEY,† D. C. HENNY,† AND J. G. TRIPP.†

A. FLORIS,‡ ESQ. (by letter).§—In his paper‖ Mr. Wiley deals briefly with the relative merits of the various types of high dams. In doing so he arrives at certain conclusions which are at variance with the views commonly held by hydraulic engineers. The present discussion will be confined merely to the action of water in the pores of gravity dams.

The problem of water pressure within a gravity dam has been attacked successfully by Professor Paul Fillunger.¶ His treatment of the subject is based on a rigorous mathematical analysis the results of which agree substantially with the tests. His findings are contrary to the opinions advanced by Mr. Wiley, so that it is worth while to compare them.

Mr. Wiley** reasons that uplift can be produced either in a horizontal seam in the rock, or in an open horizontal construction joint in the dam itself. Therefore, if the dam has no such planes of weakness, no consideration should be given to the possible action of uplift. On the other hand, the water pressure within the pores of the concrete exerts no influence on the static equilibrium of the structure. This view is supported by calling attention to the fact that the pressure in the pores is equal in all directions, so that to an upward pressure within a pore is opposed an equal pressure in the adjoining pores, causing all forces to be cancelled. The fallacy of these statements can be seen by referring to Professor Fillunger's scientific investigations.

* Discussion of the Symposium on High Dams continued from May, 1930, *Proceedings*.

† Authors' closures.

‡ Civ. Engr., Los Angeles, Calif.

§ Received by the Secretary, March 31, 1930.

‖ *Proceedings*, Am. Soc. C. E., November, 1929, Papers and Discussions, p. 2318.

¶ "Der Auftrieb in Talsperren," von Dr.-Ing. Paul Fillunger, *Oesterreichische Wochenschrift für den öffentlichen Baudienst*, 1913, Nos. 31, 32, 33 and 34; "Neuere Grundlagen für die statische Berechnung von Talsperren," von Dr.-Ing. Paul Fillunger, *Zeitschrift* des Oesterreichischen Ingenieur und Architekten-Vereins, 1914, No. 23; also, reprint.

** *Proceedings*, Am. Soc. C. E., November, 1929, Papers and Discussions, p. 2323.

If the pressure in the pores of a body is the same in all directions, but can vary from pore to pore, it is evident that no uplift can be produced. However, if the water meets greater resistance in one direction than in another an uplift pressure invariably will result. This case is present in a gravity dam, in which some parts are more porous than the others. The uplift in a concrete dam depends, therefore, on the porosity of the aggregates and of the cement paste, respectively. Since the difference in the porosities of these two materials in a dam is comparatively high, it follows that uplift cannot be neglected.

Since a rigid body submerged in an incompressible liquid, will have an equal pressure upon its surface in all directions, the resultant of the weight of the body and of the total pressure of the liquid, acting upon it, will be directed upward or downward, depending on the relative magnitudes of these two forces. On the other hand, if the water is surrounded by a rigid body, the same resultant acts on the surface of this body but has an opposite direction. This last case is of importance in the application of this law to gravity dams. In both cases the forces acting on a surface are replaced by forces acting in space, or, in other words, by forces denoting mass.

The field intensity, p, of the pore pressure in the interior of a gravity dam is smaller than the field intensity of the water in the reservoir, that is, the hydrostatic head, which is given by,

$$p_0 = \gamma\, h \qquad (3)$$

in which,

γ = specific weight of water; and

h = height of water level measured from the point under consideration.

At the down-stream face of a porous dam the field intensity of the pore pressure will be necessarily zero, or,

$$p_2 = 0 \qquad (4)$$

Near the up-stream face the field intensity of the pore pressure will be a little less than the hydrostatic head, or,

$$p_1 < p_0 \qquad (5)$$

From this expression it follows that the water in the pores of a dam will be in a state of steady flow. Assuming that the dam is water-tight at the down-stream face as well as at the base, then the infiltrating water will reach equilibrium when all pores in the concrete are filled with water. In this case the intensity of the pore pressure will be the same as the pressure in the reservoir at equal elevations. In an ordinary case, however, in which neither the dam nor its rock foundation is water-tight these lines of equal intensity or gradients will deflect toward the base and toward the down-stream face because of the variable intensities of the pore pressure (Fig. 8).

If a sufficiently small portion of the dam with the volume, v, is considered, then the resultant, P, of the water pressure on the walls of all the pores of this portion, will be expressed (according to Professor Fillunger) by,

$$P = (\mu - \nu)\frac{\partial p}{\partial z}\, v \qquad (6)$$

in which,

$\frac{\partial p}{\partial z}$ = gradient of pore pressure,

μ = coefficient of porosity of aggregates; and

ν = coefficient of porosity of cement paste.

By dividing both sides of Equation (6) by the volume, v, the pore pressure per unit of volume will be obtained.

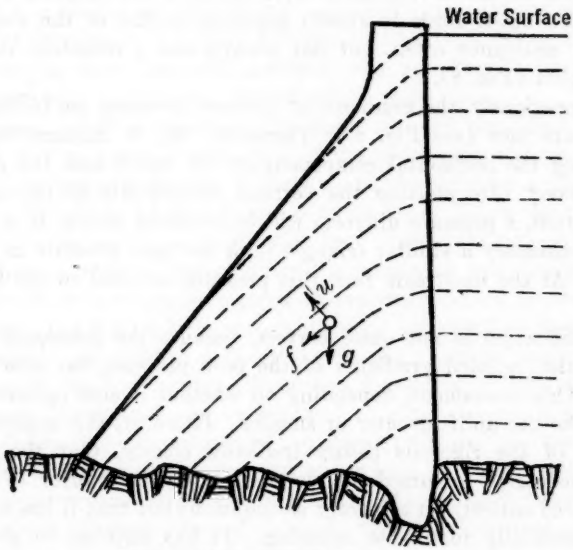

FIG. 8.—GRADIENTS OF PORE PRESSURE.

Since the porosity of the aggregates is usually less than the porosity of the cement paste, it is evident from Equation (6) that the pore pressures of this small portion will have a negative resultant. Consequently, within a dam there is always present a force with a direction opposite to that of the weight of the water. This is, however, none other than the uplift pressure itself.

Assume, for example, that it is possible to build a gravity dam with cement, but without the addition of aggregates; then, obviously, $\mu = \nu$, and Equation (6) becomes $P = 0$. In this case, therefore, uplift cannot be produced in the dam. In ordinary cases, however, the consideration of uplift in the stress analysis of gravity dams should never be omitted in spite of Mr. Wiley's assertions* to the contrary. If cobblestones are to be used, this consideration is rendered more imperative, due to their larger size.

The steady flow in a gravity dam caused by the difference of pore pressure in the dam and the hydrostatic head in the reservoir will be maintained by the frictional resistance of the water on the walls of the pores. This resisting force was determined by Professor Fillunger and was found to depend on the

* *Proceedings*, Am. Soc. C. E., November, 1929, Papers and Discussions, p. 2323.

porosity of the aggregates, on the gradients of the gravitational field (weight), and on those of the pore pressure, respectively. Therefore, within a gravity dam that is exposed to water pressure there are two forces acting, besides the weight, g. These are, the uplift, u, and the frictional resistance, f (Fig. 8). The weight is an actual force of mass with a vertical direction, the uplift is normal to the gradients of the pore pressure, and the frictional resistance is perpendicular to it. Consequently, the uplift can be considered as a normal stress, while the frictional resistance corresponds to the shearing stress. The direction of the uplift force is always opposite to that of the weight, while the frictional resistance often, but not always, has a direction the same as that of the uplift (Fig. 8).

As stated previously, the gradients of the pore pressure are inclined toward the down-stream face (see Fig. 8). Therefore, Dr. K. Kammüller has suggested omitting the horizontal component of the uplift and the entire frictional resistance.* By plotting the vertical components of the uplift in a horizontal section, a pressure diagram will be obtained which, in a triangular dam, is approximately a similar triangle with the zero pressure at the down-stream face. At the up-stream face this pressure is equal to the hydrostatic head.

This simplification is not quite correct, because the frictional resistance acting along the inclined gradients of the pore pressure has also a vertical component. This component, depending on whether it acts upward or downward, will make the uplift greater or smaller. However, this somewhat crude simplification of the rigorous theory indicates clearly, that the triangular distribution ordinarily assumed, of the uplift in the interior of a gravity dam, is not based entirely on arbitrary assumptions but that it has to a certain extent a scientifically justifiable meaning. It has nothing to do with the opening of a joint, as Mr. Wiley is inclined to assume, but it is the necessary consequence of the pore pressure distribution in the structure itself.

The foregoing theory explains in a natural way the mechanical action of water in the pores of a dam and is exact and free from assumptions. It is true that the analysis depends largely on the properties of the materials used in the construction of dams and of their rock foundation for which the various coefficients of porosity, etc., must be known in advance with a reasonable degree of accuracy. However, these are problems which confront the engineer in all his activities.

In order to determine whether the water pressure exerts a bursting action within the pores of a material by overcoming its tensile strength a test specimen can be subjected to water pressure of equal intensity and, at the same time, to a pull acting along its longitudinal axis.

Rudeloff, Panzerbieter, and Professor Fillunger tested cylinders of cement mortar to failure by an axial pull.† In these specimens water was forced into the pores. In some specimens this penetration was prevented by means of a

* "Die Theorie der Gewichtstaumauern unter Rücksicht auf die neueren Ergebnisse der Festigkeitslehre," von Dr.-Ing. K. Kammüller, Berlin, 1929, p. 13.

† "Versuche über den Porendruck des Wassers im Mauerwerk," von M. Rudeloff und Panzerbieter, *Mitteilungen* aus dem königlichen Materialprüfungsamt zu Gross-Lichterfelde, 1912, Ergänzungsheft No. 1.

rubber membrane. Other specimens were subjected to a water pressure on the outer face of the cylinder. This outer surface in some specimens was also protected by a membrane; in other specimens it was not. In all cases the water pressure was raised to a certain limit and then the axial pull was increased until the material failed. In addition to these, other specimens were tested by an axial pull only without subjecting them to a water pressure.

In these tests it was found that by increasing the water pressure the decrease of the tensile strength of the material was very rapid so that under a pressure of 8 to 10 atmospheres, it was practically zero. If the results of these tests are correct then, as Professor Fillunger states,* gravity dams higher than 250 ft. cannot be safe, because a material with zero tensile strength will also have an uncertain compressive and shearing resistance.

The reason for this apparent difference between tests and experience is easily seen if it is realized that each of the specimens was prepared in four horizontal layers perpendicular to the axis of the cylinder. Obviously, these planes were the points of weakness. Between these layers the cement mortar was very porous so that it resisted uplift very little while in the remaining more dense and sound parts of the cylinder the resistance was considerable. This unequal distribution of the pore pressure evidently caused the failure.

Sometimes the failure of a porous material subjected to water pressure in all directions is interpreted as being due to the bursting action of the water in the pores of the body under pressure.† According to Professor Fillunger, in such cases the rupture can be assigned to other causes than tension.‡ It is very probable that the release of the water pressure was too rapid so that the water in the inner pores remained under high pressure, while in the outer pores the pressure was reduced considerably. The abrupt change of the pressure within the different parts of the body causes an unequal pressure distribution which results in failure by tension.

Specimens of standard briquettes consisting of the porous Portland cement mortar, less porous slag-cement mortar, and non-porous sealing wax, were subjected by Professor Fillunger to a water pressure in all directions of 100 to 200 atmospheres, respectively.§ Some specimens were tested without being under water pressure. Simultaneously, a steadily increasing axial pull was applied by the aid of a level arm, exactly as in testing cement mortar briquettes.

In all cases no appreciable changes in the tensile strength of these specimens of different porosities could be observed. The water penetrating into the pores of the body did not produce a bursting action, because otherwise under increasing water pressure the tensile strength of the material must decrease correspondingly.

* "Neuere Grundlagen für die statische Berechnung von Talsperren," Reprint, p. 10.

† "Permeazioni d'acqua e loro effetti nei muri di ritenuta," by Oscar Hoffman, Milan, 1928; also, "Zur Frage des Auftriebes in Talsperren," von Oscar Hoffman, *Die Wasserwirtschaft*, 1929, No. 33, and "Zur Frage des Auftriebes in Talsperren," von Paul Fillunger, *Die Wasserwirtschaft*, No. 4, 1930.

‡ "Auftrieb und Unterdruck in Talsperren," von Dr.-Ing. Paul Fillunger, *Die Wasserwirtschaft*, 1929, Nos. 18, 20, and 21, p. 334.

§ "Versuche über die Zugfestigkeit bei allseitigem Wasserdruck," von Dr.-Ing. Paul Fillunger, *Oesterreichische Wochenschrift für den öffentlichen Baudienst*, 1915, No. 29; also, reprint.

Professor Fillunger's theory shows that the water pressure penetrating into the pores of the dam produces uplift and friction. The respective vectors of these forces must be combined, analytically or by the aid of a funicular polygon, with that of the weight of the dam in order to determine the stresses produced by the water in the reservoir. It is obvious, therefore, that the water pressure in the pores of the dam is an important item which cannot be neglected in a correct analysis of stress. Consequently, Mr. Wiley's claim* that stresses are not produced by the action of the water in the pores of a gravity dam—not being sustained by theory and tests—cannot be accepted as correct.

JOSEPH WRIGHT,† M. AM. SOC. C. E. (by letter).‡—The basic papers§ by Messrs. Wiley, Henny, and Ransome are exceedingly interesting and instructive, and the discussion thus far published has added much to their value. During two years since 1928 dams and their safety have occupied a prominent place in engineering literature and in engineering thought, and it is hoped that this opportunity for a full and free discussion of the subject will bring out the consensus of opinion upon the numerous mooted questions and unknown factors entering into the design of all dams.

One question which has aroused some discussion and has brought out definite opinions from a few is that of initial arch resistance of arched gravity dams. Too many unknown quantities are involved in this question to warrant much confidence in a mathematical analysis; it can only be settled by observations and tests upon existing dams. Such an analysis as is possible may be somewhat enlightening, but it also serves to emphasize current ignorance on the subject in the lack of established facts to substitute assumptions founded upon more or less far-fetched inferences. Why not go at the question in a practical way? Core borings with the drill following the joints between successive blocks, and temperature readings within the hearts of well-seasoned dams, would throw much light on the subject, and other inexpensive tests could be devised that for all practical purposes would settle the question.

Little is known about the temperature changes in the heart of a thick dam; less is known about the effect and extent of water-soaking of the concrete. Engineers are equally ignorant as to the effect of the usual method of construction, that is, the casting of alternate blocks between blocks that have hardened in air, and have given up at least a small part of their chemical heat; and they know little or nothing as to "plastic flow", or "permanent set." Besides, what they know of the modulus of elasticity of concrete is worth little upon the assumption of its uniformity in a dam. Why, therefore, attempt to juggle figures that mean so little?

The fact that the St. Francis Dam, built without vertical construction joints (as a monolith), cracked, is not conclusive to the writer. It has been well established by the "post mortem" that the abutments were incapable of sustaining heavy thrusts. This and the fact that the dam was built as a

* *Proceedings*, Am. Soc. C. E., November, 1929, Papers and Discussions, p. 2323.
† Chattanooga, Tenn.
‡ Received by the Secretary, June 13, 1930.
§ *Proceedings*, Am. Soc. C. E., November, 1929, Papers and Discussions, p. 2317 *et seq.*

monolith would predicate a behavior different from that of dams constructed in the usual manner between competent abutments.

As in most questions of controversy, however, much can be said on both sides. Certainly, the St. Francis Dam should not have been built as an arched dam, with or without compensation for the convergence of radial planes, because of incompetent abutments, and its rather great length; although from all that is now known about it, it would have failed just as surely had it been so compensated. Yet no reasonable man who has seen their foundations and profiles will say that such dams as the Arrowrock, the Exchequer, and some others which have been constructed in narrow canyons are unsafe by reason of the lack of compensation for the convergence of the radial planes; nor would he say that a straight dam of a gravity section presented less hazard as to ultimate failure. That would appear to be the crux of the question—fit the dam to the physical conditions of the site. It is well understood that a narrow canyon having competent abrupt walls is the only suitable site for a single arch of any description. Accepted principles of dam design are rooted in meager knowledge, yet it is feared that this fact is too often overlooked.

In the analysis of the gravity dam, designers know the pressure upon its face; they know, approximately, its weight; and exceedingly roughly they know the strength of concrete made under certain ideal conditions—just the bare skeleton of the problem. The remaining factors have to be assumed; yet they talk about factors of safety as if they were known definitely. In the analysis of the arched dam the engineer's confidence in reaching correct conclusions from false premises through the medium of clever mathematical gymnastics approaches the sublime! He starts with the assumption that concrete is a homogeneous material, uniformly elastic, and that it behaves uniformly under the influence of temperature changes—all of which is known to be false, or to say the least, decidedly inexact. He forms certain fundamental equations as to stresses based upon fallacies, or inexact theories of arch resistance. Then, making certain convenient assumptions with a view of simplifying the solution, he blandly proceeds to compute the stresses in all parts of the dam, ignoring the fact that an arched dam is not a true arch in any sense, and that in no two dams having different profiles would the stresses be at all comparable!

Within the last few years the tests upon models and upon the Stevenson Creek Dam, supplemented by measured strains in existing dams, have cleared the problem of the arched dam somewhat and designers now need not labor wholly in the dark; but more than anything else these tests have shown that previously accepted theories gave results widely different from the facts, and that engineers must rely largely upon models and tests of existing dams to arrive at an approximately correct design.

In considering the stability of dams, therefore, it is well to remember that all so-called factors of safety are based on assumptions that are generally unsupported by adequate reliable experimental data; and this would seem to be very pertinent to the question of the safety of higher and still higher dams, designed in accordance with accepted theories. The design of all dams

is at present, and, for some time, must remain largely in the field of empirics, and any too radical a departure from experience is fraught with more or less danger. One safeguard that the writer would suggest is provision for drawing down the reservoir wherever at all practical, even at large expense.

Probably the majority of gravity masonry dams fail by sliding upon, or beneath, their bases. Strictly analyzed, several factors are involved in such failures of which little is known. As to frictional resistance to sliding upon the base, it is generally conceded that this may be from 55 to 70% of the pressure upon the base, or perhaps a little more in exceptional cases. This opinion seems to be fairly general. On the other hand, what can be said of bonding and of shear? It is well known that some existing dams are not safe against sliding as determined by the theory of frictional resistance alone, with 70% as the upper limit, yet few have expressed an opinion as to the probability of extensive bonding, and as to what reliance may be placed upon it. Mr. Wiley mentions two instances in which more or less bonding was demonstrated, but he fails to state whether or not it was general and expresses no opinion as to its reliability in the general case.

During the construction of the Wilson Dam the writer had occasion to build a pier which, later, had to be removed. It was founded upon horizontally bedded sound limestone of a highly siliceous character that was entirely free from cracks, and was comparatively smooth. The surface was thoroughly flaked and nicked by hammering with 4-point heavy drill-bits, and was made perfectly clean by repeated washing with high-pressure jets of air and water. A good coat of neat cement grout was then applied and broomed in and the pier was constructed of a good quality of 1:2:4 concrete. The writer was very anxious to know to what extent bonding had been effected, but was not much surprised to learn that only 4% of the area of the base had bonded with the rock surface. This too took place directly under the point from which concrete was dropped into the forms in the initial stage of construction, and there seemed no other reason why it should have bonded at this point rather than elsewhere. This point was not at the center of the pier, and the surface there was apparently no rougher than elsewhere.

It is the writer's theory that upon smooth horizontally bedded rock initial bonding takes place quite generally, if the rock is perfectly clean and sound, but that in the process of crystallization the concrete pulls away before adequate strength is developed. However, this is only a theory, although in the case of the pier mentioned he thought he could detect slight evidence that this had happened. The shape of the foundation may have been a factor in the result. The base of the pier was only 4 ft. wide and 24 ft. long.

Advocates of the theory of extensive bonding sometimes base their opinion upon results obtained in brick and stone structures. In this, they overlook the fact that, in such a case, the mortar generally constitutes a very thin sheet and lacks the necessary cohesive strength to pull away, and that cracks form in the sheet instead. This might possibly explain the bonding observed by Mr. Wiley in the overturned sections of the St. Francis Dam.* Weakness

* *Proceedings*, Am. Soc. C. E., November, 1929, Papers and Discussions, p. 2323.

in the planes of lamination or the joint planes of the mica schist may have yielded to the cohesive strength of the concrete. However, it is easy to believe that if the foundation presents a sufficient number of prominent projections, such as would generally be the case if the stratification was upon edge, or dipped strongly, effective bonding may be obtained to a very considerable extent, with care in cleaning and grouting.

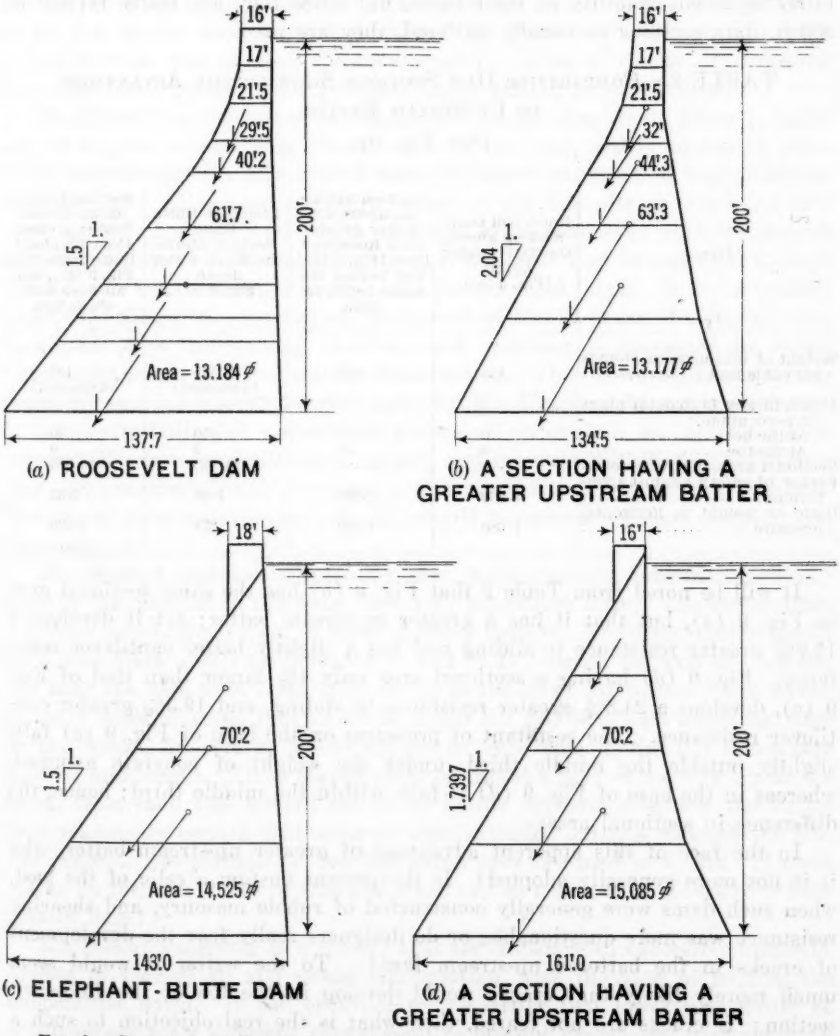

FIG. 9.—SECTIONS OF ROOSEVELT AND ELEPHANT BUTTE DAMS COMPARED WITH DAM SECTIONS HAVING GREATER UP-STREAM BATTER.

Lacking evidence as to the extent upon which effective bonding may be depended, it is necessary to rely upon frictional resistance. For this, greater pressure on the base is demanded, and the writer has often wondered why

dam designers have not made greater use of up-stream batter in the case of gravity sections. Fig. 9 and Table 2 are presented to show that greater up-stream batter is not inconsistent with cantilever resistance and minimum sections. Fig. 9 (*a*) and Fig. 9 (*c*) represent sections of the Roosevelt and the Elephant Butte Dams* from their crests to a water depth of 200 ft. Since both these dams are much higher than 200 ft., the diagrams shown may not fairly represent stability at their bases, but since they are fairly typical of 200-ft. dam sections as usually battered, they are used.

TABLE 2.—COMPARATIVE DAM SECTIONS SHOWING THE ADVANTAGES OF UP-STREAM BATTER.

(See Fig. 9.)

Item.	Roosevelt Dam: Section above the 200-ft. water depth. (Fig. 9 (*a*)).	Section having an up-stream batter greater than Roosevelt Dam (Fig. 9 (*b*)), but having the same sectional area.	Elephant Butte Dam: Section above the 200-ft. water depth. (Fig. 9 (*c*)).	Section having an up-stream batter greater than Elephant Butte Dam (see Fig. 9 (*d*)), and an area only 4% larger.
Weight of masonry, in pounds per cubic foot	155	155	145 (assumed)	145 (assumed)
Uplift, in any horizontal plane (in percentage):				
At the heel	0	0	50	50
At the toe	0	0	0	0
Sectional area, in square feet	.....	.....	14 525	15 085
Factor of safety against overturning	2.260	2.263	1.93	2.32
Ratio of weight to horizontal pressure	1.680	1.839	1.378	1.626

It will be noted from Table 2 that Fig. 9 (*b*) has the same sectional area as Fig. 9 (*a*), but that it has a greater up-stream batter; yet it develops a 15.9% greater resistance to sliding and has a slightly better cantilever resistance. Fig. 9 (*d*) having a sectional area only 4% larger than that of Fig. 9 (*c*), develops a 24.8% greater resistance to sliding, and 19.5% greater cantilever resistance. The resultant of pressures on the base of Fig. 9 (*c*) falls slightly outside the middle third, under the weight of concrete assumed, whereas in the base of Fig. 9 (*d*) it falls within the middle third; hence, the difference in sectional areas.

In the face of this apparent advantage of greater up-stream batter, why is it not more generally adopted? Is the present custom a relic of the past, when such dams were generally constructed of rubble masonry, and shearing resistance was more questionable, or do designers really fear the development of cracks in the battered up-stream face? To the writer it would seem much morely likely that cracks would develop in the toe of the Roosevelt section. If cracks are not feared, then what is the real objection to such a section?

Hydrostatic uplift is another question about which there seems to be no unanimity of opinion. All agree that there must be more or less uplift, but

* As taken from "Construction of Masonry Dams", by Chester W. Smith, pp. 247 and 253.

they differ as to its probable amount. Naturally this will vary greatly on different foundations, and it is not expected that this feature of design could be standardized; nor would it be desirable. Data on uplift are gradually being accumulated from measurements on existing dams, and within a few years something may be known about the subject. In the meantime, however, would it not be well to assume that with the best of foundations the uplift may be as much as full reservoir pressure at the heel and tail-water pressure at the toe, acting upon 50% of the base? In the writer's opinion this gives a dam section that is none too conservative. This, of course, is assuming that effective drainage is not provided for, or cannot be relied upon.

Mr. Wiley* has touched upon leakage, or seepage along "cold joints". Leakage, or seepage, is decidedly objectionable in any dam, and is, of course, especially objectionable in thin arched dams, or others composed of comparatively thin sections. All concrete workers appreciate the difficulty of treating a cold joint so as to make it water-tight, and they know that, even with the utmost care in picking, cleaning, and grouting, some joints may be expected to seep, if not leak. One remedy, of course, is to have no cold joints. It is the writer's experience that, if the succeeding lift be cast within 24 hours of stopping, only reasonable care in cleaning, wetting, and grouting is necessary to secure water-tight joints in a moderately thick section. This, however, is not always practical, and for reasons touched upon by Mr. Henny, may not be desirable. In a recently built thin arched dam plate-steel water-stops set into the superimposed lifts were used with good results, and the writer knows no good reason why the practice should not become general—particularly in thin sections. The use of these water-stops, however, should not excuse careless cleaning and grouting.

Mr. Henny† expresses a suspicion that cracks through construction blocks more or less parallel to the water face may sometimes form. The writer fears that they generally do form in dams of heavy section. The construction blocks of Wilson Dam were alternately 38 ft. and 54 ft. long on the axis of the dam, and near the base were from 75 to 90 ft. in the other dimension. The writer in many cases observed that when the upper lift had been cast about ten days or two weeks, fine cracks would appear on each side of the lift somewhere near its mid-length. These cracks could be traced a part of the way across the block, from each side toward the crack in the opposite side, and the writer feels reasonably certain that some at least extended entirely through the lift. The concrete was perhaps the best the writer ever saw used in a dam. If these cracks do form in blocks of this size what can be expected of blocks of a high gravity dam?

Mr. Henny suggests‡ as a remedy that the thickness of successive blocks be limited to 4 or 4½ ft. and slower construction, with the view of holding the temperature down. It had occurred to the writer that lower lifts would be of advantage for a different reason—not to prevent cracks, but with the hope that cracks in the superimposed lifts would break joints, and would not lie

* *Proceedings*, Am. Soc. C. E., November, 1929, Papers and Discussions, p. 2324.
† *Loc. cit.*, p. 2333.
‡ *Loc. cit.*, p. 2334.

in the same vertical plane. Thinner lifts would favor lower temperatures, but they would also favor more rapid cooling and drying, which would tend to produce cracks. Besides, in most cases, thin lifts would be most vexatious and costly in their effect upon speed of construction, and the greater number of "cold joints". When, in the construction of Wilson Dam, the 12-ft. lifts were reduced to 6 ft., progress was reduced by at least 25%, and the cost of placing raised to a greater extent, although the great length of the dam was especially favorable to the use of thin lifts. In this case, since forms were handled by traveling cranes in large sections (an entire side at a time), it was easy to construct and hold the forms for a 12-ft. lift. The reduction of lifts to 6 ft., therefore, cost considerable money, and, in this case, was probably of doubtful advantage.

Shrinkage and cracking are troublesome factors to concrete designers, and it occurs to the writer that the cement manufacturer and his chemists could probably contribute much to remedy this condition. Early cracks and planes of weakness seem to form much more readily in concrete made of certain cements than in that made of other brands. Aging of the cement of course improves this condition, but it is probable that the chemist could find means more effective. The use of pre-cooled mixing water during warm weather, as suggested by Mr. Henny, would certainly contribute to the desired result.

As to temperature stresses in well-seasoned concrete, do engineers really know much about these? The coefficients of expansion of different aggregates are given by some, as follows: Limestone, 0.00000251; granite, 0.00000868; and quartz, 0.0000101. These show a wide variation. Therefore, is it not certain that concretes made of different aggregates would show almost an equal variation in coefficients of expansion, not to mention concretes of different densities made of the same aggregates?

Mr. Henny states* that,

"Concrete with a known strength of 3 000, 4 000, and even 5 000 lb. per sq. in. in 28 days is now readily secured, and there seems to be no reason why concrete in the base of a dam could not be safely subjected to compressive stresses of 1 000 lb. per sq. in., or even more (such as would obtain in a dam about 630 ft. in height), provided a source of satisfactory aggregate is at hand, and a proper mix is used."

The statement as worded is carefully guarded, but it may be well to emphasize its limitations by introducing a few more definite provisions: First, that the maximum stresses can be known within a reasonable degree of approximation and that they may not exceed this limit, temperature stresses being considered; second, that such stresses be not used in thin sections subjected to water-soaking, and to wide temperature changes; third, that designers do not deceive themselves as to the feasibility of securing the early strengths specified; and, fourth, that reliance can be placed on the permanent quality of the concrete. All these may be implied in the statement, but its limitations cannot be too strongly emphasized. However, taken as it stands, if the base of a dam of gravity section is meant, it can do no harm for the reason that, at present, economics would bar the construction of a 630-ft. dam except in narrow

* *Proceedings*, Am. Soc. C. E., November, 1929, Papers and Discussions, p. 2330.

canyons having sound abutments, where its resistance to punching shear would be a very large factor in its stability.

As to the manufacturing of concrete and present knowledge of it, have engineers really made as much progress within the past few years as they sometimes imagine? In the writer's opinion, some of the more conservative engineers are entirely too confident of securing a specified strength. Density and water-tightness are not synonymous with strength, although they are important elements in the durability of concrete, which fact is sometimes overlooked.

Within the last several years the writer has had reason to make an occasional visit to a rather high dam that was under construction, in which high stresses were expected and for which claims of especially high-grade concrete were being made. Undoubtedly those who made such claims believed them to be true, but it was the writer's observation, confirmed by several visits, that the control exercised was entirely too lax to expect such results, and that in parts of the structure, at least, they most certainly were not secured. Some of the aggregate was too dirty for use in any structure; some of the batches that went out, appeared to be practically all mortar; occasionally, batches went out that were sloppier than the writer ever approved in the most reckless period of concrete history; and when the dam neared completion places could be pointed out where excess water was used.

In another case, the writer is told that a certain high dam which is known to have been built under rigid specifications, based upon Professor Abrams' discoveries as to strength, fineness modulus, and slump, leaks and seeps to an unusual extent.

With these experiences and others equally convincing, it is best not to be deceived in the idea that it is now easy to secure a concrete of predetermined strength and density. It is not easy, and with the utmost care the results are sometimes disappointing.

Therefore, the writer advises caution in the matter of increasing the working stresses in high dams—particularly in high, thin dams. Twenty-five years hence something may be known about concrete. At present, there are very many things that engineers do not know.

A. J. WILEY,* M. AM. SOC. C. E. (by letter).†—Professor Switzer begins his discussion‡ by citing three points in the main paper with which he cannot agree. The writer accepts these points as fair presentations of the claims made in his paper.

With reference to the first point Professor Switzer takes the case of a vertical slice of a dam, 1 ft. thick at the up-stream face and converging on radial planes, with full water pressure upon the up-stream face. The length of base is computed to bring the resultant, with reservoir full, at the third point for a dam, 1 ft. thick at the face, with parallel sides. He then shows that in the slice with converging radii and with the same water pressure

* Cons. Hydr. Engr., Boise, Idaho.

† Received by the Secretary, June 27, 1930.

‡ *Proceedings*, Am. Soc. C. E., January, 1930, Papers and Discussions, p. 183.

there will be tension in the up-stream face, a conclusion for which neither argument nor calculation is needed.

Professor Switzer then calculates the deflection of the cantilever with reservoir full and concludes that it is so small that "the convergence of the radii would not close up any contraction joints such as are used ordinarily between blocks of the dam", and that, therefore, no arch action can develop.

It would seem from the quotation that Professor Switzer believes that the contraction joints in a dam are built like those in a sidewalk, or pavement, with an actual opening filled with some plastic substance to provide for expansion. Nothing could be farther from the fact which is, of course, that concrete is tamped direct against concrete, with not even the thickness of a sheet of paper in the joint, and with the adjacent faces interlocking by offsets in one face into which the concrete of the other is poured.

Professor Switzer then states that arch action can be expected only when the contraction joints are grouted at times of low temperature, or when the arch has a comparatively short radius, and concludes that "arched action in a curved gravity dam can not be expected."

The writer disagrees radically with both the statements and the conclusion of Professor Switzer as cited in the preceding paragraph. He is aware that there is much support for Professor Switzer's attitude in recent engineering literature, but he thinks that much of it is from prejudiced sources. In a few actual cases perhaps a very thin arched dam contracted due to the shrinkage of concrete, because it set in air, or because it was so thin that it was fully subject to seasonal or even daily temperature changes and contracted with the cold.

There may be a case in which a thicker dam has shrunk while the concrete was setting, due to a combination of very rapid pouring, an excessive use of water in the mix, or a poorly graded and unwashed aggregate.

In the vast majority of arched gravity dams there is more effective arch action than in the thinner dams which depend for stability principally upon arch action. The more effective arch action in thick arched gravity dams comes from the fact that any mass of concrete setting in air has an exterior coat which shrinks in setting because of the lack of moisture necessary for complete hydration. The interior, protected by this outer skin from evaporation, remains moist for a longer period so that hydration of the coarser part of the cement and consequent expansion of the interior mass continues long after this action is suspended on the faces.

This dried out and contracted skin, which is inert so far as arch action is concerned, is a much larger proportion of a thin arch dam than of the thick arched gravity dam so that a larger proportion of the latter is effective as an arch.

From the nature of the situation, inspection of the contraction joints in a thick dam is confined to the vicinity of the faces of the dam, and to surfaces of galleries in the dam, either of which is a case of setting in air and is apt to show cracks due to the shrinkage which follows from this condition.

There is much evidence in the many thick arched gravity dams to show that the opening of the expansion joints extends only from 10 to 20 ft. into the dam, beyond which distance the dam is practically a monolith. This is proved by the absence of leaks from the contraction joints at the down-stream face; in the case of the Mulholland Dam it was shown by pressure tests.

Professor Switzer also dissents from the writer's claim that there is no uplift due to water under-pressure in the voids of the concrete. There seems to be some difference of opinion as Mr. Jakobsen* and Mr. Floris† have also dissented from the writer's views on this subject.

To clarify the matter it will be assumed that in some way all the voids in the concrete eventually become filled with water and that the water in the voids is under full reservoir pressure at the up-stream face, gradually reducing to tail-water pressure at the down-stream face. On a vertical column of unit cross-section at the up-stream face, assume the concrete to have a proportion of voids represented by p. In each unit of height in the column of unit area the sum of the volume of the voids will be p. It will be assumed that the voids are cubes and that they are uniformly spaced both vertically and horizontally, with n voids per lin. ft. There would be n^3 voids per cu. ft.; the volume of a single void would be $\frac{p}{n^3}$; its side or height would be $\left(\frac{p}{n^3}\right)^{\frac{1}{3}}$; its area would be $\left(\frac{p}{n^3}\right)^{\frac{2}{3}}$; and the area of all the voids in a single layer would be $n^2\left(\frac{p}{n^3}\right)^{\frac{2}{3}}$.

Assume that at any given point, H ft. below the reservoir surface, the dam is cut by an imaginary plane passing through the bottom of a layer of voids. In this set of voids the upward pressure is:

$$U = 62.5\left[\left(H - \left(\frac{p}{n^3}\right)^{\frac{1}{3}}\right] n^2\left(\frac{p}{n^3}\right)^{\frac{2}{3}} = 62.5\, H\, n^2\left(\frac{p}{n^3}\right)^{\frac{2}{3}} - 62.5\left(\frac{p}{n^3}\right)^{\frac{1}{3}} n^2\left(\frac{p}{n^3}\right)^{\frac{2}{3}} \quad \ldots\ldots(7)$$

In the next set of voids $\frac{1}{n}$ above the cutting plane, the downward pressure is:

$$D = 62.5\left(H - \frac{1}{n}\right) n^2\left(\frac{p}{n^3}\right)^{\frac{2}{3}} = 62.5\, H\, n^2\left(\frac{p}{n^3}\right)^{\frac{2}{3}} - 62.5\, n\left(\frac{p}{n^3}\right)^{\frac{2}{3}} \quad \ldots(8)$$

The uplift from water under pressure in the voids of the column of unit area at the up-stream face is the excess of the upward pressure over the downward pressure at the imaginary plane, or,

$$U - D = 62.5\, n\left(\frac{p}{n^3}\right)^{\frac{2}{3}}\left[1 - \left(\frac{p}{n^3}\right)^{\frac{1}{3}}\right] \quad \ldots\ldots(9)$$

If $p = 0.06$ and $n = 20$, $U - D = 1\,250\ (0.0000075)^{\frac{2}{3}} \times (1 - 20)\ (0.0000075)^{\frac{1}{3}}$, or 0.2915 lb., which is the uplift per square foot due to

* *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 867.

† See p. 1659.

water pressure in the voids. This is a constant for all heights of the dam and is seen to be negligible; but it is based upon the assumption of homogeneity in the concrete and would not apply to porous layers or any other departure from uniformity.

It appears from this that water pressure against the up-stream face does not affect the condition of stress in the concrete, but the assumption that voids are filled with water would add, for 6% of voids, 3.75 lb. per cu. ft. to the weight of the concrete. The net result of water pressure in the voids is to increase rather than to decrease the stability of a gravity type of concrete dam.

Mr. Noetzli* expresses the opinion that contraction cracks parallel to the axis are likely to form in massive concrete gravity dams, and he proposes to provide for them by joints parallel to the down-stream face. The writer believes that there is no evidence of such cracks in any dam except in the open galleries where they may be purely local and due to shrinkage from setting in air. The writer recalls no indication of such cracks as those indicated in Fig. 7.† This diagram shows a crack in the large slab which broke from the down-stream face for some distance up from the base. The writer examined this slab very critically in the field only a few days after the failure for the purpose of finding if there was any line of demarcation between the slab and the dam from which it broke. There was positively none, and the plane of fracture was perfectly new and fresh. It was exactly such a fracture as might have occurred in a dam which was so highly stressed as a vertical cantilever as to be on the point of overturning about its down-stream toe.

There was no evidence visible to the writer of the extension of the plane of fracture above the slab to indicate it as a part of a system of cracks, as it is shown in Mr. Noetzli's section. Figs. 10 and 11 are from photographs of the St. Francis Dam taken under the direction of the State Commission of California only a few days after the failure. The photographs show very clearly all the evidence with respect to the alleged axial cracks in the dam, and close examination under high magnification convinces the writer that what Mr. Noetzli takes to be axial cracks are simply the boundaries of radial contraction cracks which had broken part way through the dam a considerable time before the failure. The writer recalls that there were areas of water-stained surface in the freshly broken concrete which were plainly due to old, nearly radial, contraction cracks which formed planes of weakness along which separation took place at the time of failure. The lines in the photograph which resemble cracks, the writer believes to be lines between the water-stained and the freshly broken concrete, and it will be observed that the most strongly marked lines are in a nearly horizontal direction ending at the up-stream face.

The writer feels that the evidence of the St. Francis Dam is certainly not convincing enough to warrant the introduction of inclined joints, with their new problems, in a concrete dam of the gravity type; nor does he think that the formation of cracks in the thin buttresses of multiple-arch and

* *Proceedings*, Am. Soc. C. E., February, 1930, Papers and Discussions, p. 371.
† *Loc. cit.*, p. 372.

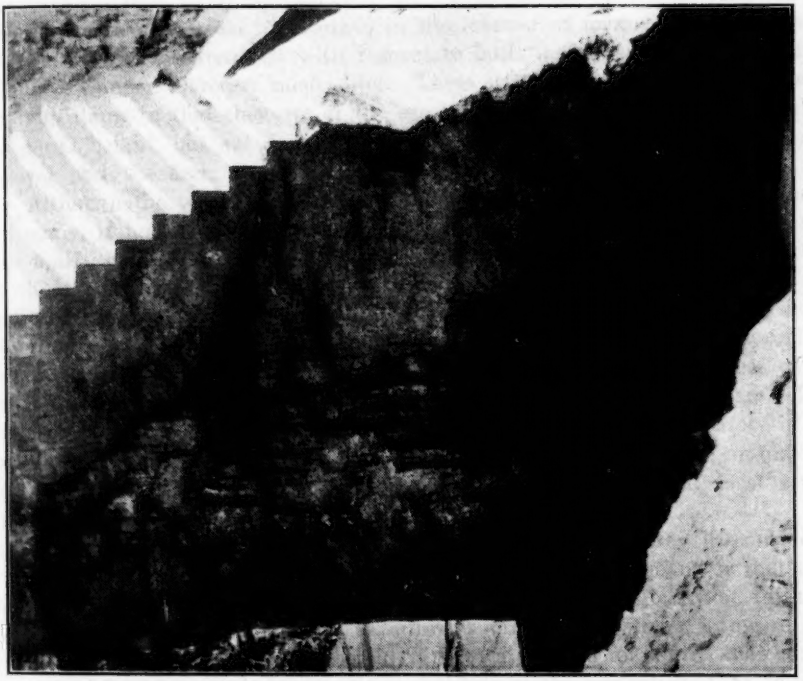

FIG. 11.—CLOSE-UP VIEW OF ST. FRANCIS DAM, SHOWING CRACKS.

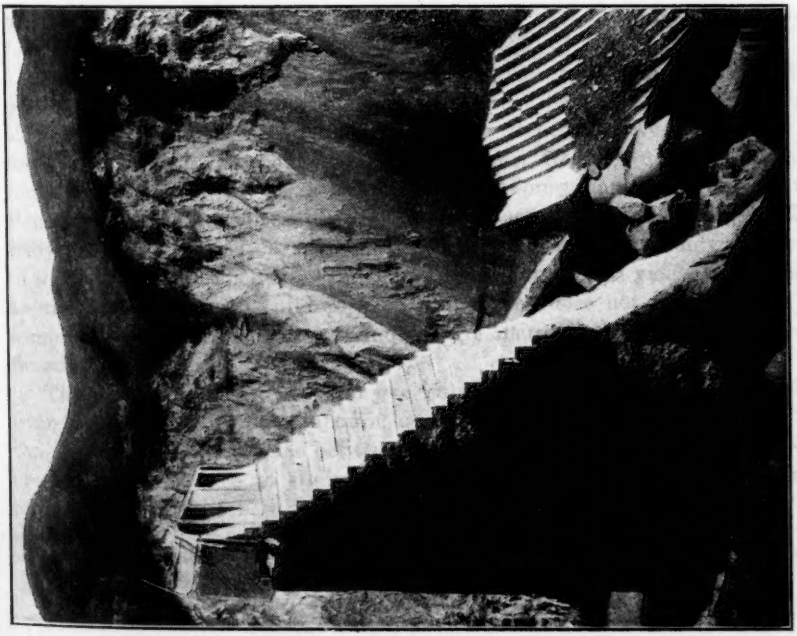

FIG. 10.—VIEW OF ST. FRANCIS DAM, SHOWING CRACKS IN DAM STRUCTURE.

Ambursen dams, where shrinkage from setting in air occurs, is proof that similar shrinkage cracks are formed in the interior of massive gravity dams.

Mr. Noetzli's statements with respect to both the Owyhee Dam and the Boulder Canyon Dam are misleading. These structures are distinctly arched gravity dams in that they are of the same section as a straight gravity dam of equal height, but are designed so as to divide the load between the cantilever and the arch, thus reducing the stresses in both systems.

Although the writer made the statement as quoted by Mr. Godfrey* that in many, if not in most, straight gravity dams, no allowance has been made for uplift, he did not mean to imply his approval of this practice. He agrees with Mr. Godfrey that uplift is a force to be recognized in the design of the gravity type, masonry dams, but he contends that where suitable physical conditions in the shape of a reasonably narrow canyon are found, security against uplift forces can be provided by designing the dam as if there were no uplift and then building it as an arch, or as an arched gravity dam. He also agrees with Mr. Godfrey in not being willing to trust the safety of a straight gravity dam against uplift to the effectiveness of cut-offs or underdrainage, although they both increase the factor of safety, at least temporarily.

However, the writer does not at all agree with Mr. Godfrey in considering uplift as the principal source of failure in gravity type dams or that the allowance for uplift in the design would remove the possibility of failure. Moreover, there has never been a failure, so far as the writer is aware, of even a straight gravity dam designed without allowance for uplift, that could not be fully accounted for by defects, either in the construction or in the foundation, so serious that even the additional weight of a dam designed for uplift, although it might have delayed, would not have prevented, its ultimate failure.

The writer does not wish to be understood as advocating the design of gravity dams without provision for uplift either by additional weight in a straight dam or preferably by the use of the arched gravity type where the topography will permit, but in his opinion Mr. Godfrey neglects the most important requirement for a gravity type masonry dam, which is that it should be founded upon a bed-rock at least as strong and durable and as truly elastic as the masonry, in the sense that it should yield only elastically to the load imposed upon it by the dam. If the foundation yields inelastically, as any material except solid rock will, and thus does not recover from the compression when the load is removed, the material does not make a safe foundation.

The writer called attention to two important elements of strength gained from building gravity dams upon a hard solid rock foundation resulting from the union which takes place between the concrete and the bed-rock. This increases the resistance to sliding on the base and also eliminates uplift on all that part of the base where the concrete is in proper contact with the rock.

* *Proceedings*, Am. Soc. C. E., February, 1930, Papers and Discussions, p. 375.

Mr. Godfrey compares porous concrete to a tube made of sponge with an outer skin which is elastic and impervious and attached to the body of the sponge. He then introduces water under pressure into the interior of the sponge-filled tube and finds, of course, that the resistance of the sponge to the expansion of the tube causes tension in its fibers. The principle stated by the writer was that no uplift was caused in a homogenous mass of concrete by water under pressure in its pores. The writer did not claim that if a vertical column of the concrete were enclosed in a water-tight tube there would be no tension in its walls if water under pressure were introduced into the tube; nor did he claim that there would be no horizontal tension in the concrete if it were attached to and made a part of the tube.

The writer cannot follow Mr. Godfrey's reasoning in citing as an example of uplift due to pressure in the pores of concrete, the case of a pier which he says was lifted by water pressure that entered a horizontal construction joint. He states that,

"The great weight of the pier * * * as well as what tensile strength the concrete possessed, were not sufficient to overcome the water pressure in the pores of this concrete, which, by Mr. Wiley's statement of the case, should be balanced and equal to zero."

It is not clear whether Mr. Godfrey thinks the uplift was due to water in the horizontal construction joint as he first indicates, or to water pressure in the pores of the solid concrete as he next asserts. Mr. Godfrey is also mistaken in attributing to the writer a statement that there would be no uplift in an open horizontal construction joint. The writer specifically pointed out "that every seam or void in a masonry dam is filled with water under pressure". In another instance he stated, "in the case of an open horizontal joint in the concrete * * * there is uplift for which allowance should be made in the designs * * *".

The writer also differs radically from Mr. Godfrey's views about the function and the value of grouting. In the first place it is not generally considered as preventing uplift, but acts like the cut-off wall in reducing it. Mr. Godfrey's conception of grouting seems to be that it consists of pouring mortar into an exposed and visible opening and his statements that: (1) "It would be quite impossible to grout up a crack"; (2) "it stands to reason that solid cement or colloidal cement will not penetrate thin crevices where water may freely go"; and (3) "some of these [rock seams] may be in a horizontal direction just beneath the base of a dam, and drilling would not disclose their presence. The cracks most difficult to grout are the horizontal ones and these are the greatest menace to the stability of the dam". All these quotations would apply if the process of grouting were to be confined to pouring mortar or grout into foundation defects exposed by the excavation, but they are not relevant to the process of "pressure grouting" which consists in forcing a thin mixture of cement grout under high pressures through drill holes extending to considerable depths below the excavated foundation. This is the process universally used for high masonry dam foundations and although the writer does not claim that it removes the possi-

bility of upward pressure entirely, it certainly reduces it by increasing the imperviousness of the rock at the heel of the dam.

Mr. Godfrey's criticism of drainage as a relief for uplift has some justification, the writer thinks, partly because of the possibility of the sealing of the drains by mineral deposits, and partly because of the danger of deterioration of the concrete or of the foundation by the short path which it offers for infiltrating water.

In Mr. Godfrey's concluding paragraph he accuses the writer of overlooking the necessity of providing "gigantic abutments" for arched dams where such dams are not built in "rock gorges", and thinks that if the cost of such abutments were added to the cost of the dam the economy of the arch dam would vanish.

An arched concrete dam is always built in a rock gorge and every part of the dam is always supposed to be on a solid rock foundation, the sides of the gorge constituting the massive abutments which Mr. Godfrey correctly claims to be necessary.

It seems to the writer that Mr. Godfrey has failed to give proper weight to the character of the foundation and that he assumes that defects in this vital part can be remedied by increasing the dimensions of the dam to provide for uplift. The writer believes that provision should be made for uplift either by arch action or by gravity, but he does not believe that either will make a safe dam on the kind of foundation that Mr. Godfrey discusses, one in which, for instance, "excessive soil pressure due to weight * * * would merely make the dam sink", or another in which "it would be impossible for rock, no matter how poor a quality, to issue in a blow-out. Only fluid or semi-fluid soil could be forced out from beneath the base of a dam".

It seems to the writer that no better description than these quotations could be given of an absolutely unsuitable material for the foundation of a high masonry dam, and that increasing the dimensions of a masonry dam on such a foundation would only delay the time of failure.

Mr. Jakobsen* objects to the writer's reference to the Exchequer Dam as an arched gravity type because in the paper an arched gravity dam has been defined as one having a section capable of carrying the full load by its weight alone and having, in addition thereto, the resistance of the massive horizontal arch. Mr. Jakobsen insists upon analyzing the gravity action of an arched gravity dam by considering the water pressure as applied to its developed up-stream face while its weight is reduced by the convergence of the radii. The writer contends that instead of applying the water pressure to the developed length of the up-stream face of the arch it should be applied to its projection upon the plane of the chord, which would give the same total water load tending to slide or overturn the arched dam as there would be on the corresponding straight dam, while the volume of the arched dam of the same section would be considerably larger and its resistance to either sliding or overturning, without considering arch action, would be greater than for the straight dam. It is only when the water pressure upon an element of the

* *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 867.

curved face is considered as resisted only by the converging radial slice of that element, that there appears to be more gravity resistance in the straight dam than in an arched dam of the same section. The analysis which Mr. Jakobsen uses to show that an arched gravity dam has less stability than a straight dam of the same section, assumes the arched dam to be converted into a straight dam of length equal to the developed up-stream face of the arch, the water pressure upon the straight face being resisted by a series of independent buttresses, each consisting of a unit section touching only at the up-stream face and formed by converging radial planes. This would lead to what the writer considers the illogical conclusion that an arched gravity dam could with equal stability be expanded into a straight dam closing a valley as much wider than the actual valley as the curved face of the arch is longer than its chord. The writer's objection is that Mr. Jakobsen neglects the fact that the water pressure upon the face of an arched dam can be resolved into two components, one acting normal to the chord of the arc and resisted by sliding friction, or shear, and cantilever, or overturning moments, and the other acting parallel to the chord of the arch and causing only axial stresses in the dam.

Mr. Jakobsen contends that a transfer of some part of the load from the cantilever to the arch, even where there are no open joints extending entirely through the dam, may be prevented by a state of tension in the concrete due to shrinkage. The writer thinks it only reasonable to assume that where radial contraction joints have been provided—as is now almost universal practice—tension in the concrete between these joints may be considered as eliminated. The writer also thinks that in a relatively thick arched gravity dam shrinkage due either to temperature or to setting in air is confined to the concrete near the faces, while the interior, which is protected from temperature changes and from setting in air, tends to expand due to subsequent hydration of the coarser particles of the cement clinker, rather than to shrink in the process of setting.

Mr. Jakobsen intimates that there is an inconsistency between the writer's present position in this respect and the quotation to the effect that some years ago he considered all stresses in arched dams to be transmitted either in horizontal or vertical planes. The writer cannot see that these views are inconsistent.

The shrinkage crack in the Mulholland Dam mentioned by Mr. Jakobsen has been previously referred to by the writer as having been shown by test to be a superficial crack with a depth of less than 20 ft.

In explanation of Mr. Jakobsen's quotation from an article by the writer to the effect that if contraction joints are not provided in high concrete dams contraction cracks will form, the writer is still of the same opinion, except that later experience has convinced him that even where contraction joints have been provided in arched gravity dams the opening of the joints is superficial and probably not deeper than 20 ft.

Mr. Jakobsen questions the writer's statement that the pressure grouting of contraction joints would result only in filling them for a distance of 15 or 20 ft. from the surface, a conclusion which follows from the writer's

previously discussed opinion that the opening of such joints does not extend more than this distance beyond the faces. The writer recognizes the fact that grout might be forced with an intensity sufficient to tear apart a closed contraction joint, and that in a case where, due to construction methods, open cracks are found to extend entirely through an arched dam, pressure grouting may be made to fill the entire joint. His conclusions regarding pressure grouting referred to ordinary practice.

Mr. Jakobsen asks for further explanation, illustrated by an example, of the writer's conclusion that if, as claimed by Mr. Jakobsen, the contraction joints should open so as to prevent arch action, the back pressure of the reservoir head upon the joints would compensate for the absence of arch action. The writer has already discussed the assumption previously made by Mr. Jakobsen that there might be a state of tension with no open crack, and has stated his conviction that it was untenable. In this case, too, there is the further condition that tension between open joints would not affect the result. In the case of an arched gravity dam, as the pressure in the reservoir is maintained, there must be some point where the joints are tight. The most unfavorable case for arch action would be that in which the joints were open all the way through, except for a tight seal at the down-stream face. The water pressure in the open joints normal to the faces of the radial sides of the segments between contraction joints can be resolved into components normal to and parallel to the center line of the segment. The normal components acting on the opposite sides of the segment will only cause compressive stresses in the segment. The parallel components will offset the water pressure on the projection of the converging sides of the segment, leaving as unbalanced water pressure only an area of each segment equal to its length at the down-stream face, so that each unit of the up-stream face exposed to unbalanced water pressure would be resisted by a gravity section with parallel sides, and would have the same stability as a straight gravity dam of the same section.

The writer wishes it understood that he is not advocating the construction of arched gravity dams with open radial contraction joints sealed at the down-stream face. He is simply attempting to show that, even if what he believes to be the untenable assumption—that an arched gravity dam cannot act as an arch because of open contraction joints—were correct, the dam would still have stability equal to that of a straight gravity dam of the same section because of back pressure in the open radial contraction joints.

Mr. Parsons* questions whether arch action in an arched gravity dam having the same section as a straight gravity dam would offset the reduction in gravity action due to the convergence of the radii. The writer thinks he has shown in his reply to Mr. Jakobsen that, neglecting arch action, an arched gravity dam of the same section, does not have less resistance than a straight gravity dam when proper relation is maintained between the water pressure and the resistance of each dam. In fact, the arched gravity dam has the greater stability because the water pressure tending to cause sliding or overturning is the same in either case and the volume of the arched dam

* *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 872.

is considerably greater for the same vertical projection of the straight and arched dams.

The writer regrets that he has given Mr. Parsons and others who have discussed his paper the impression that he did not consider uplift with enough seriousness. This probably came from the fact that he was considering it with relation to one of his sub-headings, "Past Experience",* and was trying to account for its general neglect in the past with apparent immunity from failure where other conditions were favorable. Uplift is one of the most illusive of phenomena, and it may vary under different conditions in the same dam from 100% of the head on 100% of the area to zero. A gauge tapped into a uniform concrete near the up-stream face of a dam would probably show reservoir pressure, but, as the writer has attempted to show, there would be no uplift in the body of the concrete. Gauges or pressure pipes tapped into the bed-rock at the base of the dam which show pressure head do not necessarily indicate uplift on the base, but show potential pressure which may become active pressure, or uplift, in a defective bed-rock connection, just as the potential pressure shown by the gauge embedded in the body of the concrete may become active pressure in a non-homogenous condition of the concrete, such as its intersection by an open horizontal joint or by a layer of porous concrete. For this reason uplift should be provided against, and the writer thinks that in the deep and narrow valleys or gorges where high dams are usually built, this can be done most effectively by designing the dam so as to take advantage of its resistance both as a horizontal arch and as a gravity cantilever.

The writer agrees with Mr. Werner† that it would not be advisable to design an arched gravity dam with open contraction joints sealed at the down-stream face. He does not wish to be understood as advocating such a design any more than Mr. Werner advocates the design of a curved dam with a convex down-stream face just because he called attention to the fact that under certain conditions it might be more stable than a straight dam of the same section.

The writer agrees with Mr. Werner's views as to non-existence of uplift in the voids of a homogeneous mass of concrete, and as to there being an unbalanced pressure between strata of unequal porosity. The writer does not agree with the view that this would lead to uplift at a point of perfect contact between the masonry and a rock that is free from seams and voids.

Mr. Jorgensen‡ questions the writer's conclusion that "the contraction joints of an arched gravity dam do not prevent arch action", and states that most large arched gravity dams have open contraction joints in late winter. He refers especially to the Exchequer Dam and states that it has tension in the up-stream face and that it is physically impossible for it to receive any help from arch action in the winter because of open contraction joints.

The tension in the up-stream face of the Exchequer Dam, asserted by Mr. Jorgensen, only exists upon the assumptions that there is no arch action

* *Proceedings*, Am. Soc. C. E., November, 1929, Papers and Discussions, p. 2319.
† *Loc. cit.*, May, 1930, Papers and Discussions, p. 1041.
‡ *Loc. cit.*, p. 1051.

and that its volume should be calculated with converging radii while the water pressure is considered as acting upon the developed up-stream face instead of upon its projection upon the chord. This is an assumption which has been discussed by the writer in his reply to Mr. Jakobsen.

The absence of arch action in the Exchequer Dam asserted by Mr. Jorgensen on account of alleged open contraction joints has also been discussed by the writer in his reply to Mr. Jakobsen.

Mr. Creager's discussion* of the possible inadequacy of the old theories of design when applied to the much higher dams contemplated for the future, is very timely and well considered. The writer believes that the greater difficulty of providing an ample margin of safety against sliding and against tension as the height of dams is increased can be most efficiently provided by designs in which arch resistance is used in addition to gravity action.

Mr. Floris† attempts to show upon the basis of the theory of Professor Fillunger, that the writer's claims with respect to the absence of uplift due to water pressure in the voids of a concrete dam are erroneous. His explanation of the theory is not entirely clear, nor is his statement of the writer's views entirely correct. His assertion that the writer limits uplift in a dam to horizontal seams in the rock foundation or to horizontal construction joints in the concrete is not consistent with the writer's claims which are that "there is no uplift in the body of a homogeneous mass of concrete, * * * nor in a perfect connection between the dam and the bed-rock, except that which may be due to a different degree of porosity in the concrete and the rock."

If the writer understands Mr. Floris correctly his interpretation of Professor Fillunger's theory is that there would be no uplift from pressure in the pores of a concrete dam unless there is greater resistance in one direction than in another, such as would result from one part being more porous than another, or from the difference in porosity of the cement paste and the aggregates. This is not essentially different from the writer's theory which is based entirely on the homogeneity of the mass of concrete. It is impossible from Mr. Floris' discussion to evaluate the amount of uplift which he claims would result from the difference in porosity of the various constituents of the concrete, but from the fact that the voids in the concrete would probably not exceed 6% of the volume and that it is only to the area of this small volume of voids that the difference in uplift (if there should be any) would apply, it seems evident that it would be a negligible amount.

The conclusion of Professor Fillunger from his experiments that dams exceeding 250 ft. in height cannot be safe, is startling, but fortunately Mr. Floris explains the apparent error by giving some details of the experiments from which it appears that cylinders made of alternate disks of tight and porous mortar were found to have practically no tensile strength in excess of that required to offset the unbalanced pressures due to water pressures of 8 to 10 atmospheres in the voids.

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 1052.

† See p. 1661 *et seq.*

On the other hand, the tensile strength of homogenous samples of both porous and non-porous mortar was not affected by exposure to water pressures as great as 100 to 200 atmospheres. In the first case there was unbalanced water pressure in the pores due to alternate layers of porous and tight mortar, and in the other the pressure in the pores of the homogenous mortar was balanced so that no interior pressure affected the strength of the mortar.

So far as the writer is able to determine there is no conflict between his own conclusions and those of Professor Fillunger, except with respect to whether concrete as used in high dam construction can be considered as similar to the alternate layers of tight and porous mortar designed to give unbalanced pore pressures in Professor Fillunger's experiment first quoted, or whether it more nearly resembles the uniform mortar used in the last quoted test.

Mr. Wright* gives the commonly accepted opinion of the resistance to sliding due to friction of a dam upon its base or upon seams in the rock beneath the base, as 55 to 70% of the weight. The writer believes that the most common cause of failure of straight gravity dams is sliding upon a more or less plastic seam in the rock a short distance below the base. In such cases the sliding coefficient may be very much less than the assumed minimum of 55%, but failure in that case results from a defective foundation which no design might be able to provide against.

Mr. Wright cites a case in which practically no bond was found between concrete and limestone bed-rock under a pier at the Wilson Dam. There is a theory that old concrete which has set in air forms a surface of calcium carbonate with which new concrete will not make a bond. Experience seems to confirm this theory, and Mr. Wright's discussion indicates that, in the matter of bonding with new concrete, limestone rock acts like old concrete. From the writer's experience he would conclude that with the usual care in cleaning and bonding to bed-rock, a perfect bond could be made with other than limestone bed-rock or, at least, with mica schist and sandstone.

The writer, in closing, wishes to thank all those who have joined in the discussion of his paper.

D. C. Henny,† M. Am. Soc. C. E. (by letter).‡—The writer has followed with intense interest the valuable discussion on high dams contributed by many experts. The attention which has thus been given to a difficult subject shows the papers grouped in the Symposium to have been opportune.

It is noted that Mr. Noetzli§ and Mr. Wright‖ share the writer's opinion that there is strong probability that shrinkage cracks parallel to the water face will form in massive concrete of very high gravity dams, and that if formed they contain a serious element of danger, especially in straight gravity dams.

Mr. Noetzli defends the provision of well-drained contraction joints parallel to the dry slope contained in the specifications of the San Gabriel Dam, and

* See p. 1666.
† Cons. Engr., Portland, Ore.
‡ Received by the Secretary, June 7, 1930.
§ *Proceedings*, Am. Soc. C. E., February, 1930, Papers and Discussions, p. 371.
‖ See p. 1669.

although this dam may not be built, comments on this provision are none the less valuable.

Mr. Jakobsen,* on the other hand, questions the wisdom of such joints in connection with a dam of unprecedented height, without, however, suggesting any substitute.

Contraction joints parallel to the down-stream slope have worked well in high buttresses of multiple-arch dams. It must be admitted, however, that these have the advantage of easy side drainage, obviating dependence on the thorough interior drainage of joints necessary in gravity dams. There is at least one precedent for such joints in an arch gravity dam, namely, the Sweetwater Dam, in Southern California, which as a result of being raised has a well-drained inclined joint in its enlarged structure. In that case the problem was complicated by the great difference in age between the old rubble masonry and the new concrete, but no difficulties appear to have resulted.

It is obvious that precautions may be advisable in dams of unprecedented height, which have not had to be used in lower structures and which are consequently an innovation. The caution advocated by Mr. Jakobsen in the use of innovations in high dam construction is very proper, provided it does not lead to the mere copying of precedent in lower structures.

Attention was called in the writer's paper† to other means at hand to guard against cracking of massive concrete, namely, cooling the aggregates and mixing water or building the dam with large precast concrete blocks, to be placed in the dam after chemical heat has become fully dissipated. In summer construction, such blocks can also be pre-cooled. It is not certain that much, if any, serious extra cost is involved in the latter method, although its deviation from ordinary practice may lead contractors to bid on the safe side. At any rate, in the construction of unusually massive concrete masonry in dams, safety does demand that precautions are taken against the setting up of excessive thermal stresses in the mass and the formation of cracks which may completely change the distribution of stresses assumed in the design.

This is even more true when the tendency is considered, in the manufacture of cement, to increase the fineness of grinding, which leads to earlier development of strength and greater temperature rise. Careful study should be made of qualities of cement which will result in a reduction of temperature rise without sacrifice of ultimate strength. In that connection further research is desirable as to the effect of admixtures of siliceous substances like trass, which is understood to be freely used in Europe, and of pumicite, diatomacious earth, and similar materials used in this country.

There is some evidence to show that heat generation would be reduced by the use of such admixtures without resulting in loss of strength. Experience with sand cement in the Arrowrock and Elephant Butte Dams, however, shows that admixtures may have an unexpected effect. In these two cases there has developed a tendency to surface disintegration. Evidently, a high degree of caution must be exercised in considering the use of admixtures on a wholesale scale in high dams.

* *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 867.

† *Loc. cit.*, November, 1929, Papers and Discussions, p. 2327.

Mr. Noetzli refers to the method of constructing the San Mateo or Crystal Springs Dam, near San Francisco, in blocks cast in place. The excellent result there obtained may be partly due to that cause, but even more important may have been the slow rate of construction progress and the relative coarse grinding of the cement, 90% passing through a 50-mesh sieve.*

The mere fact of casting concrete in blocks would probably not have saved the structure from cracks if it had been built with present-day speed and cement. In fact, one fine crack across the top was observed by the writer in 1906, which apparently passes indifferently through the planes separating blocks.

Mr. Wright questions the benefits which may be secured in dam construction by reducing the height of lifts as against the admitted disadvantage of more numerous construction joints. He admits that thinner lifts would favor lower temperature, but fears as a result of rapid cooling and drying, that through shrinkage cracks would form. There need be no question of rapid drying as it is becoming common practice to require concrete surfaces to be maintained moist for an extensive period, and especially the surfaces of lifts until covered by the succeeding lift. The average time of exposure is usually not very long and even where there has been seasonal suspension of work, the writer has never observed cracks in the top surface of lifts where they were wet.

On the other hand he does not share the hope expressed by Mr. Wright that thin lifts would favor a break of joints in crack formation in successive lifts—the adherence between lifts and the tendency in concrete for cracks once formed to extend into the adjoining mass would prevent cracking in steps.

Vertical cracks approximately parallel to the water face and near the toe may introduce immediate danger in straight gravity dams. In arched dams they are strenuously to be avoided, but they do not introduce the same element of danger because they need not interfere with arch action.

Mr. Noetzli refers to the Owyhee and Boulder Canyon Dams as being of the single arch type. He is correct if it is understood that they are both designed on the assumption that loads are carried partly by arch and partly by cantilever. To insure such division of load it is necessary that especial attention be paid to available methods of minimizing thermal stresses.

Mr. Godfrey states† that it is standard practice to neglect under-pressure in gravity dam design. This may have been true in the past, but since 1920, or even as far back as 1910, it is believed that this standard has changed. He quotes the writer as having given the impression that under-pressure is easily averted or avoided. This is hardly in harmony with the fact that uplift was given considerable prominence in the writer's paper, nor with the statement‡ occurring therein that, "generous allowance for uplift in the design * * * are the only safeguards, * * *."

It may also be stated that the writer has deemed the subject of such far-reaching importance that he has urged experimental determination and

* *Engineering News-Record*, May 8, 1930, p. 768.

† *Proceedings*, Am. Soc. C. E., February, 1930, Papers and Discussions, p. 375.

‡ *Loc. cit.*, November, 1929, Papers and Discussions, p. 2335.

research wherever practicable. The knowledge thus far gathered shows in two ways that assumptions concerning uplift in recent dam design have been ample. Uplift pressures as measured have been less in each case, where tested, than allowances made in the design; and investigation indicates that the assumption that such pressure is effective over 100% of the base, is very conservative. There is good reason to believe that with careful construction this percentage is less than 40, although this opinion should not lead to cutting down the present common assumption of 100% effective uplift area because it is in this matter of uplift that an ample factor of safety is most necessary.

Mr. Godfrey fears blocking of drains by freezing at the outlet and serious ineffectiveness of drains except in their immediate vicinity. The writer has never observed such effect of freezing and believes that, where drains discharge into open gutters in drainage galleries (as is now usually the case), no danger from freezing can exist.

Research as to uplift also indicates uniformly the reasonable effectiveness of properly spaced drains wherever points to measure uplift pressures have been installed. It is, however, of value to have present-day practice questioned as to effectiveness, because this should lead to closer attention to details. One requirement should be that drains must be straight and easily accessible for cleaning, and this detail is too often disregarded.

An interesting question raised by Mr. Godfrey is that of buoyancy in the case of concrete resting on a foundation below the level of back-water. It has been the writer's practice to consider full buoyancy. He believes, however, that no complete buoyancy due to tail-water can exist, because of the adherence of concrete to rock, or to concrete, and that its assumption in itself contains a considerable element of safety.

Mr. Godfrey's statement that tests for high dams have shown under-pressure of practically 100% back-water pressure after sufficient time has elapsed, is admitted where any pressure is found to exist. It is the area over which this pressure acts that remains uncertain.

Mr. Godfrey applies the term "standard dam design" to arching gravity dams without consideration of the ability of the abutments to take arch thrust. The writer cannot agree to this and knows only of a few cases where this charge might hold true, the most notorious of which is the St. Francis Dam.

Mr. Jakobsen argues that "gravity dams should be designed as such, irrespective of arch action, and that arch dams should be designed as such irrespective of gravity action." Such conclusion appears to the writer to be ultra-conservative. There seems to be no reason why an analysis of stresses, such, for instance, as that by the trial-load method, taking into consideration temperature drop, tangential shear, and twist, should not be made the basis of design.

The complexity of the calculations might tend to create doubt as to the dependence to be placed on the result. Yet the highly satisfactory coincidence between calculated and estimated deflections in models establishes confidence in the reliability of the method. The uncertainty as to the elastic qualities of the concrete used and the seemingly unavoidable errors in some of the

underlying assumptions make it necessary to limit stresses to a fraction of known strength.

Mr. Jakobsen's statement concerning the Crystal Springs Dam is welcomed. The dam has not been brought up to its maximum intended height and, therefore, has great excess strength. This statement should have been included in the original paper, but was unintentionally omitted.

As to cracks parallel to the water face in gravity dams, Mr. Jakobsen suggests that some cracks noted in galleries may extend only a short distance into the mass. This fact is difficult to determine, although where water oozes from such cracks the probability may well be that it extends for a considerable distance. One crack was observed a few years ago in the Elephant Butte Dam, in one of the cross galleries. Its position in this case robs this fact of serious significance, as it occurs about 10 ft. down stream from the main gallery and, consequently, far up stream from the toe, where such cracks would be most dangerous. It is significant that water slowly oozing from this crack indicates that the vertical drainage system is not 100% effective. The extent or depth of this crack is unknown.

In the Bull Run Dam a crack parallel to the water face has been observed in one of the cross galleries only 40 ft. from the toe. It is very fine and extends across the gallery, but on neither side of the gallery does it reach the floor level; nor is there any visible crack in the floor. Careful scrutiny also failed to reveal any crack in the surface of the dam slope above its appearance in the gallery, and the strong probability is that the gallery crack is superficial in character and due to rapid air cooling and drying.

A careful examination of cracks, their location, and probable cause, in the major concrete dams in the West, is now being made by the U. S. Reclamation Bureau, and should yield valuable results.

The fact that uplift under the base of dams may exist on planes having a down-stream pitch, and that its horizontal component constitutes an addition to the horizontal water loading, is a valuable contribution to the discussion from Mr. Parsons.* The argument applies equally to uplift which may exist along construction joints within the body of a dam and indicates the desirability of giving construction joints in concrete an upward pitch in a down-stream direction.

This expedient as regards the down-stream half of the dam was adopted in the construction of the Bull Run Dam and is also applied to the gravity abutments of the Ariel Dam, on the Lewis River, in Washington. In both cases the more important object in mind was to avoid flat construction joints along which sliding might take place.

Mr. Jakobsen very properly calls atttention to the error which may be involved in the assumption that the vertical stress distribution across a horizontal plane in a concrete dam is linear. The 1908, the Ottley and Brightmere, as well as the Wilson and Gore, experiments with models, previously mentioned, indicate that this assumption, while it may be approximately correct at higher elevations, is in error near and at the base. In themselves these

* *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 873.

experiments do not show that curved distribution of vertical stresses causes the maximum stress in a horizontal plane to exceed the maximum found with linear distribution. Nor do they necessarily indicate tension at the water face where compression was previously found; but they raise a serious doubt. This doubt is understood to be confirmed as a result of Mr. Jakobsen's application of the theory of least work to this problem.

These uncertainties may well have a bearing on the justifiability of arching a gravity dam in regard to which Mr. Werner* and Mr. Hanna† argue that additional expense is not warranted. To the writer it seems that, whether or not a curved gravity dam, after cooling, acts in all its parts as an arch dam when loaded, is not the only question. A relatively high dam will probably do so, in various parts of its height, as a result of cantilever deflection. The behavior will depend on numerous factors, especially temperature drop after arch closure or after pressure grouting of contraction joints. However, assuming that the dam does not initially act as an arch and that actual downstream movement on its base or on higher planes must take place to put arch action into full play, and considering that it is only by sliding that gravity dams have failed, then it appears that, if the abutments are capable of taking arch thrust, a greater degree of safety must result from arching a gravity dam to a reasonably effective radius. This added margin of safety is, in the writer's opinion, worth considerable extra expense, at least until the forces due to uplift and those opposing sliding, as well as the interior stresses in the concrete, especially near the water face, can be estimated with greater certainty than at present.

The writer agrees with Mr. Jorgensen‡ that there is no reason why an arch dam should not be built as high as any gravity dam on sites where it would fit—that is, where the nature of the rock abutments as well as the topographic conditions are suitable.

Mr. Creager§ argues that the tensile strength of concrete at the up-stream face of a gravity dam, as well as the shearing strength along sliding planes, if treated as margins of safety, are of far smaller relative value in a high than in a low dam. This argument is sound and should lead to the adoption of relatively safer sections for dams of increased height.

Mr. Hazen‖ has performed a real service in urging increased caution in the design of dams of unprecedented height.

J. G. TRIPP,¶ M. AM. SOC. C. E. (by letter).**—The construction of dams involves both skill and judgment to create in finality the thought of the designing engineer. Drawings and specifications when well done, attempt to delineate and determine, accurately, the masses to be opposed to the water.

The writer has confined his efforts to the technique of construction, and the technology of temporary structures for use in carrying out the designer's

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 1041.
† *Loc. cit.*, p. 1047.
‡ *Loc. cit.*, p. 1051.
§ *Loc. cit.*, p. 1052.
‖ *Loc. cit.*, p. 1053.
¶ Contr., St. Louis, Mo.
** Received by the Secretary, June 16, 1930.

intention for the permanent dam. In constructing several dams he has observed a wide divergence of opinion between designers as to the effect of concrete cracking and uplift pressures. Each designer will set forth his best conclusion in the plans and specifications, which to the builder are often enlightening and sometimes contentious and disappointing. It is unfortunately true, in the writer's opinion, that many dams are cheapened for financial reasons beyond the limits of sound engineering. He has observed all too many such structures; however, he is also sensible to the fact that an engineer worthy of the name must "do with one dollar what any one can do with two."

Because of capital investment the tendency to-day in concrete dams is to design for strength at the sacrifice of impermeability, particularly in high dams where the cubic yardage is a progressive ratio function of the height. This is a mistake. The addition of pumice to produce lower permeability and overcome laitance deposits should be encouraged. It is claimed that the addition of pumice will reduce shrinkage which, of course, is much to be desired and investigations to prove this point should not be delayed.

Shrinkage cracking in the mass can be controlled to a great extent by attention to water-cement ratio, proper relationships in aggregates and their size, and by the methods and times of deposit in the forms. The addition of ample reinforcing steel, while costly, is effective. Properly embedded steel will last indefinitely. The shape of the pouring block and its dimensions and speed of construction are matters of controversy, and, even with such a splendid example as the Crystal Springs Dam, some substantial experimental work should be done in this field to determine the proper geometrical shapes before any decision as to good practice should be adopted.

The writer agrees with Mr. Noetzli* that the theory of principal inclined stresses is valid. During the construction of several important dams, he has seen the actuality of dangerous cracking, which could have been controlled by the application of this theory in the design. The writer has seen cracks develop rapidly both transversely and longitudinally during construction of large and small dams, in speedily as well as in slowly built work. He concludes, therefore, that speedy construction is not necessarily a concomitant cause for shrinkage cracking. Different cements and aggregates and water have marked effects, as do times of day in which the concrete is poured. Protection from sun and wind and sufficient curing water applied between initial and final setting minimizes "drying out" or primary shrinkage.

The writer's observations, working under the specifications of eminent authorities have led him to conclude that while many of the vagaries of construction may be blamed for partial failures, there is still a large amount of guessing as to what constitutes good practice, and not enough conclusive experimental data available as to what is good practice in the matter of concrete for either gravity or other types of dams. Among other things, he believes that to specify never less than a barrel of cement to the yard is good practice.

In the matter of foundation uplift, the writer believes that the structural geology of the sites answers grouting criticisms. Basalt will require a differ-

* *Proceedings*, Am. Soc. C. E., February, 1930, Papers and Discussions, p. 372.

ent treatment from an upturned sedimentary or a massive felsite. The writer has seen large masses of sedimentary rock disturbed by even low-pressure grouting, as suggested by Mr. Parsons.* The writer agrees with other observers that uplift on foundations is a very potent destructive force and in all cases, under his observation, it has been considered in, or nullified by, the design. He does not agree, however, that proper drainage and either cement or asphalt grouting cannot control this force to a large extent.

Uplift between construction joints is easily controlled by horizontal copper water-stops, proper joint cleaning, and pouring of succeeding layers of concrete. It is certain that the contact between a thoroughly cleaned bed-rock and concrete is always tight and that uplift is of no possible consequence in that plane. The joints and seams below the dam are the danger spots. The shaping of the foundation assuredly has the effect desired by the designer, where the rock masses are suitable. Resultant forces can be directed to advantage by careful attention to this detail.

Finally, the construction of the dam must be manipulated and controlled with adequate and necessary plant. The fitting of plant to design conditions and specification limitations, should be understood by the directing engineer to the end that design and construction should work to the final result of safety and economy.

Without a better understanding of the hidden facts to be revealed by further investigation, it does not seem wise to predict either the type or the height of future dams.

* *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 873.

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CONSTRUCTION OF THE JAMES RIVER BRIDGE PROJECT

Discussion*

By R. C. Wilson,† Esq., and Herbert B. Pope,† Assoc. M. Am. Soc. C. E.

R. C. Wilson,‡ Esq., and Herbert B. Pope,§ Assoc. M. Am. Soc. C. E. (by letter).‖—The discussions have really amplified the contents of this paper and, therefore, there is no need for a reply by the authors except to express appreciation to those who submitted them.

Mr. Ballard¶ has brought out very clearly the fact that the various formulas in general use cannot be relied upon to govern the driving or determine the safe carrying capacity of the large concrete piles which were used in connection with this work.

The discussion by Mr. Willoughby** has clarified certain details of design that had merely been outlined in the paper. His comments in regard to the machinery for operating the lift span are of particular interest in view of the first applications of direct drive to large lift spans.

* Discussion on the paper by R. C. Wilson, Esq., and Herbert B. Pope, Assoc. M. Am. Soc. C. E., continued from May, 1930, *Proceedings*.

† Authors' closure.

‡ Vice-Pres., Turner Constr. Co., New York, N. Y.

§ Supt. of Constr., Turner Constr. Co., Asbury Park, N. J.

‖ Received by the Secretary, June 27, 1930.

¶ *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 875.

** *Loc. cit.*, May, 1930, Papers and Discussions, p. 1059.

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EFFECT OF TURBULENCE ON THE REGISTRATION OF CURRENT METERS

Discussion*

By Dr.-Ing. Ludwig A. Ott

Dr.-Ing. Ludwig A. Ott,† (by letter).‡—The solution to numerous problems concerning discharge measurements is eagerly awaited by many engineers. This paper affords a welcome opportunity to discuss one of those problems, namely, the behavior of current meters with runners in turbulent currents.

During the sixteen years since 1914 the writer has repeatedly made practical experiments on this subject and has in general arrived at results similar to those described by the authors. The writer's experience has convinced him, however, that in studying current-meter behavior in oblique currents (always assuming the meter rigidly supported at the measuring section), it is inexpedient to distinguish between "screw wheels" and "cup wheels," the type of runner effecting a difference in grade only but not in principle. The essential feature is not the shape of the runner, but the position of its axis in relation to the plane of the measuring section.

With the shape of any runner fixed, the number of revolutions for a definite flow velocity represents a function of the angle formed by the direction of flow and the runner axis, and in principle all the directions situated on a conic surface around this axis are equivalent. Moreover, only in one position of the axis, that is, perpendicular to the measuring section, is the number of revolutions a function of the angle between the direction of flow and the measuring section, as must be demanded of an instrument for determining the discharge.

Whether the runner is better provided with screw blades or cups is a question that cannot be answered offhand as long as behavior in the oblique current is the only consideration. Perhaps some other important factors must be considered, such as the disturbance of the flow, the sensitiveness of recording,

* Discussion of the paper by David L. Yarnell and Floyd A. Nagler, Members, Am. Soc. C. E., continued from August, 1930, *Proceedings*.

† Math. Mech. Institut, Kempten, Bavaria, Germany.

‡ Received by the Secretary, May 17, 1930.

the behavior near the surface or the bottom, etc. In either case, it is certain that a cup-wheel should be mounted in the same manner as a screw-wheel, with its axis of rotation vertical to the measuring section. Consequently, the plane of rotation should be parallel to the section.

In oblique currents, cup meters with their axes of rotation set parallel to the profile, show qualities similar to screw meters with their axes placed crosswise, provided the screw-wheel is of such a shape that it will revolve in this position. The writer has made a number of experiments with such runners. They were shaped like five-threaded twist drills and were 1 in. in diameter, $3\frac{1}{2}$ in. long, and had a pitch of 6 in. The generatrix of the helicoidal surface of one of the screws was a straight line, whereas those of the second and third were convex and concave curves, respectively. In the normal position, with the axis perpendicular to the measuring section, all three runners turned with equal numbers of revolutions like any other screw meter of suitable construction. When the axis was placed parallel to the profile the first runner remained at rest. The second and third turned forward and backward, respectively, with an increased speed (because the diameter was small compared with the pitch) analogous to that of right or left-turning cup meters.

To recapitulate: It is not the shape of the runner, but the position of its axis that is essential in researches on the current-meter behavior in oblique currents.

In most of his researches the writer has placed the current meters at various angles in flowing water. Presumably Treviranus* was the first to adopt this method in his experiments. Experience has proved, however, that sufficiently accurate quantitative evidence is not obtained in all cases. There are errors to be anticipated in oblique currents, because in the equation of current meters:†

$$V = A + B\,N \ldots\ldots (1)$$

the coefficients, A and B, do not, as a rule, change at the same rate for any given angular position of the axis.

Assume that a meter is rated by towing, keeping the axis parallel to the direction of the towing car. (See Fig. 26.) The result is expressed by Equation (1). Then the same meter is rated again with the axis at an angle, α, with the direction of the car (see Fig. 27). The result is then expressed by:

$$V = a + b\,n \ldots\ldots (2)$$

The component coinciding with the meter axis is thus shown to be,

$$v = V \cos \alpha \ldots\ldots (3)$$

or, substituting the value of V in Equation (2),

$$v = (a + b\,n) \cos \alpha \ldots\ldots (4)$$

One can see at a glance that Equations (1) and (2) for any angle, α, can

* Treviranus, "Uber Verbesserungen in der Konstruktion und im Gebrauch des Woltman'schen Fluegels" ("On Improvements in the Construction and Application of the Woltman Current Meter"), *Allgemeine Bauzeitung*, Bd. 26, 1861, pp. 125–164.

† With regard to the theoretical derivation of this formula as a simplification of the general equation of the second order, see, Dr. L. A. Ott, "Theorie und Konstantenbestimmung des hydrometrischen Fluegels" ("Theory and Determining the Constants of Current Meters"), Berlin, 1925.

only yield identical values for velocities in the direction of the axis when both the ratios, $\frac{B}{b}$ and $\frac{A}{a}$, are equal to the value, $\cos \alpha$. Consequently, in theory this is the fundamental requirement for a so-called component meter.

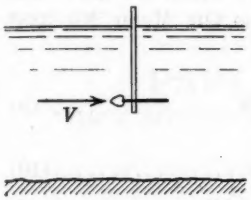

FIG. 26.

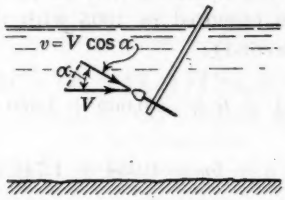

FIG. 27.

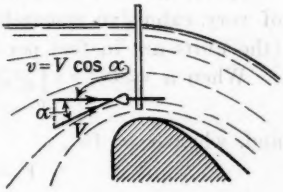

FIG. 28.

In discharge measurements similar to that in Fig. 28, Equation (4) is, as a rule, of little use, because ratings of meters at angular positions of the axis are seldom available and the angle, α, is not known. For these reasons it is desirable that in an oblique current Equation (1) furnish directly the true result, namely,

$$v = A + B\,n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which, v denotes the component of the velocity perpendicular to the measuring section, and n, the number of runner revolutions per second.

For the ideal component meter the ratio, $\frac{n}{N}$, of the number of revolutions in oblique to the number in parallel flow (with equal absolute water velocities) is determined from Equations (1) and (2), as follows:

$$\frac{n}{N} = \cos \alpha - \frac{A}{B\,N}(1 - \cos \alpha) \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

An instrument which for ratings by Equations (1) and (2) are available has an $\frac{n}{N}$-ratio of,

$$\frac{n}{N} = \frac{B}{b} - \frac{a}{b\,N}\left(1 - \frac{A}{a}\right) \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

These equations show that the cosine curve for the ratio, $\frac{n}{N}$, is only approximate in revealing the qualities expected from a component meter. They further explain the fact expressed by the authors* that for different absolute velocities the divergences of the ratio, $\frac{n}{N}$, from the cosine curve are different.

If ratings have been carried out according to Fig. 27, or Equation (2), respectively, it is quite easy to determine the percentage of error which occurs in practice by applying Equation (5) instead of Equation (4). If v denotes

* *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2631, and Fig. 14, p. 2632.

the velocity component computed from Equation (5) and Δv, the necessary correction, the result is expressed as follows:

$$\frac{\Delta v}{v} = \frac{b}{B} \cos \alpha - 1 - \frac{A}{v}\left(\frac{b}{B} - \frac{a}{A}\right) \cos \alpha \ldots\ldots\ldots\ldots (8)$$

As an instance the following equations may be quoted which are the results of very extensive researches executed in 1905 with the Ott Meter No. 385* (the units are in feet per second):

When $\alpha = 0°$,

$$V = A + B\,N = 0.064 + 1.660\,N \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

and, when $\alpha = 15°$,

$$V = a + bn = 0.034 + 1.746\,n \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Applying Equation (8) when using this meter in an oblique current of 15°, the percentage of correction for the flow component in the direction of the meter axis is,

$$100\,\frac{(\Delta v)}{v} = 1.5 - \frac{3.2}{v} \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

As will be seen, the correction can be either positive or negative. It is + 1%, 0%, and — 1%, respectively, for the velocities, 6.4 ft. per sec., 2.1 ft. per sec., and 1.3 ft. per sec.

The writer has studied particularly the range from 0° to 20° at intervals of 2°, both to the right and the left. These small intervals were deemed necessary, because otherwise irregularities in the run of the curve may easily be overlooked or assumed to be errors in measuring. Since experience has proved that in practice larger obliquities of current can nearly always be avoided, it appears to be sufficient to limit the range to 20° when determining the cosine qualities of a current meter.

The writer cannot fully agree with the statement in the "Introduction"† of the paper restricting the use of current meters to measurements in natural streams and open artificial channels. In Europe, they are the most favored instruments for accurate discharge measurement in large pipe lines. In July, 1929, comparative discharge measurements by various methods (the Allen salt velocity method, the Gibson method, with Pitot tubes and Ott current meters) were undertaken in one of the penstocks of the Walchenseewerk‡ under the impartial management of the Research Institute for Hydraulic and Hydro-Electric Structures in Munich. The measurements proper were carried out by experienced German and English crews. The results clearly showed the superiority of the current meter for this very kind of gauging, thus increasing every one's confidence in these instruments.

* Eger, Dix, Seifert, "Die Versuchsanstalt für Wasserbau und Schiffbau, Berlin. Versuche mit Stromflügeln" ("Laboratory for Hydraulics and Shipbuilding in Berlin. Researches on Current Meters"), *Zeitschrift für Bauwesen*, Vol. 57, 1907, p. 266.

† *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2611.

‡ Dr. Kirschmer, "Vergleichs-Wassermessungen am Walchenseewerk" ("Comparative Discharge Measurements at the Walchenseewerk"), *Zeitschrift* des Vereins deutscher Ingenieure, Vol. 74, 1930, pp. 521–528.

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GENERAL SPECIFICATIONS FOR STEEL RAILWAY BRIDGES

PREPARED BY CONFERENCE COMMITTEES FROM THE AMERICAN SOCIETY OF CIVIL ENGINEERS AND THE AMERICAN RAILWAY ENGINEERING ASSOCIATION

Discussion*

BY MESSRS. DAVID A. MOLITOR, HENRY S. PRICHARD, AND R. H. CHAPMAN

DAVID A. MOLITOR,† M. AM. SOC. C. E. (by letter).‡—When the Conference Committees§ submit new general specifications for steel railway bridges, it is natural to expect the product to be one which represents the last word on the subject. Unless something at least the equal of good modern practice is presented, the Society can scarcely expect those specializing in this branch of the profession to adopt such specifications; and the effort to harmonize and unify the practice on a plane commensurate with the present status of the subject, will have been defeated.

The specifications make no mention of movable bridges and supposedly apply to spans of all lengths. The unit stresses have been slightly raised and the quality of steel has been lowered. The proposed live load differs from anything now in use; a new impact formula is given; and wind loads have received unusual attention.

The time-honored custom of directing the most attention to standard loading, impact, and column formulas has been followed, while many matters of lesser importance have been over-emphasized. Economy receives little or no consideration.

* Discussion of the General Specifications for Steel Railway Bridges, continued from August, 1930, *Proceedings*.

† Structural Engr., Albert Kahn, Inc., Detroit, Mich.

‡ Received by the Secretary, May 21, 1930.

§ *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2641, *et seq.*

It is surprising that the quality of steel should have been lowered to a grade commonly used for rivet steel, when a medium grade of 60 000 to 70 000 lb. per sq. in. tensile strength with an elastic limit of 35 000 lb. per sq. in. is generally used for railway bridges. This, in conjunction with such provisions as those contained in Articles 300 and 302, should cause one to hesitate before subscribing to the new specifications.

The great railroad systems and the leading bridge engineers of the United States will continue to use their own specifications, for the very good reason that the new specifications do not appeal to them as a step in advance.

The following comments, while incomplete, are submitted as constructive criticism. The specifications should be limited to spans up to 450 ft. Larger structures usually involve details not applicable to bridges of the class herein considered. The general clauses under Section A might well be omitted as irrelevant to the subject, or should be elaborated into a complete set of "general conditions." Various changes in the form of the specifications seem desirable.

Article 101.—Bridges shall be made of medium steel of 60 000 to 70 000 lb. per sq. in. tensile strength. Rivets shall be of soft steel of 50 000 to 60 000 lb. per sq. in. tensile strength.

Article 102.—Limiting span lengths shall be: Rolled beams for spans up to 30 ft.; plate girders for spans up to 120 ft.; riveted trusses for spans 120 to 250 ft.; and pin-connected, or combination, trusses for spans 250 to 450 ft. long.

Article 104.—The depth of trusses shall be economic, but not less than one-tenth of the span, and through trusses shall be governed by the clearance diagram. For plate girders, use the economic depth, but not less than one-twelfth the span.

Article 105.—The distance, center to center of tracks, shall be 13 ft., to be increased 4 in. per degree of curve for tracks on a curve.

Article 203.—If the Cooper loadings are to be discarded it would appear that a better standard loading might be selected.

Article 204.—The formula given is no particular improvement over the well-established formula including the factor, $\frac{165}{150 + l}$, which is simpler and gives about 5% more impact for 400-ft. spans, otherwise following closely the newer type. To use one-third as much impact for electric traction seems rather low.

Articles 211, 212, 213, 300, and 302.—These clauses being interrelated, the writer proposes an entirely different group arranged as follows:

Article 211.—Combination of Stresses.—All bridges shall be designed for the maximum and minimum combination of stress in each member due to dead load, live load, and impact, to which the centrifugal load shall be added as a part of the live load for bridges on curves. Tractive forces, except for trestles, will affect the floor-beams only and must be carried from the stringers or track to the loaded chord through the lateral system at each end panel of the span. Except the transferred wind load in end posts of through bridges,

and tower legs, no wind stresses in any main truss members need be considered for spans up to 200 ft. in length. For spans exceeding 200 ft., wind stresses in truss members shall be considered in combination with the dead load, live load, and impact stresses, allowing an increase of 20% in the unit working stresses; and the areas hereby required shall govern if they exceed the areas found for the dead load, live load, and impact without wind load and using normal working stresses.

Article 212.—Combined Axial and Bending Stresses.—In the extreme fiber of any piece these stresses shall not exceed the allowable unit stresses in tension or compression. The cross-section, F, required for any member subjected to an axial stress, S, and a bending moment, M, is given by the formula, $F = \frac{l}{f}\left(S + \frac{My}{r^2}\right)$, in which, f is the allowable unit stress, reduced for $\frac{l}{r}$ when compressive; y is the distance from the neutral axis through the center of gravity of the section to the extreme fiber; and r is the radius of gyration in the direction of bending. All units are in pounds and inches.

Article 213(a).—Members subjected to dead and live load stresses of the same character and in which the maximum live load stress is produced by the maximum live load covering the whole span, shall be proportioned for the sum of the dead load, live load, and impact stresses, on the basis of the allowable unit stresses given in Article 301, as the case requires. This applies generally to chords and end posts.

With the section thus obtained and a total stress, S, equal to the dead load stress plus twice the live load stress plus twice the impact stress, or $S = dl + 2\ (ll + I)$, find the unit stress, f', resulting from 100% overload in the live load. Also, find the factor, $k = \frac{f'}{f}$ (always less than 2), which represents the ratio of increase in the unit stress, f, due to 100% overload. This factor, k, will necessarily be a constant for all members of this class and is to be used as directed in Article 213(*b*).

Article 213(b).—Members subjected to dead and live load stresses of the same or opposite character and in which the live load stress is produced by a partial loading of the span, as for web members generally, shall be proportioned for the algebraic sum of the dead load stress and twice the live load stress and impact, using a unit stress, kf, in which, f is the allowable unit stress in tension or compression as the case requires. Where a reversal in live load stress is possible, the sectional area should be computed separately for each combination with the dead load stress, and the larger area will govern the design. A structure so designed will have a uniform factor of safety well within the elastic limit, for an overload of 100% in the live load; or will have a factor of safety of more than 2 on the elastic limit for the assumed live load with impact.

Article 213(c).—Members subjected only to dead load stresses of the same or alternating character for different positions of the structure when of the movable type, shall be proportioned for the maximum stress which gives the

larger section, increased by 20% for impact. This shall also apply to tower legs and structures carrying sheaves or other machinery.

Article 214.—Since all secondary stresses, as ordinarily defined, are due to distortion of a member, it is not clear to which of the other secondary stresses the statement refers.

Article 300.—This has already been covered in Article 211.

Article 301.—The unit stresses herein given are either equal to or slightly higher than those commonly used for medium steel having an elastic limit of 35 000 lb. per sq. in. If adopted in accordance with Articles 211, 212, and 213, as here presented, then the dead load should not be excepted. The reference to high-tension steel should be replaced by complete details as given for other steel. Under Article 301, the value, 0.4 kip per sq. in., given for sandstone is somewhat high, but limestone is quite equal to concrete at 0.6 kip per sq. in.

Article 302.—Provision for Overload.—This is covered by Article 213(*a*) and Article 213(*b*).

Article 401.—The minimum thickness of metal shall be $\frac{3}{8}$ in., except for fillers. Gussets shall not be less than $\frac{7}{16}$ in. thick.

Article 408.—This specification appears to be too complicated for practical use.

Article 411.—The penalty for long rivets is not sufficiently severe. It is suggested that the increase be 1% per $\frac{1}{8}$-in. increase in length beyond 3 in.

Article 419.—While any designing method will be complicated, it may be more practical to give a table of sizes of standard lacing as used in manufacturers' standards and elsewhere.

Article 433.—Sizes of intermediate stiffeners should be standardized once for all.

Part II.—Materials.—It would appear sufficient to state the chemical requirements as to sulfur, phosphorus, and manganese and the physical properties as in Table 2,* for each kind of steel to be used under the specifications, leaving the details of the tests to the Standard Specifications of the American Society for Testing Materials. This would save about eight pages.

In conclusion, the writer suggests that these specifications might have been made to cover all types of fixed and movable steel bridges, concrete structures, and trestles for either highway or railway loading, as was done by the Ontario Railway and Municipal Board in 1916.†

HENRY S. PRICHARD,‡ M. AM. SOC. C. E. (by letter).§—These specifications‖ are admirable in many respects. In reading them the following questions have occurred to the writer.

1.—Articles 300 and 301 permit a unit stress of 24 000 lb. per sq. in. In structural steel with a yield point at 30 000 lb., a dead load stress of 24 700 lb. per sq. in. is permissible if the material is not rejected by Articles 300, 301, and 702. Are not these values too high? A stress of 24 700 lb. is only 82.3% of the specified yield point.

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* *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2668.

† Standard Specification for Bridges, Viaducts, Trestles, or Other Structures, 1916. Toronto, Ont., Canada.

‡ Analytical Engr., Am. Bridge Co., Pittsburgh, Pa.

§ Received by the Secretary, June 2, 1930.

‖ *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2641, *et seq.*

2.—Is the yield point as determined by the drop of the beam in ordinary mill testing (see Articles 807 and 809 for Structural Steel) sufficiently reliable as a criterion of strength?

J. A. L. Waddell, M. Am. Soc. C. E., has recorded* yield points in specimens from the interior of 1-in. universal plates from the same heat, but tested at different mills and the Drexel Institute, and at different rates of speed, as given in Table 16.

TABLE 16.—Test of Similar Specimens of 1-Inch Universal Plates from the Same Heat.

Place of test.	Rate at which tested.	Pounds per square inch.
Yield point at Drexel Institute	1 in. in 6 min.	27 800
Drop of beam at Drexel Institute	1 in. in 6 min.	30 300
Drop of beam at mill	1 in. in 3 min.	28 500
Drop of beam at another mill	1 in. in 15 sec.	38 500

The yield point at 28 500 lb. per sq. in. at one mill is only 74% of that at 38 500 lb. per sq. in. at the other.

In Article 907, under "Section 9.—Silicon Steel," after stating that "the yield point shall be determined by the drop of the beam of the testing machine", the Committees add "when operated at a cross-head speed not greater than $\frac{1}{2}$ in. per min. The testing machine shall not be stopped to obtain the drop of the beam." This is a qualification not given for structural steel or nickel steel. If the rate at which this test is made is too rapid near the yield point, the beam is likely to indicate the yield point too high, whether the steel is structural, silicon, or nickel.

3.—The formula for axial compression (Article 301(*a*)) follows a form, first proposed by Tredgold, which fell into disuse but was revived by Lewis Gordon. Afterward it was analyzed by Rankine, who made the mistake of applying to columns a formula for deflection of beams which is not applicable, and thereby obtained a formula which is not only decidedly erroneous theoretically, but which cannot be fitted to any set of tests covering any considerable range. It is surprising that a man of Rankine's genius should have made, or acquiesced in, such an error in regard to the deflection of columns. A centrally loaded column, made of perfectly elastic material and having an initially straight axis, is truly a spring, and Rankine has proved the theorem:†

"That a spring of a given length and section, to the ends of whose neutral surface a pair of forces are applied, will not be bent if those forces are less than a certain finite magnitude."

He develops the equation, $P = \frac{\pi^2 E I}{l^2}$ (which is Euler's), and adds, "and this finite quantity is the smallest force which will bend the given spring in the manner proposed."

Shall the general specifications for Steel Railway Bridges perpetuate such an error?

* *Transactions*, Am. Soc. C. E., Vol. LXIII (June, 1909), p. 222.

† "Applied Mechanics," Eleventh Edition, pp. 351–352.

4.—The percentage of increase in capacity to sustain compression which an increase in ultimate strength, yield point, and useful limit point (these are co-ordinated to some extent) gives to short columns, grows less and less in columns of moderate length (when other conditions remain constant) as slenderness increases until, in long columns, it disappears entirely. This is graphically illustrated in Fig. 17, which was first presented by the writer in connection with another paper.* Would it not be well to recognize the fact cited, and thus illustrated, instead of stating (Article 301(*b*)), unqualifiedly, that "the allowable stresses may be increased in proportion to the specified minimum yield point of the stronger steel"?

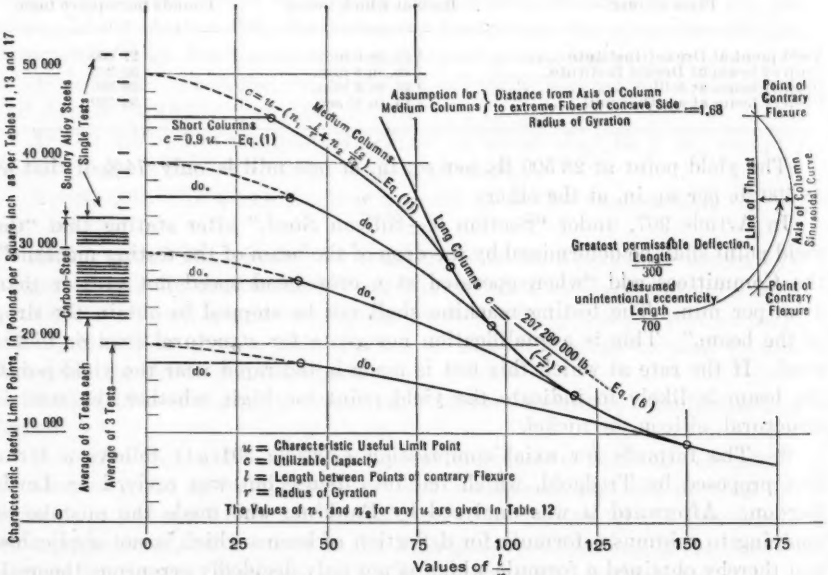

FIG. 17.

5.—The qualification "provided such yield point is not more than 70% of the ultimate strength" (Article 301(*b*)) is different from that suggested in the paragraph immediately preceding. In view of the facts cited in a previous paper by the writer,† would it not be well to reduce the 70 to 60 per cent.?

6.—The U. S. Board for Testing Materials, appointed by the President of the United States in 1875, made a report‡ in which it describes its investigations and includes the statement:

"* * * as important differences exist in the proportionate strength of different-sized bars made of the same material, which are due entirely to differences in the processes by which they are manufactured, and as the elimination or reduction of such differences would necessitate such a great and expensive change in the system by which the bars are produced that it is not

* *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1292, Fig. 6.
† *Loc. cit.*, p. 1235.
‡ Watertown Arsenal Rept., Vol. 1, 1881, p. 45.

probable that it will be often attempted, it is necessary that these differences should be taken into consideration when estimates of the strength of any structure in which rolled wrought iron of different sizes is introduced are made, and in all tables of strength based upon the strength of such bars."

The Watertown Arsenal* made twenty-six tests of bars, forged down from an open hearth ingot in the direction of its length at various temperatures and with various amounts of reduction. The tests showed variations in "elastic limit" from 47 000 to 72 000 lb. per sq. in., depending on the temperature at which the steel was reduced and the amount of reduction. The U. L. P.'s ("Useful Limit Points"), which vary from 46 000 to 72 000 lb. per sq. in. (and, in some cases, are 2 000 lb. per sq. in. less than what the report terms the "Elastic Limit"), and data regarding tensile strength, temperature, and reduction, are compiled in the paper by the writer, previously cited.† Notwithstanding the facts given therein no specifications from the time iron and steel were first rolled, as far as the writer knows, have made any differences in unit stresses in consequence of the facts cited or of similar facts otherwise developed. Other cases galore could be cited to show that strength varies greatly in consequence of facts of manufacture not practicable to control so that the results will give the uniformity in strength desired. Would it not be well to recognize in some way, in specifications, the fact that a 3-in. rod, as usually manufactured, will not have the same strength per square inch as a ¼-in. rod of the same material, and that a plate, 2 in. thick, will not have the same strength per square inch as one ¼ in. thick?

R. H. CHAPMAN,‡ ESQ. (by letter)§—The detailed specifications of the American Railway Engineering Association have been used on the South Australian Railways for a number of years. The Engineering Department now proposes following the modifications that have been made in the later specifications‖ by the Conference Committees.

On the South Australian Railways it is current practice, however, to follow the Australian Standard Specifications for structural steel. Those specifications stipulate an ultimate strength within the limits of 62 700 and 73 900 lb. per sq. in., and allow a unit working stress of 17 900 lb. per sq. in. This seems to be more economical than to use the mild steel within an ultimate strength ranging from 55 000 to 65 000 lb. per sq. in., which allows a unit working stress of only 16 000 lb. per sq. in.

Furthermore, the report of the English Bridge Stress Committee (1928) has placed the phenomenon of impact on a more scientific and satisfactory basis than has hitherto been the case. The substance of this report has been adopted by all Australian railways as a basis for the calculation of impact values for all future work. At present, data are being compiled concerning the hammer-blow, and other, characteristics of Australian locomotives. The object in view is the use of the methods given in the report mentioned, on

* Watertown Arsenal Rept., Vol. 3, 1909, pp. 869–896.
† *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1268.
‡ Chf. Engr., South Australian Rys., Adelaide, South Australia.
§ Received by the Secretary, July 8, 1930.
‖ *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2641, *et seq.*

bridge-strengthening problems, and the adoption of standard conventional locomotives of various types, having definite impact characteristics, for use in calculating impacts for all future railway bridge design.

The latest conference of Australian Railway engineers was definitely of the opinion that an empirical formula is not logical if it excludes the vital questions of bridge types and the characteristics of the locomotive concerned.

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PAPERS AND DISCUSSIONS

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RAINFALL CHARACTERISTICS AND THEIR RELATION TO SOILS AND RUN-OFF

Discussion*

By George C. Danforth, M. Am. Soc. C. E.

George C. Danforth,† M. Am. Soc. C. E. (by letter).‡—Among points which stand out in this paper are: The great value of the condensed data in Table 1 (Appendix I);§ a feeling that its value could be measurably increased by the inclusion of certain additional data; and the fact that uncertainties in connection with some of the place names bring out the importance of including the usual geographic co-ordinates for each station.

Incidentally, it seems desirable to express an appreciation of the value of "World Weather Records",‖ which is the source of the major part of the data listed in Table 1 (Appendix I). In this, stations are listed alphabetically by countries, and monthly and yearly values of precipitation, temperature, and pressure are given in the unit used in the original record. While there may be some question as to the advisability of abandoning the 10° squares of latitude and longitude of the Réseau Mondial, an index on that system is appended.

In Table 1 (Appendix I) the monthly precipitation is given for the "normal year"; that is, the monthly means for the period of the record, and the stations are arranged in a novel and exceedingly useful form for the study of variations over large areas. While the table is valuable as it stands, it is felt that the engineer interested in run-off would greatly appreciate the inclusion of two additional "years" in the record, namely, monthly precipitation for an average year; and monthly precipitation for a minimum year.

In the United States the excellent data of the Geological Survey leaves little reason for attempting to derive a run-off from rainfall, but the greater

* Discussion of the paper by C. S. Jarvis, M. Am. Soc. C. E., continued from August, 1930, *Proceedings*.

† Hydr. Engr., Augusta, Me.

‡ Received by the Secretary, April 30, 1930.

§ *Proceedings*, Am. Soc. C. E., January, 1930, Papers and Discussions, p. 32 *et seq.*

‖ Smithsonian Miscellaneous Collections, Vol. 79, 1927.

part of the world has no available record of stream flow, and rainfall records must be used in preliminary estimates of available power. From this point of view it is felt that records for the average and minimum years are of greater importance than the record for the "normal year."

The importance of the minimum year record will be obvious. The need of figures for the average year (a year of the record the total of which equals or approximates the mean yearly total) is due to the fact that the "normal" year given in Table 1 (Appendix I) is fictitious; it probably never has occurred, and never will actually occur. The method of obtaining the "normal" year is one that smooths out the high spots. The fact that, in a given locality, six or eight months are dry, may not appear in its figures unless the dry months are the same year after year; this is rarely the case in fact. To illustrate the difference in the types of year under discussion, the monthly rainfall for two stations in Brazil is given in Table 9 for the "normal", average, and minimum year of the record. Alto da Serra is in the State of São Paulo, on the coastal side of the Serra da Mar, 2 625 ft. (800 m.) above sea level, latitude 23°47′S. It is in the path of the southeast trade winds, has a sub-tropical climate, and a high rainfall. Quixeramobim, in the State of Ceará, is in the interior of the dry northeast at an elevation of 679 ft. (207 m.), above sea level, latitude 5°16′ S. Its climate is tropical. These data are taken from "World Weather Records."

TABLE 9.—MEAN MONTHLY RAINFALL, IN INCHES,* OBSERVED AT TWO STATIONS IN BRAZIL.

Month.	ALTO DA SERRA.			QUIXERAMOBIM.		
	Normal.	Average.	Minimum.	Normal.	Average.	Minimum.
January	16.6	15.4	7.0	3.1	0.8	4.7
February	15.4	20.1	10.3	3.5	5.1	0.3
March	14.4	10.4	14.8	5.7	8.4	1.9
April	12.1	13.0	10.4	4.8	4.3	0.6
May	8.4	10.5	1.1	3.7	2.6	0.6
June	7.7	7.2	0	1.5	2.1	0.2
July	6.7	5.3	7.0	0.8	1.3	1.3
August	7.8	13.9	5.1	0.4	0	0.7
September	10.7	7.0	2.5	0.08	0.1	0.08
October	12.4	13.7	5.4	0.04	0	0
November	14.1	14.1	8.6	0.2	0.3	0
December	14.8	11.0	17.0	1.1	0	0
Total	141.5	141.5	89.3	25.1	25.0	10.3

* Original data in millimeters.

The stations in Table 1 (Appendix I) seem to be well chosen to give a general idea of climatic conditions. One exception to this may be noted in the case of Brazil.* No station is listed to indicate in any way the unusual condition prevailing in the so-called "sertão," the dry interior of the northeast. The coastal stations listed show a normal precipitation and the high rainfall of the Amazon Basin is near; nevertheless, the greater part of the States of

* *Proceedings*, Am. Soc. C. E., January, 1930, Papers and Discussions, p. 38, Table 1 (Appendix I), Stations 375 to 380, inclusive.

Maranhão, Piauhy, Ceará, Rio Grande do Norte, Parahyba, Pernambuco, and Bahia have normally seven dry months or more a year, at which time the country assumes the aspect of a desert. At times, there have been 1-year or 2-year droughts, with heavy loss of life and cattle, and large emigration to other areas, notably during the "Great Drought" of 1877-79 in Ceará. The writer has had occasion to estimate the minimum flow on certain streams in Pernambuco. One appeared to have less than 1 cu. m. per sec. (35.3 sec-ft.) on an area of 2 980 sq. km. (1 852 sq. miles); and on 2 580 sq. km. (1 603 sq. miles) a minimum flow of zero. One station showing these conditions is listed in "World Weather Records", but was omitted from Table 1 (Appendix I). The area is amply covered in reports of the Directoria de Meteorologia de Brasil.

While the addition of monthly temperature and pressure records might not have a value sufficient to balance the loss of compactness, it is felt that the mean annual temperature and pressure would be extremely useful. The temperature is of considerable interest to both the engineer and climatologist. The pressure is of interest in relation to the world belts of high and low pressure, such as the general high pressure (30.0 to 30.1 in.) of 30° latitude, falling off to 29.8° in the equatorial belt and toward either pole (29.9 at 60° N. and 29.3 at 60°S.)*

The value of a meteorological record depends much on ability to locate the station with reference to the coast line, mountain ranges, and prevailing winds. At many stations the exact location becomes of importance. For example, Alto da Serra (Station 376) in the State of São Paulo, Brazil, faces the sea near the top of the coastal range, intercepting rainfall from the prevalent southeast trades with resulting heavy precipitation, while the plateau, about six miles inland, is relatively dry.

A casual examination of the remainder of Table 1 (Appendix I) shows that for some stations in Africa the country is given; for others, merely, "E. Africa", "W. Africa," etc. Certain stations in Java, Sumatra, and Timor are listed rather vaguely as "East Indies." The station listed as "Pará, near Belem, Brazil" is decidedly uncertain as to location. Belem is the city which has been called Pará erroneously on some English and American maps in the same way that Recife has been called Pernambuco, and Forteleza has been called Ceará, using in each case the name of the State for its capital city. Belem is about 100 miles up the Amazon (this southern branch of the delta is actually the Rio Pará). There seems no way of determining from data at hand whether this station is 100 or more miles inland, or actually on the coast at Salinas, a third-class station listed by the Directoria de Meteorologia. In addition to a possible confusion resulting from errors or vagueness in place names, may be noted: (1) The possible duplication of names in the same country; (2) the fact that many towns and cities have changed their names (a common occurrence in South America); and (3) that many places have two names, one only appearing on this or that map. If the co-ordinates were given, these sources of uncertainty would be eliminated.

* "World Weather," by H. H. Clayton, p. 29.

The statement is made* that the greatest rainfall intensity occurs in sub-tropical and not in equatorial regions. While many of the stations in Table 1 (Appendix I) show no data on daily or hourly maximums, it appears to the writer that the high records are found rather generally in tropical areas. Table 1 (Appendix I) lists ten stations with a mean annual rainfall of more than 200 in. Of these, eight are tropical; the Formosa Station (494) is just inside or just outside the tropics (the name does not appear on available maps), with an elevation of 1 378 ft. The highest record listed is that of Cherrapunji (Station 515), in the Khasi Hills of Assam (Elevation 4 309). Its climate is sub-tropical (latitude 25° 16′ N.) and the mean rainfall given is 456 in., approximately one-half of which falls in June and July, and with a 24-hour precipitation of 40 in. (June 12, 1876). Similar conditions exist on the Western Ghats, a mountain barrier to the southwest monsoon, where precipitation increases with altitude to 250 in. at Mahabaleshwar, while the tableland of Deccan to the east is subject to drought. Baguio (Station 492), in Northern Luzon, with a yearly average of 182.0 in. and a daily maximum of 46.0 in. (July 15, 1911), and with an elevation of 4 959 ft. (latitude about 17° N.), is on a similar mountain barrier to the same monsoon. The wind is from the southeast, but both here and in India the movement during the northern summer is from the sea toward the heated land area of Southeastern Asia.

Of fourteen stations listed with a mean annual rainfall between 150 and 200 in., eleven are tropical and three are temperate, the latter with elevations between 1 100 and 4 400 ft. The highest record shown for North America and South America is that at Buenaventura, on the Choco Coast of Colombia (Station 346), with an average of 280.6 in. Mr. Leo Miller, of the American Museum of Natural History, is authority for the statement that the average rainfall on the San Juan River Basin just north of Buenaventura (latitude about 3° N.), is more than 400 in.

Of these twenty-five high records, twenty are definitely tropical, two are either tropical or slightly over the border, and all five exceptions have elevations between 1 000 and 4 400 ft. It seems probable, however, that the location of these high records in the tropics is due to the fact that monsoon or trade winds moving over a wide expanse of sea are commonly found in tropical areas. When these winds of high humidity meet a physical obstruction, such as the Serra do Mar in Brazil, the hill country in India, or the elevations in Hawaii in the path of the northeast trades, forcing moisture-laden air to cooler altitudes, heavy precipitation occurs, the opposite side of the obstruction being comparatively dry. The forcing of moisture-laden air to cooler altitudes with resulting precipitation may also be a result of atmospheric conditions, such as the meeting of southeast and northeast trade winds on the Lower Amazon.

In discussing altitude and precipitation A. J. Henry, Meteorologist, U. S. Weather Bureau, stated,† "The most favorable conditions for an increase in precipitation with increase in altitude, *viz.*, saturated air and relatively high temperature, are found in the tropics." The word, "tropics," in this case

* *Proceedings*, Am. Soc. C. E., January, 1930, Papers and Discussions, p. 20.
† *Monthly Weather Review*, January, 1919, p. 34.

presumably refers to the area between 23° 27′ north and south latitude, which is the geographic definition, and the writer has used it with that meaning. From a climatic point of view, however, it seems reasonable to consider it as lying between the two high pressure belts at about 30°, which separate the trade wind areas from the west to east cyclonic movements of the temperate zones. While the equatorial rain belt oscillates roughly between 20° N. and 20° S., the 30° limit would include also the monsoon areas and all the high rainfall records except in the mountains of Wales.

Evaporation.—The evaporation data given in Table 2* are of some interest, together with the inference that they are given for use as indices of evaporation from a free water surface. This table shows for only a small percentage of the stations listed whether the record was from a land or a floating pan and in no instance is information given concerning the size of the pan and the conditions under which it was exposed.

Some confusion still exists (as a perusal of some recent textbooks will prove), concerning the relation between the actual evaporation from open water and that obtained from floating and land pans. Much valuable information on this subject is available in papers presented before the Society, particularly in the discussions on the paper by the late Edwin Duryea, Jr., and H. L. Haehl, Members, Am. Soc. C. E., entitled "A Study of the Depth of Annual Evaporation from Lake Conchos, Mexico,"† and in the paper on "Evaporation on United States Reclamation Projects," by Ivan E. Houk, M. Am. Soc. C. E.,‡ and the discussions thereon. The writer has reached the conclusion, from these data, and from his own experiments, that a floating pan properly installed and of such depth that the temperature of the water in the pan is the same as that in the reservoir, will approximate the evaporation from the reservoir. He has also reached the conclusion that the land pan is useless without definite information as to its size and method of exposure, and that its records are of doubtful value unless a coefficient has been obtained by comparison with a properly installed floating pan at the same site. If monthly records are to be used, the coefficient must be obtained from monthly comparisons as it varies decidedly throughout the year.

* *Proceedings*, Am. Soc. C. E., January, 1930, Papers and Discussions, p. 10.
† *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1829, *et seq.*
‡ *Loc. cit.*, Vol. 90 (1927), p. 266.

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PLASTIC FLOW IN CONCRETE ARCHES

Discussion*

BY PAUL BAUMAN, M. AM. SOC. C. E.

PAUL BAUMAN,† M. AM. SOC. C. E. (by letter).‡—The introduction of Bach's exponential formula in the fundamental equations of internal equilibrium of a concrete structural member leads to the results shown in Figs. 9 to 13,§ inclusive. In assuming the exponent, m, to be 1.30, the author exaggerates the influence of the commonly committed error due to m = unity in concrete design; but his elaborate paper is undoubtedly of greater interest from a theoretical than a practical standpoint, thereby making any limit between minus and plus infinity permissible. In this connection it may be noted, however, that the exponential formula, Equation (9),‖ is meant to be an interpretation of experimental results rather than a physical law, as the answer that k becomes infinitely great for an infinitely small stress, for example, is doubtlessly wrong. Furthermore, it is evident that the elastic behavior of concrete is not fully described by the exponential formula, as Poisson's ratio is also variable, thereby requiring supplementary information.

Both the linear and the exponential relations between stress and strain are approximations, the latter merely being a closer one, provided the exponent, m, is correctly chosen or experimentally determined. The merit of the first one is that it errs on the side of safety. As long as the physical process of deformation due to stress is as little explored as it is to-day, particularly with a material as erratic as concrete, it seems advisable to keep on erring by the application of Hooke's law and the law of superposition in structural design.

The tendency of self-preservation in concrete if subjected to a bending test is evident as well as in any other crystalline conglomerate. Imme-

* Discussion of the paper by Lorenz G. Straub, Jun. Am. Soc. C. E., continued from August, 1930, *Proceedings*.

† Chf. Designing Engr., Quinton, Code & Hill—Leeds & Barnard, Los Angeles, Calif.

‡ Received by the Secretary, June 9, 1930.

§ *Proceedings*, Am. Soc. C. E., January, 1930, Papers and Discussions, p. 65 *et seq.*

‖ *Loc. cit.*, p. 60.

diately before failure occurs lasting deformations take place, starting at the extreme fibers and gradually moving toward the neutral axis. The stress distribution then corresponds to a relatively great exponent, m, resulting in an increase in stress areas as compared to the triangular distribution, thereby permitting the section, relatively speaking, to resist a greater moment under smaller maximum stresses. Simultaneously, the neutral axis, due to a difference in strain between tension and compression, which becomes more pronounced near the point of exhaustion of tensile strength, will shift from the axis of gravity so as to disengage the critical fibers somewhat on the tension side.

Recent research on crystals at the Institute of Technology at Pasadena, Calif., has probably shed more light on what is going on when deformation takes place than any similar work heretofore. According to these findings crystals are built up of small elements or blocks, each consisting of several molecules which undergo translatory and rotary motions when deformation takes place, thereby governing the behavior of the crystal. These elements are either plastic or brittle and form the basis of classification of crystals into: (1) Macroscopic plastic and microscopic plastic; (2) macroscopic plastic and microscopic brittle; (3) macroscopic brittle and microscopic plastic; and (4) macroscopic brittle and microscopic brittle.

Needless to say these findings could not be directly applied to such complicated crystalline conglomerates as concrete, but they are indicative nevertheless of what is going on when deformation takes place. In all probability concrete would be classified as macroscopic plastic and microscopic brittle; that is, the elastic limit of elementary blocks coincides with the breaking strength.

In view of the uncertainty of the tensile strength of concrete and of the steadily growing tendency to reduce the size of structural members with a simultaneous increase in span, it will perhaps be a relatively short time only before plain concrete will be barred from use in members subject to failure due to tensile stresses originating from any cause whatsoever. One cannot set a safe limit to tension in plain concrete if there is a possibility of serious vibrations affecting it, which, unfortunately, is the case everywhere on earth.

In 1903, a reinforced concrete frame bridge, as shown more or less diagrammatically in Fig. 32, was built for a National Exhibition in Dresden, Germany, and subsequently was subjected to a test* resulting in its failure. A concentrated load varying from zero to a little more than 77 500 lb. (Fig. 33) was applied at the center of the girder span and records were kept of the vertical deflection, d, of these girders directly under the load. The strain, E_s, in the reinforcing bars directly under the load was measured and from these values the steel stresses were computed. Finally, the deformation of the posts, that is, the angular deflection at the top, horizontal deflection at center, and settling of bases, was observed.

As in this discussion the change of modulus of elasticity with the change of loading is of prime interest, it will suffice to confine these studies to the top girder only.

* Dr.-Ing. W. Gehler, Betonbeiblatt No. 9, *Deutsche Bauzeitung*, 1904.

In Fig. 33 the deflections, d, and the corresponding steel stresses are given for P varying from zero to 77 500 lb. Lines A, B, C, D and A, B', C', D' very closely approximate the observed deflections and steel stresses, respec-

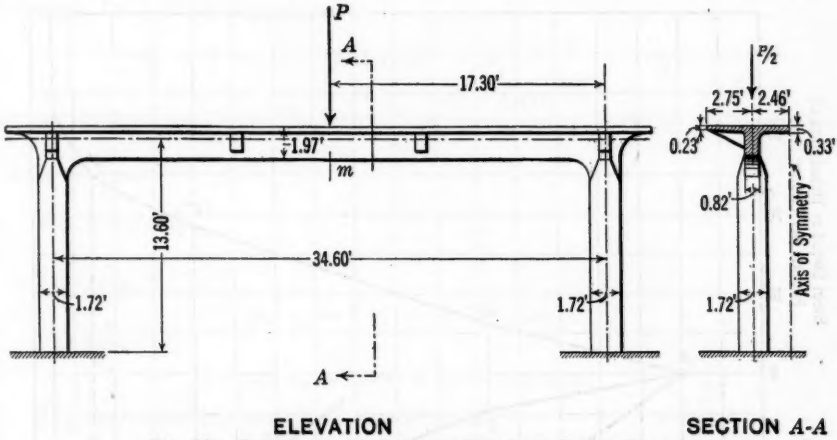

FIG. 32.—SKETCH OF REINFORCED CONCRETE FRAME BRIDGE.

tively. The significance of these straight-line zones becomes evident if it is remembered that, according to the old theory of elasticity, the deflection of the top girder, due to bending, is:

$$d = \int \frac{M\, d\, M}{E\, I\, d\, P}\, d\, s = \frac{P\, l^3}{48\, E\, I} - \frac{M\, l^2}{8\, E\, I} \ldots\ldots\ldots\ldots (119)$$

or that it is equal to the deflection, due to $M_0 = \frac{P\, l}{4}$, of a simple beam of equal span, minus the influence of the corner moment, $-M$. Substituting $\mu\, M_0 = \frac{\mu\, P\, l}{4}$ for $-M$:

$$d = \left(\frac{2 + 3\,\mu}{96}\right) \frac{P\, l^3}{E\, I} = C \frac{P\, l^3}{E\, I} \ldots\ldots\ldots\ldots\ldots (120)$$

because μ is, for one and the same frame structure, a constant. This is the equation of the curve through Lines A, B, C, and D (Fig. 33) and,

$$\tan \alpha = \frac{d\,(d)}{d\, P} = C \frac{l^3}{E\, I} \ldots\ldots\ldots\ldots\ldots\ldots (121)$$

is its slope. Since this slope is constant between Lines A and B, B and C, and C and D, respectively, it follows that the product, $E\, I$, is also constant within each of the three zones. The physical meaning of this is that in Zone A-B the full concrete section is active on the tension as well as the compression side of the neutral axis; in Zone B-C the first hair-cracks occur in the concrete under tension, partly eliminating its activity; whereas, in Zone C-D, the limit of proportional deformation of steel is reached and failure begins. As the rigidity of the structure inside each zone remains unchanged (tan α = constant), the product, $E\, I$, is an index of rigidity. To determine

E of the compound material, that is, for concrete and steel or reinforced concrete, Equation (121) may be written, $E \tan \alpha = \frac{C\,l^3}{I}$.

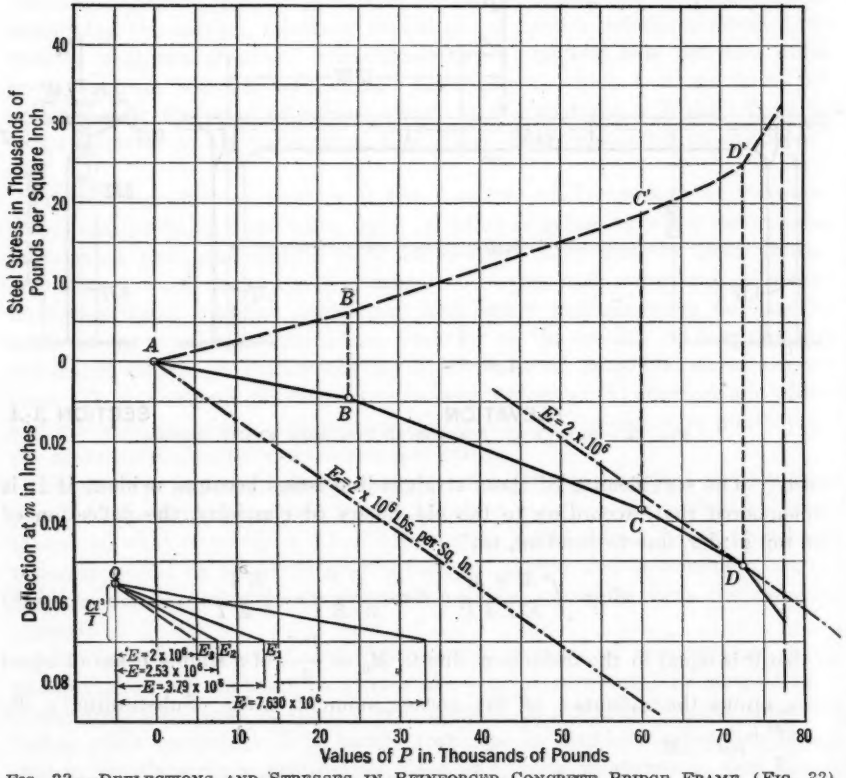

FIG. 33.—DEFLECTIONS AND STRESSES IN REINFORCED CONCRETE BRIDGE FRAME (FIG. 32) DUE TO CONCENTRATED LOADS AT THE CENTER.

In Fig. 32, $I = 22\,930$ in.^{4}; that is, the moment of inertia of the full concrete section of the top grider; $l = 34.60$ ft. $= 415.30$ in.; and $C = \frac{2 + 3\,\mu}{96}$. Using Gehler's "short-cut" for the determination of μ,

$$\mu = \frac{h\,I}{l\,I_p} \qquad (122)$$

in which, $I_p = 15\,140$ in.^{4} and $h = 13.62$ ft. $= 163.40$ in. Therefore,

$$\mu = \frac{163.40 \times 22.930}{415.30 \times 15\,140} = 0.597$$

As there is always some uncertainty as to the degree of restraint of the posts at the base, the average between fully restrained and fully hinged is taken.

For full restraint:

$$\rho_1 = 2 + \mu = 2.597 \text{ and } -\mu_1 = 0.385 = \frac{1}{\rho_1}$$

whereas, for a fully hinged base,

$$\rho_2 = 3 + 2_\mu = 4.194 \text{ and } -\mu_2 = \frac{3}{2\,\rho_2} = 0.358$$

or an average of $\mu = -0.372$. Hence, $C = \frac{2 - 1.116}{96} = 0.00922$. The slopes, $\tan \alpha = \frac{d}{P}$, are:

For Line *A-B*:

$$\tan \alpha_1 = 3.81 \times 10^{-6} \text{ in. per lb.}$$

For Line *B-C*:

$$\tan \alpha_2 = 7.67 \times 10^{-6} \text{ in. per lb.}$$

For Line *C-D*:

$$\tan \alpha_3 = 11.50 \times 10^{-6} \text{ in. per lb.}$$

The corresponding moduli of elasticity are:

$$E_1 = \frac{C\,l^3}{I \tan \alpha} = \frac{9\,220 \times 10^{-6} \times 415.30^3}{3.81 \times 10^{-6} \times 22\,930} = \frac{29.10 \times 10^6}{3.81}$$
$$= 7\,630\,000 \text{ lb. per sq. in.}$$

$$E_2 = \frac{29.10 \times 10^6}{7.67} = 3\,790\,000 \text{ lb. per sq. in.}$$

and,

$$E_3 = \frac{29.10 \times 10^6}{11.50} = 2\,530\,000 \text{ lb. per sq. in.}$$

To show this graphically $\frac{C\,l^3}{I}$ is plotted in Fig. 33(a) and rays are drawn parallel to Zones *A-B*, *B-C*, and *C-D*, respectively, through 0, intersecting the base at the values of E_1, E_2, and E_3, respectively. Finally, $E = 2\,000\,000$ lb. per sq. in. is plotted, and a parallel to the corresponding ray is drawn as a tangent to the Curve *A*, *B*, *C*, *D*. The point of contact approximately coincides with the end of Zone *C-D;* that is, with the beginning of failure, thereby justifying the usual assumption of $E = 2\,000\,000$ in concrete design as it approaches the lower limit. This value, however, leads to an exaggeration of deflections as indicated by a parallel line through Point *A*. At the end of the first zone, that is, at Point *B*, the tension in concrete for a load of 23 800 lb. was computed and found to be 350 lb. per sq. in., or approximately one-tenth the ultimate compressive strength of concrete. The maximum compressive stress at the center of the span, due to the ultimate load of 77 500 lb. was computed at 800 lb. per sq. in., whereas near the support it was in excess of 1 800 lb. per sq. in. Failure, however, occurred in the steel at m (Fig. 32).

It would be interesting, indeed, to have comparative data based on the author's new theory of elasticity.

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LAMINATED ARCH DAMS WITH FORKED ABUTMENTS

Discussion*

By Messrs. Paul Bauman and F. W. Hanna

Paul Bauman,† M. Am. Soc. C. E. (by letter).‡—The author has endeavored to eliminate complexities in the interior stability of thick concrete arches by designing laminations to furnish a simple substitute for the cumbersome trial-load method. Unfortunately, his new type which, in principle, is a "telescoped" Dordogne Dam,§ involves serious physical uncertainties which might readily impair the safety of the structure.

The assumption, for example, that the flexibility of the compound structure is governed by that of the individual rings is hardly justifiable as long as the faces of the rings are in contact. The relative motion of molecules (relative lengthening and shortening) at contact points would add to the restraining influence of sliding friction so as to render the effective frictional coefficient very uncertain.

Furthermore, this coefficient is not a constant for one and the same point; it is a maximum at the time that movement begins (end of rest), and diminishes as movement continues. This feature would tend to cause slips between the laminæ, resulting in "dam quakes", just as it produces slips between geological strata resulting in earthquakes. The ensuing vibrations would not be desirable to say the least.

On the other hand, it cannot be denied that the lamination would enhance the flexibility of the compound structure and, therefore, would tend to reduce the eccentricity of the line of pressure. As the load on a dam, under normal conditions, is gradually increasing during the initial filling of the reservoir, the up-stream ring would be engaged first; then the next ring would be engaged, etc.

If the spacing between the down-stream face of the up-stream ring and the up-stream face of the one adjacent to it were such as to cause uniform contact

* Discussion of the paper by Fred A. Noetzli, M. Am. Soc. C. E., continued from August, 1930, *Proceedings*.

† Chf. Designing Engr., Quinton, Code & Hill—Leeds & Barnard, Los Angeles, Calif.

‡ Received by the Secretary, July 11, 1930.

§ *Proceedings*, Am. Soc. C. E., February, 1930, Papers and Discussions, p. 285.

as soon as the up-stream ring has its share of the full load on the dam, its elastic behavior, up to this point, would undoubtedly be independent of the remainder of the laminæ. Based on the same premises, a further increase of the load would then engage the first plus the second ring, then the first plus the second plus the third, etc. The nth lamina would then be engaged for a load equal to $\frac{n-1}{n}$ times the total load. As soon as contact takes place between two faces, relative motion could undoubtedly still develop, but, even if such motion were disregarded, it is evident that the eccentricity of the line of pressure—and, consequently, the extreme fiber stresses of n combined laminæ—would be materially smaller than that of a monolithic dam of equal cross-section. The structural difficulties involved in such a spacing of faces to provide for deflection would be very serious indeed.

The elastic behavior of a laminated arch is dependent upon the load. The greater the load the smaller its flexibility. This is a distinguishing characteristic.

The flexibility of an arch slice is described through its ellipse of elasticity or, in other words, through the moments of inertia of its elastic weights about the principal axes of gravity. The fundamental problem in the design of a laminated arch, therefore, is the determination of the effective moment of inertia of its elastic weights which is limited by the elastic behavior of the most flexible, up-stream lamina as a maximum and the elastic behavior of the solid arch with a thickness equal to n laminæ as a minimum.

Assuming for the moment a symmetrical layout and numbering the laminæ consecutively, beginning at the up-stream face, the elastic center, elastic weights, and moments of inertia of Arches 1, 1 + 2, 1 + 2 + 3, etc., are determined.

Their centers of gravity are readily found, the principal axes are located, and the moments of inertia about these axes are determined. The effective moment of inertia of the combined laminæ is $\frac{1}{n}$ times the sum of the moments of inertia of the individual groups (1, 1 + 2, 1 + 2 + 3, etc.), because each of the n combinations contributes $\frac{1}{n}$ parts of its flexibility to the elastic behavior of the entire section.

In Fig. 17 this hypothesis is illustrated for a symmetrical section with three laminæ of equal thickness. It shows clearly the diminishing of the ellipse of elasticity; that is, the decrease of flexibility as the number of laminæ engaged to carry the load increases. (Arches 1, 1 + 2, and 1 + 2 + 3 are designated A, B, and C, respectively, in Fig. 17.)

The elastic weights, W, of Arches A, B, and C, are:

$$W_A = \frac{24\left(R_u - \frac{t}{2}\right)\phi_0}{E\,t^3} \qquad (13)$$

$$W_B = \frac{3\,(R_u - t)\,\phi_0}{E\,t^3} \qquad (14)$$

and,

$$W_C = \frac{8\left(R_u - \frac{3\,t}{2}\right)\phi_0}{9\,E\,t^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

The distances, y_s''', of the elastic centers from the common center of curvature, C, are:

$$y_A = \frac{\left(R_u - \frac{t}{2}\right)\sin\phi_0}{\phi_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

$$y_B = \frac{(R_u - t)\sin\phi_0}{\phi_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

and,

$$y_C = \frac{\left(R_u - \frac{3\,t}{2}\right)\sin\phi_0}{\phi_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

The moment of inertia of the elastic weight, W (about the x-x axis), was found graphically as $E\,I\,W = 3.33$ (see Fig. 17).

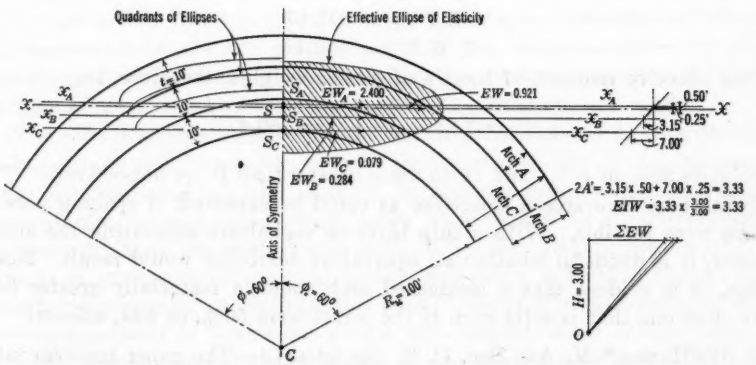

FIG. 17.

The moments of inertia of Arches A, B, and C (about the x-x axis), are (very approximately),*

$$I'_A = \frac{24\left(R_u - \frac{t}{2}\right)^3}{E\,t^3}\,K' \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

$$I'_B = \frac{3\,(R_u - t)^3}{E\,t^3}\,K' \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

$$I'_C = \frac{8\left(R_u - \frac{3\,t}{2}\right)^3}{E\,t^3}\,K' \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

* *Transactions*, Am. Soc. C. E., Vol. 93 (1929), top of p. 1675.

in which,

$$K' = \frac{\phi_0}{2} + \frac{\sin 2\,\phi_0}{4} - \frac{\sin^2 \phi_0}{\phi_0}$$

About the y-y axis, the moments of inertia are:

$$I''_A = \frac{24\left(R_u - \frac{t}{2}\right)^3}{E\,t^3} K'' \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

$$I''_B = \frac{3\,(R_u - t)^3}{E\,t^3} K'' \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

and,

$$I''_C = \frac{8\left(R_u - \frac{3\,t}{2}\right)}{9\,E\,t^3} K'' \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

in which,

$$K'' = \frac{\phi}{2} - \frac{\sin 2\,\phi_0}{4}$$

Introducing numerical values as shown in Fig. 17 there results:

$$E\,I''_A = 493.00$$
$$E\,I''_B = 52.35$$
$$E\,I''_C = 13.07$$
$$\Sigma\,E\,I'' = 558.42$$

The effective moment of inertia of elastic weights, therefore, is:

$$I' = 3.33 + \frac{558.42}{3} = 189.47$$

which is 38.30% of I''_A, and 14.45 times greater than I''_C, respectively. This is probably as favorable a condition as could be expected, if spacing between laminæ were feasible. With a thin layer of asphaltum separating the arches, however, it is doubtful whether an equivalent flexibility would result. Nevertheless, it is evident that a laminated arch shows a materially greater flexibility than one that is solid even if the joints were 50%, or less, efficient.

F. W. Hanna,* M. Am. Soc. C. E. (by letter).†—The paper involves interesting questions in arch dam design; as to the superiority of stress analysis for the thin arch ring; the layout problem for dams in irregular canyons; the use of forked abutments to decrease arch spans; the effect of overhang on the layout and stresses; and the substitution of a series of thin arches for a single thick one.

Designers of arch dams are familiar with the difficulties attending the analysis of thick arch rings near the bottom of high dams. When arch formulas are used, tension appears to be involved, both at the crown and at the abutments; while, as a matter of fact, it is probable that combined arch and beam action exist in a complicated form. Undoubtedly, on account of the shortness of the span and depth of the beam, shear provides the bulk of the required resistance. Even if the arch rings at the base were laminated, it is probable

* Chf. Engr. and Gen. Mgr., East Bay Municipal Utility Dist., Oakland, Calif.
† Received by the Secretary, July 21, 1930.

that shear along the canyon bottom and walls would still furnish the principal resistance.

The author proposes to use a cone layout for arch dams, but this involves an intricate manipulation of central angles, radius lengths, and overhang. Up-stream conic surfaces of arch dams must consist of sections of the surfaces of parent cones. Such sections are parallelogrammatic, trapezoidal, triangular, parabolic, or hyperbolic in nature and are roughly the same as the possible canyon sections suitable for arch dam sites.

In the case of a parallelogrammatic section, the conical surface will be cylindrical as the cylinder is a cone with its vertex at infinity. Both the central angle and radii from top to bottom of the dam can be kept constant, and difficulty with overhang can be obviated. In the case of a triangular section, the vertex of the cone will be at foundation level, and the central angle can be kept constant from top to bottom of the dam, the radii will decrease uniformly from the top to the bottom of the dam, and difficulty will be experienced with overhand. In the case of a trapezoidal section, the vertex of the cone will be below foundation level, the central angle can be kept constant from top to bottom of the dam, the radii will vary uniformly from a maximum at the top to a minimum at the bottom of the dam, and difficulty will be experienced with overhang. In the parabolic or hyperbolic section, the vertex of the cone will be below foundation level, both the central angle and the radii will decrease from the top to the bottom of the dam, and difficulty will be experienced with overhang. Consequently, the up-stream surface of the dam must generally either be warped or subject to overhang.

It is also desirable to have all the crown points and arch ring centers in the same plane and, if possible, to have the plane vertical. If the cross-section of the dam site is an isosceles triangle, an isosceles trapezoid, an equi-angular parallelogram, or any symmetrical curve with respect to a vertical axis, all the crown points in the neutral axes of the arch rings and the centers of curvatures or of symmetry of the neutral axis will lie in a vertical plane. If the triangle or trapezoid is oblique, the parallelogram non-equi-angular, or the curve symmetrical in horizontal planes with respect to an oblique axis, then the plane containing the crown points of the neutral axes and their centers of gravity or of symmetry will be in an oblique plane. In either case the horizontal arch rings will be symmetrical and symmetrically loaded.

If, however, on account of irregularities in the sides of the canyon walls, the cross-section does not conform to the rectilinear boundary of the triangle, trapezoid, or parallelogram, or to the boundary of a curve symmetrical in horizontal planes with respect to an oblique plane, the various arch rings cannot be made symmetrical with reference to a single vertical or inclined plane. If the distances of the crown points at the neutral axes or their centers of curvature from a vertical or an inclined plane are small, only slight errors will be involved in considering them symmetrical and symmetrically loaded.

As stated by Mr. Noetzli,* the use of forked abutments in the design of arch dams will promote general symmetry of the dam and will decrease the span and the radii in the upper section.

* *Proceedings*, Am. Soc. C. E., February, 1930, Papers and Discussions, p. 274.

The excessive weathering of the canyon walls near the top of most dam sites so increases the width as to make the design of the upper part of a high arch dam either uneconomical or infeasible without the construction of abutments to take the place of the missing canyon walls. In order to meet the walls in wide canyons at suitable angles the arch has to be too flat for economy; and, if tangential abutments of full height are used, they are sometimes so nearly parallel to the canyon walls as to make them miss the walls entirely or to make their dimensions uneconomical. To overcome this difficulty the designers of arch dams have used the principle of forked abutments, the up-stream fork serving in part as a gravity dam of short length to close the reservoir and also sometimes in part as a buttress to resist arch thrust, and the down-stream fork serving only as a buttress to take the remaining thrust of the arch. Usually the buttresses are designed with outward sloping tops so that the weights when combined with the arch thrusts taken by them at all points from the springing lines outward meet the adjacent horizontal planes or foundations at angles less than the friction angle. Whether the inclined arch action should be considered in the design of these buttresses is somewhat questionable, as the amount carried by them is not easily determined and their extra cost due to embodying this additional factor of safety is usually not too great.

Forked abutments somewhat similar to those suggested by the author have been used with success in the Santeetlah Dam and the Calderwood Dam built by the Aluminum Company of America. The writer used this principle also in the preliminary design of an arch dam for the Pardee Dam site for the East Bay Municipal Utility District, Oakland, Calif.

In the Santeetlah Dam the sloping buttresses of the forked abutments were made radial to the arch section at the springing lines; they were heavy enough to take the radial thrust and to throw the remaining thrust of the arch into direct line with the other forks of the abutments; these, in turn, were designed to take this thrust and, as gravity sections, to close the reservoirs at the ends of the arch. In the Calderwood Dam it was found more convenient and economical to use three forks for the abutments. The first forks consist of gravity sections to close the reservoir, running on a radial line from the springing lines at the left abutment and nearly so at the right abutment; the second forks consist of sloping spray walls running along radial lines from the springing lines, sloping downward at a 0.5 to 1 slope, and designed to take the radial thrust and to act as spray walls; the third forks consist of buttresses extending from the ends of the arch in the line of the remaining thrust and sloping downward away from the arch at a 2 to 1 slope. They are designed to take the normal thrusts at the ends of the arch. In the preliminary design of the arch dam for the Pardee Dam site, the forked abutments were computed so that the gravity sections closed the reservoir by the shortest routes to the canyon walls, and the buttresses were designed to slope downward away from the ends of the arch at a 2.5 to 1 slope and to take the entire thrust of the arch.

The effect of overhang down stream at the crown will be beneficial in the control of the central angles and radii, and in carrying the load to the abut-

ments by inclined arch action. However, it complicates the arch ring analyses. An equally important help can be obtained in the adjustment of the radii and central angles through overhang in an up-stream direction adjacent to the abutments. Abutment overhang, however, inclines the arch rings upward and carries the stresses upward at the abutments. This overhang usually occurs near the base of the dam, however, where the weight of the superstructure and the shortness of the arches counterbalance the bad effect. Advantages are obtained, however, by a combination of an overhang down stream at the crown near the top and an overhang down stream at the abutments near the base of the dam; but the overhang should never be enough to endanger the gravitational stability of the structure.

The writer is not fully convinced that proper transmission of stresses can be made across the laminations in the laminated type of dam. In the first place, imperfections in construction may be such as to cause the load from any lamination to be improperly transmitted to, and distributed on, the next succeeding lamination down stream. The arch rings under water load deflect down stream at the crown and up stream near the abutments. The deflection of the primary arch under water load down stream at the crown will transmit a certain load to the next succeeding arch at the crown, but not necessarily at the abutments. Consequently, the loads on the secondary arches will have different distributions and the deflections of the secondary arches will not conform accurately to those of the primary arch and to each other. These inequalities in loads and deflections will upset the stress calculations and make them uncertain both in the primary and in the secondary arches.

As a matter of fact, the lack of conformity in load distribution and deflections seems to be a serious defect in the laminated arch type of dam. Further, there is the possibility that cracks will form in the thin laminations of the laminated dam, as shown by the experience in the Stevenson Creek Experimental Dam. Moreover, if there is open space between the laminations due to shrinkage in the concrete and to settling under gravity of the asphaltic filling, undue strain may result in the primary arch. Over-deflection of the primary arch near the top of the dam may cause uplift and cracking through strut action in the secondary arches. On the other hand, if insufficient lubrication exists between the laminations, they will not act as separate arches; as a matter of fact, they cannot act fully independently unless there is a frictionless surface between them.

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BALDWIN FILTRATION PLANT, CLEVELAND, OHIO

Discussion*

By James H. Herron, M. Am. Soc. C. E.

James H. Herron,† M. Am. Soc. C. E. (by letter).‡—In this paper the excellent compilation of cost data is one of the most valuable features. While the improvement was made during the period of high costs, the proper index can be applied to make them available for any period.

It has been a source of gratification that the plant as finally planned has worked out so well since both the Cleveland Water Department and the Commission of Engineers, of which the writer was a member, were severely criticized for separating the Filtration Plant from the initial pumping station. Difficulty had been experienced with sub-grade conditions at one of the other plants located on the shore of the lake, and it was deemed impossible without high cost to build this plant adjacent to the Pumping Station. Since the Baldwin Reservoir had already been planned as a distributing reservoir for the low service, it was thought wise to avoid the expense of an additional filtered-water reservoir by utilizing one already available. The cost of pumpage is little different whether the water is delivered to the filtration plant through long lines and into the reservoir through short lines, or *vice versa*. The separation of the plants may have involved some additional pumping capacity, but with little greater total head. The increased investment was more than offset by the saving effected in utilizing a reservoir already planned and practically built as a clear-water basin.

In the building of the plant the utmost care was taken in the selection and handling of materials. The aggregates especially were approved only after examining all available supplies.

In the concreting operations great care was used in the grading of materials and in the quantity of water. This latter was kept at a minimum. Between 300 and 400 tests were made on concrete cubes (tests on cubes were

* Discussion of the paper by J. W. Elms, G. W. Hamlin, A. G. Levy, and J. E. A. Linders, Members, Am. Soc. C. E., continued from August, 1930, *Proceedings*.

† Cons. Engr., Pres., The James H. Herron Co., Cleveland, Ohio.

‡ Received by the Secretary July 21, 1930.

then required by the Building Department of the City of Cleveland), with the following average results:

Nominal mix.	Strength at 28 days, in pounds per square inch.
1:3:5	2 475
1:2:4	3 100
1:2.5:3.5	3 400
1:3:3	3 875

The higher strength mixes were used for the important work, and the results fully justified the care exercised.

The concreting operations, both as far as the proportions of the materials and placement were concerned, were in advance of the general state of the art then prevailing. As a result the structure has shown no signs of disintegration or other deterioration, and should give continued satisfaction.

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RELATION BETWEEN RAIL AND WATERWAY TRANSPORTATION: A SYMPOSIUM

Discussion*

By Samuel O. Dunn, Esq.

Samuel O. Dunn,† Esq. (by letter).‡—The National Administration has proposed, as one of its major activities, the widespread development of inland water transportation. In view of the importance to the United States of adequate, dependable, and efficient transportation service furnished at the lowest possible cost, of the magnitude of the proposed waterway development, and of the vast expenditure of public funds which such development would entail, some analysis of the general waterway situation, based upon the facts as disclosed by past experience, may very well be made.

Inland Waterway Development.—The Great Lakes are inland waterways, but it should be understood that in the following discussion the words "inland waterways", are used only in reference to improved rivers and canals. The Great Lakes were made deep and large by Nature, and nobody questions that transportation on them is much cheaper than by rail.

At Louisville, Ky., on October 23, 1929, President Hoover, in connection with the celebration of the completion of a 9-ft. channel in the Ohio River, outlined the general policy of his Administration to continue "the modernizing of every part of our waterways which will show economic justification in aid of our farmers and industries." No elaboration was made of the phrase "economic justification."

There are many waterway advocates who subscribe wholeheartedly to this program and who support it with optimistic opinion, theory, belief, and prophecy. Yet in fairness to the waterways, which in the past have been embarrassed by the too optimistic predictions of their friends, in fairness to

* Discussion on the Symposium on Relation Between Rail and Waterway Transportation, continued from August, 1930, *Proceedings*.

† Editor *The Railway Age*, Chicago, Ill.

‡ Received by the Secretary, June 13, 1930.

the railways with which these waterways will compete, but most of all in fairness to the general public which will "foot the bill," the basis of actual fact is the criterion by which the program should be judged.

Chief among the claims advanced in support of waterway development, and hence presumably closely linked to "economic justification", is the statement that such development will "cheapen" transportation. What is meant by "cheaper" transportation? Do the advocates of waterway development mean that waterway conveyance charges alone will be lower than the corresponding rail rates, or that total waterway costs, including both charges for carriage and the burden of capital charges, maintenance, and operating expenses borne by the taxpayers, will be less than the railway rates? These questions thus far remain officially unanswered.

In an Interim Report issued in 1927 by the Board of Engineers for Rivers and Harbors and the Shipping Board, "to render immediately available data showing the importance to the country of the improvements completed and in progress in the Ohio Valley," the following statement was made:

"The primary measure of the economic value of any waterway system is the savings in transportation costs which it affords. Usually this saving is shown by comparing the water transportation costs with the rates for moving the same goods between the same points by rail. Included in this cost of water transportation should be the charges for interest on capital investment in waterway improvements plus annual cost of operation and maintenance."

An exactly opposite position is taken by Maj.-Gen. T. Q. Ashburn, U. S. A., Chairman and Executive of the Inland Waterways Corporation, who states, in the 1929 Annual Report of that Corporation, that,

"Because hundreds of millions of dollars have been spent in making streams navigable is absolutely no reason why the upkeep of these rivers or the cost of their maintenance should be charged to this corporation or any other corporation operating fleets on these water 'rights-of-way'."

Finally, a middle ground also appears to be taken in the Interim Report mentioned, which claims—although with little convincing evidence—that certain waterway expenditures have already paid for themselves. In a report entitled "Inland Water Transportation in the United States", the Transportation Division of the U. S. Department of Commerce advances a theory, subscribed to by many, that much of the past expenditures for river improvements should simply be charged off and forgotten. Furthermore, it states that "for services on channels already made navigable or on canals already built the returns need cover only current operating expenses and necessary maintenance of channel, terminals, and floating equipment in order to justify continuance."

So far, search for an interpretation of "cheaper" transportation and its relationship to "economic justification" has brought to light only conflicting Government opinions. In order to obtain official information, inquiry as to the meaning of this phrase was made of the Hon. Patrick J. Hurley, Secretary of War, under whose authority and jurisdiction National waterway improvements are carried out. Mr. Hurley's only reply was that "The War Department has no intention of inviting controversial discussion as to the relative merits of various forms of transportation."

In the face of this official attitude, one must draw his own conclusions. It would appear that the only logical interpretation to be put upon the claim of "cheaper" water transportation is that it means that total waterway costs, regardless of who pays them, will be less than the corresponding rail charges. Total cost is the crucial test.

The only cost paid for the transportation of freight by rail is the freight rate charged. The cost of transportation by inland waterway is divisible into two parts. The first is paid by the public, and consists of interest upon the investment made in improving the waterway and building terminals, the expenses of maintenance, and certain operating costs, particularly in the case of canals or canalized streams. This public payment represents a subsidy to the waterways that results in an increase in taxes. The second part of the cost of water transportation is that incurred in the actual movement of freight.

Now, on sound principles of economics, what should be the objective? Should it be to reduce freight rates entirely regardless of the effect upon taxes? Plainly, if the development of waterways will result in reductions of freight rates, it will also result in increases in taxes. Total taxes in the United States increased 30% between 1923 and 1928, and amounted to twice as much as the total freight earnings of the railways. Can it be contended that taxes are so unimportant, and freight rates so important, that the National prosperity would be promoted by securing a saving in freight rates at the cost of an increase in taxes that would exceed the saving in freight rates?

Furthermore, the development of waterways is not the only way available in which taxes can be used to reduce freight rates. Recently, the British Government actually has secured reductions in freight rates on agricultural products by giving the railways equivalent direct subsidies from the Public Treasury. The railways of Canada receive subsidies from the Government for making low rates. This is not pointed out in advocacy of similar subsidies for the railways of the United States, but it is surely pertinent to call attention to the fact that if the total cost of inland water transportation, including the part paid in taxes, is or will be greater in this country than the total cost of providing similar transportation by rail, then any desired reduction of freight rates, either on certain commodities, or in certain parts of the country, or even on all traffic throughout the United States, obviously could be secured at a lower cost to the taxpayers by having the Government subsidize the railroads than by having it develop and maintain waterways. As regards the total cost of carrying all freight there is this further fact to consider: The greater the density of railway traffic, the cheaper the railways can handle each unit of this traffic. Therefore, any policy that hinders the increase of railway traffic also hinders the reduction of the cost of transportation by rail.

The situation thus narrows down, primarily, to a question of relative costs. How, then, do the total costs of transportation by inland waterway and by railway in this country compare, and how may they be expected to compare in the future? Actual experience can never be intelligently or safely disregarded. For many years the Government has been improving inland waterways and the country has actually had transportation by both water and rail. There

is an abundance of information regarding the costs to the public of rail transportation, but lamentably little reliable and satisfactory information regarding costs of inland water transportation. For many years the Army Board of Engineers for Rivers and Harbors and others have been giving estimates to Congress and the public regarding proposed waterway improvements, including the traffic that would be handled, the expense of improving and maintaining the waterways, and the savings in transportation costs that would result. Those who have made these estimates seldom have come forward after the improvements have been made with figures showing the extent to which results have or have not justified their estimates and expenditures.

There are three great inland waterways in the United States upon which transportation operations are now being conducted. These are the New York State Barge Canal, the Ohio River System, and the Mississippi. What does the record show?

In 1929, the New York State Barge Canal handled a freight traffic amounting to 715 551 472 ton-miles. Cost of canal maintenance and repairs amounted to $3 390 432, while interest on the investment, at 5%, amounted to $8 811 061 in that same year. On a ton-mile basis, maintenance and repair costs were 0.474 cent, and capital costs, 1.231 cents, a total "hidden" cost of 1.705 cents per ton-mile which was paid in taxes by the people of New York State. The average actual waterway conveyance charge on the Barge Canal—a figure which far too often is considered by the waterway advocate as the total cost of water transportation—is conservatively estimated at 0.450 cent per ton-mile, making the total cost of transportation on the New York State Barge Canal 2.155 cents per ton-mile. The corresponding average ton-mile revenue of the Class I railways in the Eastern District in 1929 was 1.090 cents. Frederick Stuart Greene, M. Am. Soc. C. E., Superintendent of Public Works of New York State, has said that in 1925 it cost the State $4.51 per ton for all freight loaded on the canal, regardless of length of haul, and that it would have been cheaper for the State if all the freight carried on the canal had been put in railroad cars and the State had paid the freight bills. Is this "cheaper" transportation, or economic justification?

What of the Ohio River System? It is conceded even by some of the principal advocates of waterway development that the bulk of the very large expenditures heretofore made on waterways has been wasted, because it has not resulted in the increase of water traffic and the reduction of transportation costs which it was intended to accomplish. It is stated, however, that future expenditures will be made more wisely and that it is entirely beyond question that they will result in cheapening transportation. In other words, the public is asked to do what is always unintelligent and unsafe, namely, to disregard experience. Surely, however, it cannot reasonably be asked to disregard very recent experience. The completion of the improvement of the Ohio River System is so recent that it was celebrated within the past year (1929). What does this waterway show on a cost basis?

Some very interesting figures of "savings" to the public resulting from water-transportation operations on the Ohio River System are contained in a report entitled "Transportation in the Mississippi and Ohio Valleys",

recently issued by the War Department Board of Engineers for Rivers and Harbors and the Bureau of Operations of the U. S. Shipping Board. In this report the following statement is made: "Based upon a ton-mileage of 2 649 000 000 on the Ohio River System in 1926, the savings in transportation costs amounted to about $25 000 000."

From the foregoing figures, the "savings" claimed appear to amount to 0.944 cent per ton-mile. Disregarding the fact that the ton-mile basis of comparison is peculiarly favorable to the waterways because of the fact that water routes, over any material distance, are almost invariably much more circuitous than the corresponding rail routes, it is interesting to note that in the same year (1926), the average ton-mile revenue of the eight principal Class I railroads operating in the Ohio Valley was only 0.883 cent. In other words, the waterway "savings" claimed amounted to 107% of the corresponding average rail revenue per ton-mile. At their average ton-mile rate of 0.883 cent, the adjacent railways would have carried the entire traffic moved on the Ohio River System for less than $23 400 000, or for about $1 600 000 less than the saving in transportation costs claimed.

This average of 0.883 cent per ton-mile for the adjacent railways covers all classes of traffic and is thus increased by the inclusion of much high-grade traffic which the waterway does not handle. The greater circuity of river routes also distorts the figures in favor of the waterway. On the other hand, average hauls on the Ohio River System are relatively short and, on corresponding rail hauls, average ton-mile revenue would be affected by the relatively large part of terminal expense it would have to bear. These three factors cannot be evaluated statistically. Despite this fact, it seems pardonable to view with some skepticism statistics which indicate that the transportation of freight on the Ohio River System resulted in savings to those for whom the freight was carried actually exceeding the total amount that the railways, at their average revenue per ton-mile, would have received for handling it.

From the report of the Board of Engineers previously mentioned, the total investment of the public's money in the Ohio River System appears to be about $240 000 000. A study of all the available data indicates that for every ton of freight carried 1 mile on the Ohio River System the taxpayers are paying a cost of about 5 mills; and those whose freight is carried are paying a cost of about 5.5 mills, or a total of more than 1 cent, as compared with an average ton-mile revenue of less than 9 mills on the adjacent railways. Approximately 90% of the traffic handled on the Ohio River System consists of such low-grade commodities as coal, sand, and gravel. The average revenue per ton-mile of the Chesapeake and Ohio and the Norfolk and Western Railroads, which operate in the same territory and the traffic of which consists mainly of low-grade commodities, especially coal, is only about 6.5 mills, or 40% less than the average total cost of transportation on the Ohio River System. Does this show "cheaper" water transportation, or economic justification for further waterway development?

Finally, what is the situation on the Mississippi River, the third of the Nation's great waterways? It is impossible to obtain detailed statistics regarding the Government's investments in the Mississippi for navigation purposes,

or detailed figures regarding operating costs, but the situation seems very aptly summed up in the report of the Inland Waterways Corporation for 1929, wherein General Ashburn states:

"If the cost of their [the rivers'] upkeep be charged to the few operators thereon, the only result would be to destroy their value as water 'rights-of-way' by raising the cost of transportation *via* water so high that no carrier could operate and the whole policy of Congress would be defeated."

Clearly, if the imposition upon carriers by water of charges merely to defray the cost of maintaining the waterways would make the cost of water transportation so high that the water carriers could not compete with the railways, it must be that the total cost of transportation by inland waterways is, and will be, greater than the cost of transportation by rail. In other words, water transportation is made possible only through large Government subsidies, paid, in turn, by the people at large in the form of taxes.

So much for cost! What about certain other requisites essential in an efficient transportation machine? To be specific, what about adequate and reliable service?

The following quotations from the 1929 report of the Inland Waterways Corporation are highly illuminative as to the ability of waterways to furnish adequate and dependable service.

Regarding the Upper Mississippi, the report states:

"The rainfall during the months of May and June was very deficient and by mid-June we were beginning to have trouble on account of low water. * * * Our greatest misfortune during the month of August occurred when the lower gates of the lock at the high dam between the cities of St. Paul and Minneapolis gave way, permitting the water above the dam to escape and leaving the steamer *Altair* and six barges hopelessly stranded at the Minneapolis terminal."

Similar misfortunes visited the Lower Mississippi. Witness the report:

"Within the last five months of 1929 we encountered every detrimental navigating element presented in the almost eleven earlier years of operation and in an aggravated degree, resulting in serious interruptions to schedules and revival of the service complaints which had been almost entirely eliminated. The inadequate channel was comparable to 1922 with the exception that in 1929 a far greater area was involved and the duration of the low-water period was almost twice as great. In 1929 the detention to line vessels in active service directly chargeable to fog and unavoidable channel obstructions was as follows: August, 1 913 hours; September, 3 356 hours; October, 3 323 hours; November, 743 hours; and December, 2 047 hours. Ice closed the St. Louis harbor almost an entire month earlier than usual."

As to the third operating division of the Inland Waterways Corporation, the report continues:

"On the Warrior River we experienced the greatest flood period in our career, considerably slowing up operations. Normal tows of seven barges had to be reduced to two and three barges, in order that the power might carry them up stream."

As regards the Ohio River, the following Associated Press dispatch was sent from Portsmouth, Ohio, on June 5, 1930:

"Boat traffic on the Ohio River between Pittsburgh and Cincinnati was abandoned temporarily to-day when the water dropped so low as to make navigation impossible. Lack of rainfall during the past several weeks was blamed. The *Chris Greene* went aground on a sand-bar above here last night, but succeeded in breaking loose and later attempted to return down stream to Cincinnati. A number of boats and packets tied up here rather than attempt passage through the sand-bars. The navigable stage here is 9 ft., but the level had dropped to 6.5 ft. to-day and was dropping rapidly. Above Portsmouth the water became so shallow that persons could wade across the river. The water dropped below the navigable stage at Dam 30, Greenup, Ky., and Dam 32, Vanceburg, Ky. Dam 31, near here, was of no use to river traffic as wickets had been raised several days ago to make repairs which will require three weeks. The *Senator Cordell,* bound from Cincinnati to Pittsburgh, tied up here because of the low water. Boats that stopped here were the *Ed Kenna,* pulling empty barges from Cincinnati to Huntington; the *Otto Market* and the *A. C. Ingersoll,* both bound to Cincinnati with tows of coal."

No further evidence is needed, beyond these official statements, to indicate the weakness inherent in waterways when it comes to providing adequate and dependable transportation. Whether from ice, from low water, or from high water, streams are affected by natural forces beyond the control of the engineer, be he ever so skilled. Reliable and adequate transportation service is essential to National life, in its present highly developed industrial stage. Such service the railways give to every section of the country. Such service the waterways do not, and cannot, give.

The increase which has been made in the past decade in the efficiency of railway operation—marked by cheaper service, faster service, more adequate service, and more dependable service—has been characterized as one of the outstanding achievements of the post-war era. Business leaders of the Nation have testified at length as to the value of this improved rail service, both to their own industries and to the Nation at large. This testimony is a matter of common knowledge, including in its list of benefits such factors as greatly reduced inventories; release of capital frozen in transit; reduction of factory operating expense, and stabilization of labor forces; reduction of demurrage charges; receipt of shipments in better condition; increased volume of sales; and the widening of potential markets. In fact, it has been reliably estimated that the benefits derived by general business from the improvements made in railway service since 1922 are equivalent in annual value to a reduction of at least 30% in present railway freight rates. Do the waterways offer any such service benefits to the Nation? They do not and, from their natural characteristics, they can not.

In determining "economic justification" the fact cannot be disregarded that rail service is faster, more dependable, and better in other respects than is the transportation service offered on inland waterways. Therefore, to put the water service on an economic parity with rail service, the total cost of transportation by water must be enough less than the cost by rail to offset this difference in the character and value of the service rendered. It has often been stated—and, indeed, used as a basis for calculating water rates—that rail service is at least 25% more valuable than water service and that, therefore,

the costs by water must be at least 20% less than by rail to offset this difference in the quality of service.

The available facts regarding the three principal inland waterways indicate that the total costs of transportation thereon are actually greater than the corresponding rail charges. They are thus in the position of furnishing poorer service at higher costs. Is this "economic justification" for further waterway development?

Finally, there is one further point upon which question may be raised as to the advisability of further waterway development. In the first place, these proposed improved waterways, if developed, either will or will not handle a material tonnage. If they will not, there is no legitimate reason for spending vast sums of public money on their improvement. If they will handle a material tonnage, this, of necessity, must be taken in large part from the railways. That the waterways, as their advocates claim, will develop any considerable volume of "new" tonnage is hardly susceptible of proof. This is not to deny that the development of new waterways, under some conditions, may result in increased traffic for certain railways adjacent thereto; but in the main it appears that such problematical increases in traffic on certain railways may result only through the relocation of industries, to take advantage of the possible lower water conveyance charge and to profit from the public transportation subsidy financed by the taxpayers. Under such circumstances, traffic gained by one group of railways will be traffic lost by another rail group. It is impossible to avoid the conclusion that if improved inland waterways attract the traffic which their advocates foresee, there will be a material reduction in the tonnage that would otherwise be offered to the railways for transportation.

What will this mean to rail costs and rail service? The inability of inland waterways to furnish adequate and dependable service has already been mentioned. This inability means that the railroads, as an economic necessity, must maintain surplus transportation capacity to be able to handle adequately, dependably, and efficiently the waterway traffic (or a considerable part of it) at those periods when the waterways, from causes beyond human control, cannot furnish adequate and dependable service. Such periods have occurred times without number in the past. That they will recur is beyond question.

Meanwhile, what of the railways in those years in which the waterways are able to furnish their service and in which traffic is attracted from the rail lines because of the lower direct conveyance charge to the shipper? What of the capital costs on the railway plant? How will their properties, necessary in time of waterway failure, be supported under a reduced tonnage? It is obvious that this remaining rail tonnage must either bear higher rail charges or else suffer impaired rail service. In either case the same practical effect is produced—the cost of rail transportation is increased.

This economic necessity for railroad transportation capacity to assume the carriage of water traffic in those inevitable periods when the waterways cannot function satisfactorily is in striking contrast to the second argument advanced by the waterways advocate—that further waterway development is required

to "relieve" the railways, either of current or of probable future traffic demands upon them. This argument, at least in part, was made by L. D. Cornish, M. Am. Soc. C. E.* Without attempting any detailed analysis of Mr. Cornish's article, some observations upon his conclusions, as viewed in the light of actual traffic changes on the railways in the forty years since 1890, may pertinently apply not only to Mr. Cornish's arguments, but to those of others who foresee a railway need for "relief" from an overwhelming traffic.

His conclusions† are predicated upon a forecast that in 1950 railway freight traffic will reach the staggering total of 870 000 000 000 ton-miles. Revenue ton-mileage in 1929, the highest figure ever reached, was approximately 450 000 000 000; so Mr. Cornish apparently foresees an increase of 420 000 000 000 ton-miles in the next 21 years. What in the record of the past justifies such a belief? What are the facts?

Statistics of railway revenue ton-mileage are available for the past forty years. From 1890 to 1929 (39 years) the increase in railway freight traffic was 374 000 000 000 ton-miles. Mr. Cornish's estimate for the next 21 years is, therefore, 12% in excess of the actual increase in the entire last 39 years. The increase in the 21 years from 1908 to 1929 was 232 000 000 000 ton-miles. Mr. Cornish's anticipated increase is, therefore, approximately 80% in excess of the increase that actually occurred within the last 21 years.

Even more significant is the marked change which has occurred since 1920 in the trend of railway freight traffic, a change characterized not by an increasing growth, but by a rapidly declining rate of growth. From 1890 to 1900 there was an annual average increase in railway ton-mileage amounting to 6 500 000 000. In subsequent periods the average annual increase was as follows: 1900-1910, 11 500 000 000; 1910-1920, almost 16 000 000 000; 1920-1929, only 4 000 000 000. Thus, during the last nine years, the average annual increase in railway ton-mileage was only about one-fourth as great as in 1910-1920, only about one-third as great as in 1900-1910, and only about two-thirds as great as in 1890-1900. In the light of this marked retardation since 1920 in the rate of growth of railway freight traffic, it seems extremely doubtful whether the increase within the next 21 years will exceed 125 000 000 000 ton-miles, which would make the total freight traffic available for the railways in 1950 only 575 000 000 000 ton-miles. Mr. Cornish estimates* that "railroad renewals and increased operating efficiency may provide" for the handling of about 650 000 000 000 ton-miles in 1950. It would appear, therefore, from the actual experience of the past decade, and Mr. Cornish's own estimate of the amount of traffic that can be handled by rail in 1950, that the railways not only will not need any "relief" by waterways in 1950, but will be able then, as they are now, to handle much more traffic than is available for them.

The railways have no fear of an overwhelming increase in traffic, in view of the experience of the last decade. On the contrary they fear, and with reason, that the retardation in the rate of increase in their freight traffic, caused at least in part by the loss of business to their subsidized competitors, may result in a situation wherein it may be difficult to support their properties.

* *Proceedings*, Am. Soc. C. E., March, 1930, Papers and Discussions, p. 561.
† *Loc. cit.*, p. 581.

Already retrenchments in capacity have been made in various parts of the railway plant. On the Class I lines the number of passenger-train cars has been reduced from 55 040 in 1924 to 53 097 in 1928; the number of locomotives, from 65 358 in 1924 to 59 391 in 1928; total locomotive tractive power, from 2 611 000 000 lb. in 1926 to 2 580 000 000 lb. in 1928; the number of freight-carrying cars, from 2 357 234 in 1925 to 2 297 677 in 1928; and the total carrying capacity of freight cars, from 105 953 000 tons in 1926 to 105 327 000 tons in 1928. Further, in the peak year of 1929, the average daily freight-car surplus at no time fell below 107 000. In the face of these figures it seems obvious that the railways' imperative need is not for "relief" from traffic, but for more traffic. Every ton taken from the railways increases the unit cost of rail transportation; every ton gained by them reduces this unit cost.

It thus appears from the available evidence that on the three great inland waterways the cost of transportation is greater than by rail; it appears that inland waterways are physically incapable of furnishing adequate and dependable transportation service; it appears that if the proposed development of inland waterways attracts to those waterways the volume of traffic which is predicted, there will be a material loss in the volume of tonnage which otherwise would move by rail, while at the same time the railways must maintain surplus transportation capacity against the inevitable waterway breakdowns; it appears that there is no necessity for waterway development to "relieve" the railways. In consequence, waterway development will mean that the traffic which these waterways may attract will be handled at a higher cost to the Nation in general than if it were carried by rail; that this service will be inferior to rail service; and that either railway rates will have to be increased on the remaining rail traffic or the quality of rail service reduced. In other words, the only results of further inland waterway development seem to be increased National transportation costs and impaired National transportation service.

Those who favor the development of waterways doubtless will answer that the facts presented herein do not prove the greater cost of water transportation as compared with rail transportation. If so, let them come forward with better data—data such as would be convincing to any trained student of business and economic problems. The most outstanding fact regarding this controversy concerning the comparative costs of water and rail transportation is the lack of credible and convincing evidence in support of the claims made as to the low and prospective costs of transportation on inland waterways. The public is being asked to invest hundreds of millions of dollars in waterway improvements, and in behalf of this expenditure claims as to the cheapening of transportation that will result are being made, which are not supported either by the facts derived from past experience or adequate data as to future probabilities.

The Nation has to-day a railway system which is being operated efficiently and is more than adequate to all the demands being made upon it. When it is proposed to develop another system of transportation by the expenditure of public money, which, it is claimed, will carry freight at less cost than the

existing means of transportation, the burden of proving that claim obviously rests upon those who make it. As this other means of transportation is to be provided by increasing the taxes of the public, it is obvious that its development cannot be economically justified except by showing that it will probably, and, indeed, almost certainly, cause a reduction in what the public must pay for freight transportation that will exceed the increases in taxes it must pay.

To say that the development of waterways will "cheapen" transportation because it will cause part of the Nation's freight to be carried for lower rates than it would be carried by rail simply evades the real issue. The cost of transportation, whether by airway, highway, waterway, or railway, is the total amount that has to be paid to provide such transportation, regardless of who pays all or any part of this cost. All the people directly or indirectly pay both taxes and freight charges. The question of the expediency of developing any inland waterway always, therefore, is a question of taxation as well as of freight charges. It always involves the questions, first, as to whether the increase in taxes will or will not exceed the savings in freight charges directly effected by the development of the waterway; second, as to whether there will not result a diversion of traffic from the railways that will make it necessary for them to charge higher rates on their remaining traffic than would otherwise be necessary; and, third, as to whether, if there actually is a net reduction in the total amount that must be paid for transportation by both rail and water, those who pay the increased taxes will be the beneficiaries of the reduction in freight rates.

Not until the advocates of inland waterway development squarely meet these questions with data such as would be convincing to private capitalists who were being solicited to invest their own money, will inland waterway development be discussed on its merits as a National economic and business problem. The American public is being urged to invest a large amount of its capital in inland waterways to increase the prosperity of the Nation, and not merely that of certain shippers, communities, or territories. It ought not to invest it, or be asked to invest it, on the basis of information any less complete and convincing as to probable costs and savings than that which would be demanded by business men who were being asked to invest their own capital.

This country has had more than a hundred years of experience with inland water transportation. Would a private business, after a hundred years of experience in its own field, spend a dollar for development on prophecy? Would it not search the records of the past for a guide to the future? Is it unreasonable to ask similar prudence in Government expenditures?

"Economic justification" is an adequate reason for future waterway development in the United States; but in accepting this reason it would appear, first, that the term should be interpreted as a reduction in total National transportation costs; and, second, that it be supported not by belief, opinion, theory, or prophecy, but by facts and facts alone.

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PAPERS AND DISCUSSIONS

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COMPLETION OF MOFFAT TUNNEL OF COLORADO

Discussion*

By Messrs. J. Vipond Davies and R. H. Keays

J. Vipond Davies,† M. Am. Soc. C. E. (by letter).‡—In order to give a complete story of the work, this valuable paper must be read in connection with the previous one by R. H. Keays, M. Am. Soc. C. E.§ Certain aspects which had influence and bearing on the final result, are considered in the following discussion.

In 1913, the Tunnel Commission of the City and County of Denver, by its Board of Engineers, Messrs. Brunton, Davies, and Finch, after considering carefully a geological report prepared for the Commission, reported on the project, recommending the present general location and profile of the tunnel. Further, under pre-war conditions, estimates of the probable cost of the tunnel construction were prepared. At that time the question of water supply was not looked upon as important, but to meet the requirements of the law it was contemplated that pipe conduits might be laid in the floor of the tunnel itself. It was also contemplated that the railroad tunnel would be equipped and operated by electric power. The geological report indicated no likelihood of encountering anything but sound hard rock, although it might be of varying texture and character.

In the preparation of the Moffat Tunnel Law, no reconsideration was given to the excessive advances in cost under post-war conditions, and only an arbitrary addition was made, as stated in the New Act, in the sum of $2 300 000 over that provided in the earlier law, which had been declared by the Courts of Colorado to be unconstitutional.

The Engineering Board and Staff consequently came into the problem of construction of a new project with hands already tied, on a definite location

* This discussion (of the paper by Clifford Allen Betts, M. Am. Soc. C. E., published in April, 1930, *Proceedings*), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Cons. Engr.; Pres., Jacobs & Davies, Inc., New York, N. Y.

‡ Received by the Secretary, May 27, 1930.

§ *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 63.

and elevation, with an estimated cost declared in the text of the law, and with a real problem for water supply to be included. The Commission had power to equip and electrify the tunnel and approaches for operation. In earlier years this electrification had been desired and planned by the Railroad Company, and, it was anticipated, would be installed in the completed tunnel.

Previous to the preparation of plans for the tunnel, as now constructed, the Board visited the site with the State Geologist, who indicated that the glacial valley erosion, on the west slope, might be deep, so that if a change were made in the location by a few hundred feet in a southerly direction, it would ensure the entire western half of the tunnel being in solid rock throughout. As is now known, that view was only partly correct, for although the rock existed continuously, its physical structure was most deficient. The doubt on the part of the Geologist, was that the glacial valley would be so deep as to intersect the tunnel section in the branch of Buck Creek nearest the Western Portal. There was no doubt expressed as to the quality of bed-rock where found.

With rock of more or less uniform character there appeared no reason why progress in driving should not have been equal from both ends; consequently, the grade of 0.8% originally contemplated was increased to 0.9%, with the apex of the eastern and western gradients as nearly as practicable at the middle point of the tunnel. This increased grade would be no detriment if the railroad were to be operated with electric power, particularly as the approach grades at both ends, are 2% up to the tunnel portals. The eastern gradient of 0.3% was maintained for proper drainage. The work of construction was in active progress with those established gradients, when the Railroad Company determined that it would not electrify for operation, but would use steam power, for which it considered the 0.8% gradient to be vital, and artificial ventilation to be an absolute essential.

In view of the fact that the rock generally from the Eastern Portal to the ridge of the Rocky Mountains was hard and practically dry, it was not thought that the driving of the eastern end over the apex for a longer distance would be of material consequence. From this change arose one of the serious difficulties which developed later and has been recorded.

Mr. Betts refers to the absence of diamond drill borings.* It is open to question whether such borings, if made, would have altered the situation. It appears certain that the pre-conception of the Geologist as to the depth of the glacial valleys on the western slope was probably under-estimated. An attempt was made, before finally determining the axis location, to dig pits from the surface in order to locate the rock floor below the west branch of Buck Creek, only to find glacial moraine of boulders and gravel, bearing heavy water. The rock in the tunnel on the western side of the Rockies, to outward observance is sound native material, but has been rotted, probably by water infiltration on imperfectly fused material. It seriously lacks supporting strength. Even in the small section of the Water Tunnel, it would not carry its own loads without close timber support.

* *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 687.

Further, the moraines are a vast reservoir of water from melted snow, as that is practically the only source of water to creeks or on the soil, from higher elevations. As Mr. Betts states,* while only a few thousand feet of the tunnel was ever anticipated to require artificial support, the final result proved that every bit of the western half needed both temporary and permanent support. Further, the heavy water encountered in the tunnel driven from the East Portal, was tapped practically under the summit of the Continental Divide. This clearly shows that the fissures in the rock through which the water flowed came from the great Ranch Creek Moraine on the west slope of the mountains.

As long as the railroad was expected to be electrified, there was little concern as to the ventilation of the 6¼ miles of tunnel through the mountain, particularly as the usually existing higher barometric pressure at the western end, added to the difference in actual elevation of the portals, made it certain that there would be adequate current of air to provide purity for passengers and workmen, as well as for cattle, which constitute an important element in the traffic eastward. When, however, it was finally determined that the railroad would be operated by steam power, the problem of ventilation became of vital importance, necessitating determination of those factors relative to which little or no authoritative information was available, particularly as to the frictional resistance to the movement of air in such a tunnel. The Engineering Profession is deeply indebted for these investigations to the U. S. Bureau of Mines, its engineers, and to Mr. Betts. The present method of operation and ventilation can be considered efficient only until the time is reached when traffic over this route increases so as to require electrification. At present, only a few trains per day are operated through the tunnel.

The general administration of the work was excellent throughout, both in respect of the Commission and its officers, as well as on the part of the Contractors and their staff. The high elevation and remoteness of the camps were sources of anxiety from the first. The original specifications, prepared in the hope of obtaining straight unit price bids, were greatly elaborated so as to obligate any contractor to install and maintain the most complete possible equipment, sanitary facilities, and general camp hygiene. Notwithstanding the fact that the provisions of the contract and administration of the work were completely changed, those provisions as outlined were carried out to the fullest limit, to the great advantage of the work.

R. H. KEAYS,† M. AM. SOC. C. E. (by letter).‡—This valuable contribution supplements the paper previously submitted by the writer,§ especially relating to events after 1925. The very valuable description of the ventilation experiments and the ventilation plant move one to attempt to supplement this feature and make a few general remarks on the ventilation of long railroad tunnels.

Besides the system described by the author,‖ which is dependent upon direct air pressure or suction, there is the well-known Sacchardo System of induced

* *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 687.

† Chf. Engr., Ulen & Co., Athens, Greece.

‡ Received by the Secretary, July 15, 1930.

§ "Construction Methods on the Moffat Tunnel," by R. H. Keays, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 63.

‖ *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 703.

ventilation. In this system the mass of air in the tunnel is pushed forward by the pressure caused by numerous jets of high-velocity air, introduced at one of the portals of the tunnel. These air jets are arranged in an annular ring and point into the tunnel parallel to the walls.

The Sacchardo System requires a ventilating fan to work against a much higher pressure than the direct displacement system. Because there is a rather definite and low limit to the pressure to be developed by a fan, and because the longer tunnels naturally require greater pressures, it is very difficult to apply the system to the longest tunnels.

An important objection to its application is that in the Sacchardo System the current of air is not reversible unless one has a plant at each end. The great advantage of the system, however, is that both portals of the tunnel are open at all times, so that the ventilation system will always function. With the direct-pressure system, there can be no ventilation when the closing of the curtain door is obstructed by a passing or standing train.

In any system of ventilation the question also arises as to whether to blow with or against the movement of the train. In the system as described by Mr. Betts, the ventilation is against the train. Ventilating with the movement of the train, however, is somewhat safer in that the smoke from the locomotive travels ahead rather than back among the passengers and crew. To blow with the movement of the train, however, requires much greater volume and velocity of air, unless the speed of the train is kept very slow.

In the case of the Moffat Tunnel, it is impracticable to rely on the Sacchardo System installed at either end. At the east end, the air pressures would have to be made high enough to overcome the large and almost continuous difference in the barometric pressure of the atmosphere between the two portals, while, if installed at the West Portal, the blowing would have to be in the same direction as the heavy traffic. A plant installed at the West Portal would also cost much more than one at the East Portal because the electric power, presumably used for operation, comes from a generating plant on Boulder Creek, which is some distance from the East Portal. A power line over the range would be difficult to maintain and a line through the tunnel itself would be very expensive.

The Sacchardo System, as the writer understands it, was first applied on one of the Apennine Tunnels, in Italy, and was so successful that later it was used on all other mechanically ventilated tunnels in Europe, with the single exception of the Simplon Tunnel. In each case the blowing was against the heavy traffic. In the case of the Simplon Tunnel, which is single track and more than 12 miles long, it can be imagined that some very high pressures would have been necessary with the Sacchardo System to induce sufficient draft. The only recourse, therefore, was the direct-pressure system.

The writer has had the pleasure of passing through most of the long tunnels of Europe and wishes to note here that there is apparently no longer any ventilation problem because all the railways have been electrified. Trains pass through at high speed, and one is hardly aware of the existence of the tunnel.

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FREQUENCY AND INTENSITY OF EXCESSIVE RAINFALLS AT BOSTON, MASSACHUSETTS

Discussion*

By W. W. Horner, M. Am. Soc. C. E.

W. W. Horner,† M. Am. Soc. C. E. (by letter).‡—Every designing engineer responsible for a combined or a storm sewer system would like to say:

"I am proportioning my sewers and drains to take the run-off corresponding to a 10-year frequency (or other frequency, as the case may be), because I have analyzed the conditions that will prevail when the less frequent and more intense precipitations occur.

"I find that rains corresponding to the 15-year curve will overcharge the sewers by a certain percentage, will cause a raising of the hydraulic grades which will back water into a certain percentage of the basements; and will create damage in the amount of so many dollars. I find that the still less frequent and more intense 20, 25 and 50-year types of rains will produce certain additional definite damages which I have evaluated.

"Summarizing all these damages, it appears that the average annual damage which will result from the occasional insufficiency of the sewers designed for the 10-year rain has a certain definite value, the capitalization of which is less than the cost of providing more capacious structures. Consequently, my choice of a 10-year frequency represents the exact economic solution of this particular problem."

It is highly improbable that any engineering designer has ever made any statement of this kind. Until now there has not been available a sufficiently good understanding of rainfall rates and frequencies to justify such study, nor has there been any indication that the character of land improvements within a drainage area can ever be so definitely known, and the use of the improvement so completely determined, that damages on the basis suggested could be arrived at with any reasonable approach to accuracy. Most engineers determine their choice on the knowledge of the temper of the citizenship and

* This discussion (of the paper by Charles W. Sherman, M. Am. Soc. C. E., published in April, 1930, *Proceedings*), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Chf. Engr., Sewers and Paving, St. Louis, Mo.

‡ Received by the Secretary, May 19, 1930.

build sewers of a certain size, because they have come to believe that the use of smaller sizes would result in such frequent overcharge as to produce a mass of small damages and of great aggravations; and that the people are willing to pay for this particular type of sewer rather than to undergo the inconvenience and expense which the smaller and cheaper structures will involve.

It seems most unfortunate that a better economic analysis has not been possible. Although the extremes of detail set out in the hypothetical case noted are never expected to be available, it is desirable to approach that condition. Mr. Sherman's studies and results certainly offer at least a little advance toward the end desired.

There have been studies of rainfall frequencies of short storms in the past. Many engineers have delved into the matter crudely for office purposes. One very good preliminary study of this kind has been presented in the progress reports of the Committee on Rainfall and Run-Off, of the Municipal Engineers of the City of New York, but nothing heretofore has approached Mr. Sherman's work in the logic of handling, or in the careful detail of development.

If any difference of opinion were possible, it might lie in the placing of his curves with reference to points between 30 and 70 hours' duration. It looks as if his frequency curves are somewhat higher than the data justify for these longer storms, and this possibility is further borne out by the inspection of Fig. 2.* If the storm of 1895 were considered as being possibly of the 100-year order, then his 50-year frequency envelope might well remain below the 7-in. line.

Considering the entire economic phase again, it is possible that the writer is over pessimistic as to the ultimate possibility of definite detailed analyses. While these are not likely to be used for newly developing areas, it is not impossible that matters involving relief sewers or flood channels for well developed districts of relatively unchanging use will occasionally be found to be susceptible to such treatment. When and if such occasions do arise, the author's data will be of inestimable value.

* *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 724.

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ANALYSIS OF CONTINUOUS FRAMES BY DISTRIBUTING FIXED-END MOMENTS

Discussion*

By Messrs. C. P. Vetter, L. E. Grinter, S. S. Gorman, A. A. Eremin, and Elmer F. Bruhn.

C. P. Vetter,† Assoc. M. Am. Soc. C. E. (by letter).‡—The author presents an ingenuous method for determining certain statically indeterminate quantities in continuous frames, one of the main advantages being the absence of any mathematics except the simplest arithmetic.

He illustrates the application of his theory by analyzing a certain hypothetical frame. In a way it is unfortunate that this particular type of structure has been chosen as an example, as the weaknesses of the principle are here so apparent. Had the author chosen, for example, an ordinary continuous girder, they would not have been evident, and it would have been more clearly demonstrated that he has added a new method to those already known for solving one of the most common problems in modern concrete engineering.

Considering Fig. 1,§ it is at once apparent that the vertical component of the reactions at *F* and *G* cannot be determined by the method outlined in the paper. In consequence, the direct stresses in the members, *F C* and *C G*, are unknown.

Those familiar with the theory of least work will recognize that this is because of the author's neglect of deformations due to direct stresses and shear. While the latter may often be safely ignored, neglect of the former is dangerous in many cases, the given example being one of them.

In his "Conclusion",‖ the author lists up to seventeen different points which have not been dealt with. The writer could add a great many more which have just as much, and more, to do with the subject under discussion, such as the

* This discussion (of the paper by Hardy Cross, M. Am. Soc. C. E., published in May, 1930, *Proceedings*) is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

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‡ Received by the Secretary, June 2, 1930.

§ *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 922.

‖ *Loc. cit.*, p. 927.

influence of variation in temperature, the yielding of foundations, and the shrinkage or swelling of members if built of concrete.

The writer does not agree with the author's closing remarks* concerning the value of a correct analysis of statically indeterminate structures. Stresses in most, although admittedly not as yet in all, such structures can be determined completely and correctly using those assumptions—only those assumptions—which are also used when analyzing statically determinate, or simple, structures. Short-cuts and approximations are permissible if one is in a hurry and if the error is negligible and approximately known, but the value of an approximation is determined only by the number of assumptions involved; every time a new assumption is made the value of the findings is reduced. If enough new assumptions are added, the answer may be valueless.

The theory of least work and the slope-deflection method have not added any assumptions to those generally used in all engineering, and, therefore, should be preferred wherever possible. Also, an engineer should "hitch his wagon to a star."

L. E. Grinter,† Assoc. M. Am. Soc. C. E. (by letter).‡—In presenting this paper, Professor Cross has made generally available a highly valuable method of structural analysis, which has previously been known to relatively few engineers. This method of balancing fixed-end moments was suggested to the writer a number of years ago by Professor Cross. Subsequently, he has found need to develop it to care for the balancing of shears or deflections, of thrusts or joint forces, and of angle changes or joint rotations.

Considered in its entirety, this is clearly a general method of successive corrections. In the form suggested by Professor Cross, one starts with fixed-end moments and the corrections are computed from the effect of allowing the joints to rotate in succession. The method is, therefore, either approximate or exact to any desired degree, depending only on the number of corrections one is willing to compute. It has the advantage over other methods of approximation in that the operator instinctively senses the amount of error involved in discontinuing the computation of the corrections at any given stage of the process, and naturally he continues until the desired degree of precision is reached. It has the further advantage: (1) Of speed, because one wastes no time in obtaining unnecessary precision; (2) of simplicity, because all computations involve only the use of arithmetic; and (3) of ease of remembering, since each step of the process has a definite physical significance.

These statements are borne out by the fact that the method in its simplest form—balancing moments on a continuous beam—is easily digested by junior engineering students and retained to the extent that, guided by a few suggestions, they readily make use of it in the senior year. Any one who has attempted to have senior students make use of the method of three moments, without considerable coaching, will appreciate the distinction.

In this connection, it should be noted that textbooks on strength of materials frequently do not give the three-moment theorem, except for equal spans

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 928.

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‡ Received by the Secretary, June 6, 1930.

and simplified loads. By balancing fixed-end moments, the student finds little added difficulty in handling unequal spans, complex loadings, or even a variation of moment of inertia in the several spans. Of course, variation of moment of inertia within a single span can be handled only by the advanced student.

The writer feels that much of the simplicity of this method is dependent upon a complete visualization of the corresponding physical action of the structure. In most methods of analyzing continuous frames, it is customary to allow rotation and translation to take place freely by inserting pins and sliding plates into the structure. Later in the process, continuity is reproduced by the application of forces at the cut sections. By the same reasoning it becomes clear that one may apply any number of restraining forces or moments to a structure as long as these restraints are removed later.

In the simplest case, that of balancing fixed-end moments where joint translation is not involved, the physical action is best described as the application and successive release of restraining moments at the joints. As these moments are released, each joint rotates until it takes its proper slope. When one joint is allowed to rotate, all others are held fixed. Every member meeting at the rotation joint, therefore, acts physically as a beam which is simply supported at the near end, fixed at the far end, and which, at the same time, is undergoing rotation at the near end. The resistance of such a beam to rotation is dependent upon its physical properties, or $\frac{I}{L}$ value. As Professor Cross points out, this value must be replaced by the moment necessary to produce a unit rotation at the near end, if the beam is not of uniform section. Again, the physical action of such a rotating joint readily clears up the fact that the unbalanced moment will distribute to all members meeting at the joint in direct proportion to their stiffness or resistance to rotation.

The carry-over moment may best be looked upon as another characteristic of the physical action of the structure. When a simple beam of uniform section is rotated through a given angle at one end, the far end rotates through an angle which is one-half as great. To prevent this rotation, or to rotate the far end backward to its original position, clearly requires the application at the far end of one-half the applied moment at the near end. Using the ordinary sign convention of beams, the carry-over factor is accordingly —50% of the balancing moment. For a beam of non-uniform section, or for one not fully restrained at the far end, the imaginary angle change to be corrected at the far end would not be exactly 50% of that at the near end, and the carry-over factor would be different from 50 per cent.

The writer would like to emphasize the fact that it is not necessary to consider the far end of the member as fixed. It might be allowed to rotate freely and the angular change later be taken into consideration, or its restraint may be determined and a moment carried over in proportion to this restraint. The writer makes occasional use of the conception of free rotation in a method of balancing angle changes, and frequent use of the idea of estimating the end restraint in an approximate method of balancing moments.

The viewpoint that he wishes to make clear, however, is that one may consider any joint as fixed, free, or partly restrained, at will, at any point in the process. It is only necessary that the recorded data be consistent with the physical action of the structure, that all artificial restraints be finally released, and that all discontinuities be removed.

The writer makes use of the term, "balancing," to indicate the process that Professor Cross prefers to call "distributing." This matter of terminology is of small moment, except that for clearness it has seemed desirable to reserve the term, "distributing," to explain the process of dividing a known moment in one member among the other members meeting at a joint. "Balancing" is used where an externally applied moment is divided among all members meeting at a joint. When so defined, "distributing" is a process to be applied when "running out" moments through a continuous frame of which only a single span is loaded. This method was mentioned in the preceding paragraph as an approximate method of balancing moments, where the carry-over moment is determined from the restraint at the far end of the member.

It is to be expected that among several investigators starting with the same conception as to balancing of fixed-end moments, each should finally develop a different system for accomplishing this purpose. The writer's convention of signs and method of recording the results are sufficiently different from those used by Professor Cross to justify their presentation.

The sign convention used by Professor Cross is that ordinarily given for horizontal beams in the study of strength of materials. This is ideally adapted to the discussion of moments in horizontal beams, but it has always seemed to the writer to be misplaced when applied to column moments. When thus used, some secondary sign convention must be adopted, such as the agreement that all columns will be considered as rotated through an angle of 90° in a clockwise direction. (Professor Cross suggests rotating the sheet instead.) When members meet at various angles, as in secondary stress computations, the convention of signs for strength of materials becomes still more confusing. In order to obviate this difficulty, the writer prefers to adopt the simple sign convention that positive moment at the end of any member tends to rotate the adjacent joint clockwise. This convention automatically cares for the signs of horizontal, vertical, and diagonal members.

In applying this system, it has been found to be a simple matter roughly to sketch the shape of the deflected structure with fixed ends. This process has the double advantage of fixing the signs beyond question and of enhancing the physical significance of the method. Sketches of typical deflected members are shown in Fig. 3. It is worth noting that a member which does not have a point of contraflexure has moments of opposite sign at the two ends. One point of contraflexure is indicated by moments of like sign and two points of contraflexure again by moments of opposite sign. The distinction between none and two points of contraflexure can not be drawn from the signs of the end moments, but is evident from the condition of loading.

When this convention is used, the signs of the balancing moments around any joint are unusually simple. If the resultant unbalanced moment is positive, all balancing moments are negative, and their sum is equal to the total

unbalanced moment. Thus, the total moments after balancing around the joint add up algebraically to zero. In comparison, one notices that the algebraic sum of all moments around a joint is not zero when written out according to the sign convention used by Professor Cross.

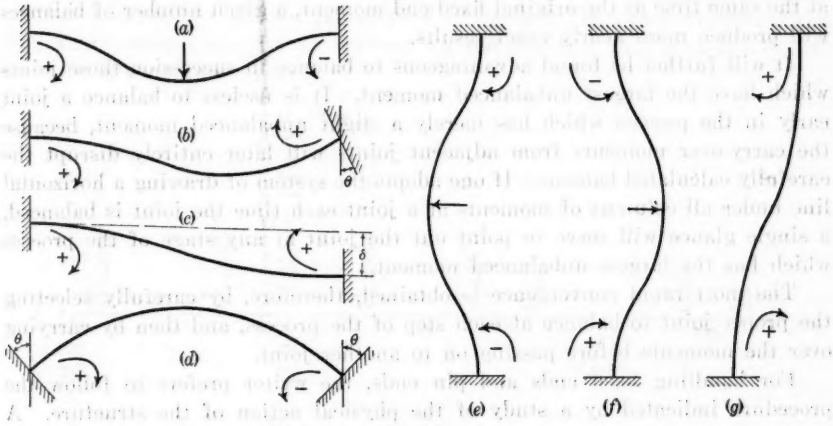

FIG. 3.

The carry-over moment, which is 50% of the balancing moment for a member of constant section, is of the same sign as the balancing moment. This fact is evident as the physical action of rotating one end of a member when the far end is restrained or fixed produces one point of contraflexure. (See Fig. 3(*b*).)

This convention of signs accordingly has the following advantages:

1.—Girders, columns, and diagonal members are handled in identically the same manner.

2.—The algebraic sum of all moments around a balanced joint is zero.

3.—All balancing moments at a joint caused by a single balance have like signs. Their sum is equal and opposite to the unbalanced joint moment.

4.—Carry-over moments are of the same sign as the balancing moment.

These advantages may seem somewhat unimportant, but they produce a simplification that reduces the possibility of error to an absolute minimum.

In finally comparing the sign convention used herein with that recommended by Professor Cross, one is naturally influenced in his decision of preference by his previous experience. One's personal system, even though more tedious, is always more satisfactory to him than another with which he is less familiar. It is of interest, however, that both sign conventions are actually used in either system. The signs set up by Professor Cross must be interpreted in terms of clockwise or counter-clockwise rotation in order to check $\Sigma M = 0$ at a joint. Similarly, the sign convention used by the writer must be transferred back into terms of fiber stresses if a moment diagram is to be drawn.

Another simplification which the writer has found advantageous is to complete the carrying over of moments from one joint before balancing the next joint. In this way one eliminates the possibility of neglecting to carry over certain moments. Furthermore, as the carry-over moment is balanced at the same time as the original fixed-end moment, a given number of balances will produce more nearly exact results.

It will further be found advantageous to balance in succession those joints which have the largest unbalanced moment. It is useless to balance a joint early in the process which has merely a slight unbalanced moment, because the carry-over moments from adjacent joints will later entirely disrupt the carefully calculated balance. If one adopts the system of drawing a horizontal line under all columns of moments at a joint each time the joint is balanced, a single glance will serve to point out the joint at any stage of the process which has the largest unbalanced moment.

The most rapid convergence is obtained, therefore, by carefully selecting the proper joint to balance at each step of the process, and then by carrying over the moments before passing on to another joint.

For handling fixed ends and pin ends, the writer prefers to follow the procedure indicated by a study of the physical action of the structure. A fixed end cannot rotate and, therefore, cannot be balanced. The original fixed-end moment plus the carry-over moments, accordingly, add up algebraically to give the final moment. A pin is entirely free to rotate and has zero moment. The fixed-end moments and all carry-over moments therefore must be released by allowing free rotation at the pin. In recording the data, this free rotation may be looked upon as produced by the application of equal and opposite moments at the pin.

In order to illustrate the system described herein, the writer has prepared Fig. 4, which is the identical problem worked out by Professor Cross. The balancing is started at Joint B where an unbalanced positive moment of 100 exists. This joint was selected because it offered as large an unbalanced moment as any other. Joint D might equally well have been chosen as a starting point. Since BC is twice as stiff as BA, the first correction moments are -67 in BC at B, and -33 in BA at B. After the balancing moments are recorded, horizontal lines are drawn under these figures to indicate that all moments above these lines are balanced; that is, they add up to zero around the joint. Now, one-half the -67 at B in BC is carried over to C with the same sign, or -33. Similarly, one-half the -33 at B in BA is carried down to A with the same sign, or -16. The next step is to balance Joint D where a large unbalanced moment of -100 exists. Balancing moments of $+63$ in DC and $+37$ in DE are recorded, corresponding to five-eighths and three-eighths, respectively, of the unbalanced moment. Horizontal balance lines are again drawn, and these moments are carried over with one-half their value and like sign to the far ends, D and E.

The process is thus continued around the frame, the joints being balanced in the following rotation: B, D, F, E, C, B, F, and C. They might have been balanced with equal satisfaction in some other sequence, but it is not likely that a more rapid convergence will be found. It is well to note here that

in returning to balance a joint for a second or third time, one balances only those figures below the last set of horizontal lines.

In Fig. 4 the first recorded figure in each group is the fixed-end moment. The last recorded figure in each group is the final moment in the adjacent member obtained by adding the column of figures. In order to clarify the diagram, column moments are written on the columns, and girder moments are written above the girders.

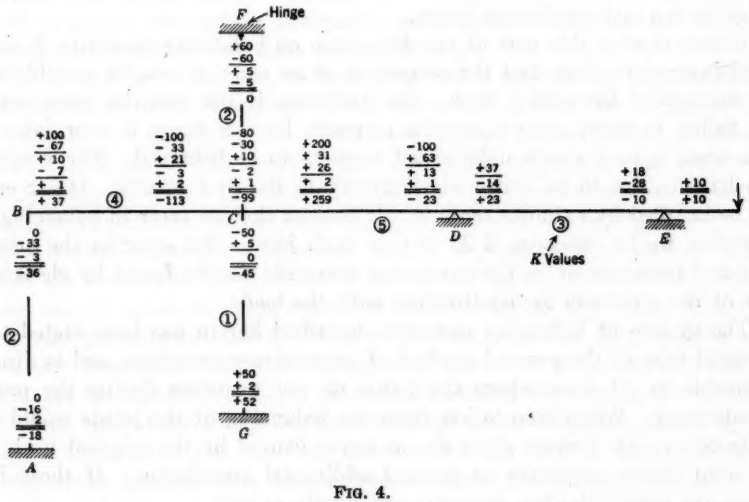

FIG. 4.

In justification of this method, it is necessary to show that the results obtained in this case by the balancing of only eight joints are within 4% of those obtained by the system suggested by Professor Cross in thirty balances. Also, after balancing each joint twice by Professor Cross' system, or a total of ten balances, one joint, *D*, still is seen to be 38% in error. A fairer comparison might be obtained by noting that Professor Cross' system gives satisfactory accuracy after balancing each joint three times, or a total of fifteen joints to be balanced. Thus, in this problem, nearly one-half the mechanical labor is saved by the system here suggested. At least a 25% reduction in labor will usually be found in the solution of all continuous frame problems.

The writer wishes to stress particularly the advantage that one sets his standard of accuracy before starting to balance joints. For instance, if whole numbers are used as in Fig. 4, an error arises from the fact that 33 is recorded instead of 33.33, when 100 is to be divided into three equal parts. There is also an error where an uneven number is multiplied by 0.5 to obtain a carry-over moment. When dealing with whole numbers, a maximum error of 2 is a possibility. If the final moments are at least equal to 50, the maximum error is thus limited to about 4 per cent. Where smaller fixed-end moments are found, it may be desirable to use three significant figures. After one becomes familiar with the method, he naturally watches the figures closely and makes the errors balance. An excellent example of this may be seen in

Fig. 4. Note that at Joint A the writer first carried down less than one-half the balancing moment and the second time he carried down more than one-half. No attempt was made to balance errors at other joints.

If these suggestions are followed, one merely continues to balance joints until the carry-over moments are smaller than unity in the last significant place used. When so reduced, the columns may be added, knowing that the results are within the limits of accuracy represented by a variation of 2 in the last significant figure. Slightly more care will reduce the variation to unity in the last significant figure.

Before closing this part of the discussion on balancing moments, it should be of interest to show that the correction of an error is usually possible without destroying the entire work. For instance, if the operator finds that he has failed to carry over a certain moment, he can carry it over later and then treat it as a single unbalanced moment to be balanced. The results of this balance are to be added algebraically to the first results. Other errors can be handled in a similar manner. It is clear that an error in balancing can be picked up by checking $\Sigma M = 0$ at each joint. An error in the original fixed-end moments or in the carry-over moments can be found by checking a part of the structure for equilibrium with the loads.

The system of balancing moments described herein has been stated to be a special case of the general method of successive corrections, and is directly applicable to all cases where the joints do not translate during the process of balancing. When translation from the balancing of the joints would normally occur, this process gives the moments caused by the original loads and the joint forces necessary to prevent additional translation. If these joint forces are negligible, the moments are nearly exact.

An interesting illustration of such a simplified case is that of secondary stress analysis. Knowing that the deflections of the pin-ended truss and of the riveted truss are essentially alike, one merely calculates the deflections of all panel points caused by the direct stresses; from these deflections one computes the fixed-end moments, and by balancing these moments arrives at the final secondary moments. It should be noted that the fixed-end moments for this case are determined from joint translations, but no additional translations of importance occur during the balancing process. In this same connection, it is interesting to note that the moments caused by a known settlement, horizontal movement, or rotation of a support of a continuous frame may be evaluated in exactly the same way.

A general system of successive corrections for solving all problems involving joint translation can now be set up, based upon the knowledge that the results of the first balancing of moments gives the moments caused by the real loads and the joint restraints. It is only necessary to determine these artificial joint restraints by statics from the calculated moments and the applied loads. All joint restraints are then balanced by equal and opposite forces, and the fixed-end moments caused by this new force system are determined and balanced. This process gives rise to a second set of joint restraints which are removed in the same way. It will be found that the relative importance of each successive set of joint restraints decreases until a set is reached

which may be called negligible. Usually, two or three successive corrections will be sufficient for engineering accuracy. The final moments are obtained by adding algebraically the moment sets caused by the several force systems.

It is highly important to study the physical action of the structure, which corresponds to this method of analysis. The first deflected position of the structure does not correspond to the proper deflected position. As each successive force system is allowed to deflect the structure with fixed joints, that structure approaches nearer and nearer its true deflected position. After several successive corrections, it almost exactly coincides with its true deflected position, and the joint restraints are negligible.

The writer has used this method with entire success in the analysis of all continuous frames, including the determination of wind stresses in building frames, open web trusses, continuous arches on elastic piers, and more unusual academic problems. The method has a valuable application to continuous arch systems, in which the idea of balancing joint forces applies directly to the elimination of the troublesome unbalanced thrust at the pier tops.

The method simplifies itself when applied to a single-story bent or culvert in that the restraining force system is merely a single horizontal force applied to the top of the bent. Successive restraining forces are found to form a convergent series, geometric in form, which has a definite mathematical sum. From this knowledge, or from the physical action of the structure, one can readily see that there is a direct proportion between the true moments caused by a given lateral force and the first balanced set of moments caused by this force. This ratio is the same as that between the applied lateral force and the difference between this lateral force and the first joint restraint. Professor Cross arrives at this conclusion in a somewhat different manner.

When discussing multi-storied bents, Professor Cross suggests* producing a unit shear successively in each story and determining the effect on adjacent stories in terms of both moment and shear. Then, by combining these effects, the proper shears may be obtained in all stories, and by combining the moment sets in the same proportion, the true final moments are obtained. When the writer first put this method into practical application, it was thought that it would be necessary to make use of simultaneous equations for combining shears. Later, however, he showed that the same result could be obtained more easily and just as accurately by the summation of a simple series of terms for each story. The development of such a series is again best visualized from the physical action of the structure. A shearing distortion in one story gives rise to shearing distortions in other stories. From the ratio of these shears a set of carry-over factors for shear is determined. The proper shears are then applied in all stories and the carry-over shears recorded. These carry-over shears are balanced, and the process is continued until they become negligible, as in balancing moments. From the physical viewpoint, one is successively distorting one story after another until they all take their final deflected position. Finally, by summing each series of terms, the shears necessary to be applied to all stories to produce the proper final shears are known, and the moment sets are combined in the same proportion.

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 927.

Although this method is convenient and entirely satisfactory, the writer is now finding the general method of successive corrections, just described, to be more useful in most cases. Its main advantage is that it lends itself directly to an approximate solution as one may neglect joint restraints to give any desired degree of approximation, thereby allowing a saving of labor. It also offers an opportunity to the experienced analyst to take advantage of his knowledge of deflections to reduce materially the amount of work required. For instance, the writer seldom finds it necessary to balance more than one set of joint restraints. The method of balancing shears suggested by Professor Cross is more valuable for cases in which the effects of several force systems are to be evaluated. When the carry-over factors for balancing shears are once determined, the moments caused by several force systems may be obtained with relatively little additional labor.

This paper constitutes an outstanding contribution to the field of structural analysis. Without question the method of balancing fixed-end moments and the other closely related methods of analysis are destined largely to supplant the old classic methods dependent upon the solution of simultaneous equations. The writer is not alone in his belief that Professor Cross may be regarded as the leader in a new school of thought in the field of structural analysis.

S. S. GORMAN,* ASSOC. M. AM. SOC. C. E. (by letter).†—This paper is a clear exposition of a simple and practical solution for many complicated problems. The method presented is very simply and easily applied and is just what its author claims it to be—a method of successive approximations, giving reasonable precision speedily. The writer has applied it to a number of analyses and has found it to be very useful, but he realizes that it must accompany sound judgment in its application.

This discussion will be confined to the method of distribution at a joint similar to *B*, in Fig. 5. While the author's description has been very general, possibly a further elaboration on such a joint will save others from the confusion experienced by the writer.

Assume an L-frame as shown (Fig. 5), in which, $p = 20\,000$ lb.; $w = 1\,000$ lb. per ft.; $L = 25$ ft.; and, $h = 20$ ft. The relative values of $\frac{I}{L}$ for the members are indicated in circles. End *A* is hinged and End *C* is fixed.

After first writing the fixed-end moments for each member, consider Joint *B*. At first, one might believe that the unbalanced moment is — 102.08, and proceed to distribute this moment, ultimately obtaining an answer which would be incorrect. The unbalanced moment is actually — 2.08, and is distributed between the members, *A B* and *B C*, as on the left side of the joint, and imaginary members of zero stiffness on the right side of the joint, giving the moments as shown in Fig. 5. This procedure is continued, as outlined

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† Received by the Secretary, June 14, 1930.

by the author, always balancing the left-side moments against zero for the right-side moments, until the final moments are obtained.

A similar condition is present in the author's illustration at the joint, *C*, Fig. 1,* if the members, *C D* and *C F*, are without load and are considered of zero stiffness. In this case the left-side moments (— 150) would be distributed so as to balance with zero on the right side of *C* (*B C* would get + 120 and *C G* would get + 30).

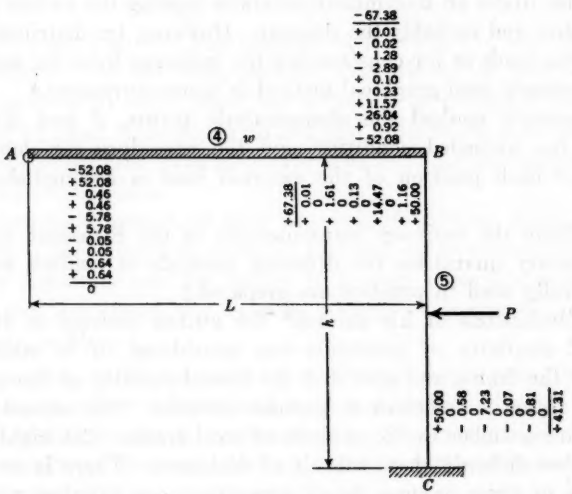

FIG. 5.

As the author has pointed out† the selection of signs is most important, and as the moments are balanced at a joint algebraically, it is absolutely essential to a proper solution of any problem that the signs are properly selected; this is fundamental. Simplification of analysis is highly commendable, but it must not be spoiled by careless application.

The writer wishes to express his gratitude to the author for his splendid presentation, which eliminates formerly cumbersome methods of analysis of statically indeterminate frames.

A. A. EREMIN,‡ ASSOC. M. AM. SOC. C. E. (by letter).§—A number of the analytical and semi-graphical methods of moment distribution in continuous frames with rigid connections have been developed in the last few decades. These methods are generally based on what is commonly known as Mohr's principle,‖ namely "the angular deformations of the elastic line at supports are equal to the reactions of the beam loaded by its moment area divided by *E I*".

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 922.
† *Loc. cit.*, p. 921.
‡ Asst. Designing Engr., Bridge Dept., State Highway Comm., Sacramento, Calif.
§ Received by the Secretary, June 16, 1930.
‖ "Neuere Methoden zur Statik der Rahmentragwerke," by A. Strassner, Vol. 1, 1921, p. 11.

This principle applies to beams resting on unyielding supports. However, when a load is applied to continuous frames the joints move from their original positions, due partly to direct stresses in the members, or lateral forces at the frame and partly to changes in the lengths of the members as the result of variations in temperature, or a combination of all these factors.

The author restricts his analysis to frames in which the joints do not move.* If it is required to determine the moments at the joints of a continuous frame under an unsymmetrical static loading the author's method will be the simplest and probably the shortest. However, for distributing moments due to moving loads or for constructing the influence lines for moments at the joints, Strassner's semi-graphical method is more convenient.†

In Strassner's method the characteristic points, J and K,† are determined for the unloaded condition and the procedure for determining the moments for each position of the external load is accomplished by simple graphics.

To facilitate the ordinary manipulations in the Strassner method, tables of the necessary quantities for different methods of loading and shapes of beams generally used in practice are prepared.‡

In the limitations of his method* the author omitted to state that the modulus of elasticity of materials was considered to be constant for all members of the frame and also that the lateral rigidity of the members of a continuous frame is sufficient to prevent bucking. The second statement is important to remember in the analysis of steel frames with rigid connections.

The author defined§ three schools of designers. There is no justification for the first of these because many experiments on existing reinforced concrete and steel frames with rigid connections have proved that there is a distribution of stresses from one member to adjacent members.

On the other hand, imperfections in workmanship and incomplete knowledge as to the actual elastic properties of structural materials prevent the development of an exact method of determining a real distribution of the stresses in frames.

In order to maintain a reasonably uniform margin of safety for all members in the frame it should be designed for stresses as determined by one of the approximate methods based on the elastic properties of materials. In the formulas for the final stresses in the members of the frame, constants may be introduced to eliminate the errors due to neglecting some minor factors in the development of the method. These constants may be determined by experimental study of the various frames with rigid connections.

ELMER F. BRUHN,‖ ASSOC. M. AM. SOC. C. E. (by letter).¶—The writer has long been familiar with this method of distributing fixed-end moments and has used it extensively. He strongly recommends it because it has developed

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, pp. 922 and 927.

† "Neuere Methoden zur Statik der Rahmentragwerke," by A. Strassner, Vol. 1, 1921.

‡ "Neuere Methoden zur Statik der Rahmentragwerke," by A. Strassner, Vol. 1, 1921; see, also, *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 167.

§ *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 928.

‖ Engr. of Tests, BIJ Aircraft Corporation, Baltimore, Md.

¶ Received by the Secretary, July 12, 1930.

for him a much better understanding of the action and relative rigidity of indeterminate structures than is given by the common methods of analysis. These latter, in general, involve the writing of sufficient elastic and static equations, after which the work is entirely an algebraic mechanical process. The structure is entirely forgotten and although the various terms in the equations may represent certain rigidity characteristics or distortional influences, it is no doubt true that this significance often escapes the designer's attention.

Using the author's method one assumes each member of the structure in a definite restrained state and then, realizing that continuity must be preserved and statics satisfied at each joint, the structure is gradually but definitely relieved, joint by joint, from its restrained and unbalanced state and is guided into a state of equilibrium. The structure is at all times the center of thought, a fact that merits emphasis.

In the aeronautic structural field the method of least work seems to be deeply rooted. For analyzing complex continuous structures it gives long and unwieldy equations and has proved to be very unsatisfactory. Of many uses, the writer has found the author's method of particular value in the following three problems:

(1) For making rapid comparative studies of continuous closed frames, a type of structure which is being used more and more in airplane bodies.

(2) For assisting in digesting test results. Government regulations generally require tests to destruction of all major airplane units and new types of structures. It is difficult to compute the ultimate strength of a continuous structure accurately. This is due, in part, to the fact that the ordinary elastic formulas apply only for stresses in the materials below the elastic limit. Tests of continuous structures generally show a higher ultimate strength than that computed. This result can be explained as follows: When a continuous structure is loaded some portions of it reach the elastic limit stress first and lose a considerable fraction of their original stiffness or rigidity. The result is that these overstressed portions are relieved of part of their load by the other portions of the structure which still retain their original stiffness. Since it is impossible or impractical to proportion structures for equal stress intensity, this drifting tendency of the internal loads to the portions of higher stiffness has considerable influence upon the ultimate strength, amounting in many cases from 25 to 40% more than the strength computed by the ordinary methods. The author's method has proved valuable in estimating this internal re-adjustment.

(3) For solving extremely complex problems as, for example, the determination of secondary moments due to deflection in a continuous beam with variable moment of inertia and carrying both transverse and axial loads. In airplane structures these moments prove of real importance. To attack such a problem with mathematics is practically impossible.

The question of correcting for joint displacements is less important in airplane work, as, in general, the loads and structures are symmetrical. On

the other hand, space frameworks are quite common. This further complicates matters because of torsional forces, but there seems to be no reason why such forces could not be distributed in a similar manner if the torsional characteristics of the members can be determined.

It seems probable that in the aeronautic structural field will be found many problems that will lend themselves to the convenience and advantages of Professor Cross' method.

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PAPERS AND DISCUSSIONS

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THE TRAINING WALL ACROSS THE LIAO BAR IN MANCHURIA

Discussion*

By R. D. Goodrich, M. Am. Soc. C. E.

R. D. Goodrich,† M. Am. Soc. C. E. (by letter).‡—A record of construction experience such as that presented in this paper, is always of value to the profession.

As shown in Fig. 1,§ there are four principal ports in the Gulfs of Liaotung and Pechihli. Dairen and Chinwangtao are located in natural indentations of the coast and are free from ice during the winter months. On the other hand, Niuchuang and Tientsin have harbors in rivers that drain large areas and they are usually closed for several months in the winter. In many respects these two drainage systems are similar.

As the author states,‖ the average fresh-water discharge of these rivers is not sufficient in itself to maintain proper channels for navigation by ocean steamers. The situation is made worse by the character of the floods, which carry enormous quantities of silt. Samples containing 10% by weight of silt are not uncommon. Commerce at Tientsin has been greatly handicapped in recent years by the silting of the channel from Tientsin to Taku, practically closing the Hai River to ocean steamers for months.

These conditions would naturally tend to the development of Dairen and Chinwangtao as the logical ports of North China. However, there are many other considerations having great weight in port development. Distribution to the interior from these two ports must of necessity be almost entirely by rail. On the other hand, Tientsin and Niuchuang have the decided advantage of inland waterways as well as railways for the collection and distribution of

* This discussion (of the paper by P. N. Fawcett, M. Am. Soc. C. E., published in May, 1930, *Proceedings*), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Prof., Civ. Eng., Coll. of Eng., Univ. of Wyoming; Senior Hydr. Engr., U. S. Engr. Office, Sacramento, Calif.

‡ Received by the Secretary, July 8, 1930.

§ *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 930.

‖ *Loc. cit.*, p. 932.

ocean freight. From Tientsin there are three waterways and three rail routes, north, south, and west, not all of which are shown on the outline map (Fig. 1). The greater cost of transportation by rail compared with that by Chinese river boats for many commodities, and the greater length of haul and consequent cost of freight from the winter ports to the points of distribution to the interior markets, largely offset the difficulty of maintaining river ports and harbors. There is, therefore, keen competition between these ocean terminals, influenced also by the large investments already made at each point. Not only must the present works be maintained, but improvements must be made in order to protect the business and investments in all these ports.

The most serious and difficult problem involved in the maintenance of these rivers and harbors is that caused by the formation of a bar outside the mouth of any of the larger streams of China. At the mouth of the Yellow River conditions are such as practically to prohibit the construction of a harbor for the interchange of freight between ocean steamers and river boats. The rate at which the coast line is being extended at this point is much greater than at the mouth of either the Hai River or the Liao River.

In 1925, the construction of training walls was also undertaken at the mouth of the Hai River. It is understood that this project contemplated the use of walls carried above high tide on both sides of the channel. The writer has always felt that a single curved jetty might have provided the desired benefits at much less cost to the interests involved. As noted by the author,* all such local improvements must be financed largely by the collection of harbor dues, over and above the regular customs rates. It is evident, therefore, that development depends almost entirely upon the initiative and enterprise of the commercial interests, both Chinese and foreign, at each individual port. In earlier days dredging and other modern harbor works were instigated by the foreign shipping interests, but due to western influence Chinese capital is also largely involved at the present time, the amount varying of course with the locality.

In view of all the difficult conditions connected with the maintenance and development of facilities for commerce in the Orient, the paper presents a very valuable record of experience and progress, and a report of the results obtained after the final completion of the Liao Harbor Jetty will be of great interest to many engineers.

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 932.

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EVAPORATION AS A FUNCTION OF INSOLATION

Discussion*

BY MESSRS. H. H. KIMBALL AND C. G. ABBOT.

H. H. KIMBALL,† ESQ. (by letter).‡—That a close relation exists between insolation and evaporation is well known. Thus, in a paper entitled "Applications of Heat Radiation Measurements to the Problems of the Evaporation from Lakes and the Heat Convection at Their Surfaces",§ A. Angström assumes a balance between (1) the insolation absorbed at a water surface; (2) the heat loss from the surface through radiation, convection, and conduction; (3) the heat energy expended in raising the temperature of the water; and (4) in the process of evaporation at its surface. It is evident that if Components (1), (2), and (3), of this balanced system, can be measured or computed, Component (4) will be known.

However, when, at a later date, in a paper entitled "On Radiation and Climate", ‖ Angström undertook to account for the expenditure of the radiant energy received from the sun, he computed, from evaporation measurements and the latent heat of evaporation, the energy expended in evaporating moisture from various surfaces, including water surfaces.

In his paper, the author goes back to Angström's earlier method, and shows how the values of various terms may be computed, and that if the quantity of insolation received at the water surface is known, the rate of evaporation may be computed; or, *vice versa,* if the rate of evaporation is known, the insolation may be computed.

In Table 11,¶ comparisons are given between observed and computed amounts of evaporation, the computed amounts being based on "insolation observed at pyrheliometer stations."

* This discussion (of the paper by Burt Richardson, Esq. published in May, 1930, *Proceedings*) is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† In Chg., Solar Radiation Investigations, U. S. Weather Bureau, Washington, D. C.

‡ Received by the Secretary, June 17, 1930.

§ *Geografiska Annaler*, 1920, H. 3.

‖ *Loc. cit.*, 1925, H. 1, och. 2.

¶ *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 957.

Pyrheliometric readings are available for only a few of the stations named in the table. Therefore, the values given for insolation cannot be considered of a high order of accuracy.

The author states* that,

"When the value of insolation is not known from pyrheliometer records it is possible to compute the insolation, I_{ex}, incident upon the exterior of the earth's atmosphere at any latitude, and, later, to convert this into actual Insolation I by means of two coefficients ρ' and F, the mean transmission coefficient of the atmosphere at a particular place and the clearness factor, respectively."

This is true if the atmospheric conditions are known, as has been shown by the writer in a recent paper entitled "Measurements of Solar Radiation Intensity and Determinations of Its Depletion by the Atmosphere"†; but it will by no means do to assume an annual mean value for ρ', which is dependent especially upon the water vapor and dust content of the atmosphere. Both these factors are subject to marked seasonal or annual variations, especially near sea level. (See, for example, values of ρ', or a_{0-1}, for Washington, D. C., determined for zenithal sun, $\psi = 0^0$.)

C. G. Abbot,§ Esq. (by letter).‖—While no doubt valuable in engineering estimates the use of the coefficient of atmospheric transmission as a single valued quantity is not, of course, exact, for it varies with the wave length, with the station, with the month, and from day to day in a very wide range. Therefore, the results reached by treating it as a single valued constant can be no more than rough average approximations.

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 957.
† *Monthly Weather Review*, Vol. 58, February, 1930, pp. 43–52.
§ Secy., Smithsonian Inst., Washington, D. C.
‖ Received by the Secretary, June 16, 1930.

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CITY PLANNING AS RELATED TO THE SMALLER CITIES

Discussion*

By Stephen Child, M. Am. Soc. C. E.

Stephen Child,† M. Am. Soc. C. E. (by letter).‡—Mr. Wood reiterates the recommendations of the National Conference on City Planning that all American municipalities should establish city planning commissions.§ He states, furthermore, that provision should be made for the establishment of a city planning office and the employment of a city planning consultant.‖ Unfortunately, smaller cities frequently become imbued with the idea that it is quite unnecessary to have a consultant, perhaps because a local architect and engineer are members of the commission. This is false economy.

City planning is a never ending job. Cities cease to need planning only when they cease to grow. There are many methods of procedure and no "hard-and-fast" rules established. A great metropolitan community must adopt the method best adapted to its own peculiar conditions. However, a large group of cities, ranging in population from 10 000 to 100 000, have much in common as regards the more general features of city planning procedure.

For example, they first need a properly organized city planning commission; then a comprehensive civic survey, to be followed by the preparation of plans for streets, zoning, parks, and other features of growth. Furthermore, and perhaps most important, all these plans and projects must be followed up patiently and continuously, year after year, until finished.

For most of the smaller communities, a separate city planning department would be out of the question. The necessary "follow-up" work is no doubt done best "at home" and not from some expert's office. Recognizing this essen-

* Discussion of the paper by E. A. Wood, M. Am. Soc. C. E., continued from August, 1930, *Proceedings*.

† Landscape Archt.; Consultant in City Planning, San Francisco, Calif.

‡ Received by the Secretary, May 24, 1930.

§ *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 962.

‖ *Loc. cit.*, p. 963.

tial, more or less complete transfer of city planning functions from the expert's offices to some one of the city's departments, it may be best to begin in another way and to proceed with the plan by fundamentally different methods.

As an example of how many mistakes are made in city planning, consider a hypothetical small town. Suppose that under the stimulus of a campaign carried on by an effective citizens' advisory committee and sponsored perhaps by the Chamber of Commerce—a city planning commission has been organized, a fund appropriated for its work, and a competent consultant employed. For a year or more everything seems to be progressing quite well. A preliminary survey is prepared; an illustrated pamphlet report, containing recommendations for a major street plan and for other important features controlling the city's growth, is published.

Then comes the inevitable slump. Opponents of the zoning plan appear; the Committee declines to appropriate money for a much needed traffic artery; a proposed bond issue to provide for a playground system, park, or civic center is voted down; and, finally, the funds for the city planning commission itself are either not appropriated at all or are radically cut.

By this time the expert has gone on to new fields of endeavor. Possibly some of the valuable data and plans he has prepared are buried in the vaults of the city clerk's office and forgotten. Much material remains in the possession of the expert, whose office is perhaps a thousand miles away. The city then drifts along in practically the same shiftless, inefficient manner that it has for years. Small and large subdivisions continue to be added and haphazard growth goes on, with little guiding or planning.

Then follows another "spasm" of civic righteousness—the result, perhaps, of some grade-crossing accident or other occurrence that arouses public interest. There is a scurrying around in search of mislaid data and the original expert and city engineer are not now near at hand to help in the re-educating work.

The foregoing remarks will serve as an example of conditions that are working against good city planning, particularly in the smaller cities. The ideal steps proposed for the same situation might be analyzed somewhat as follows:

Consider a city of about 30 000 population. Often with the support of the Chamber of Commerce, a Citizens' Advisory Committee is appointed and this Committee employs a consultant who helps in the preparation and enactment of city planning ordinances. The ordinances will include the general features proposed by the Advisory Committee on City Planning and Zoning of the United States Department of Commerce as recommended by Mr. Wood.* This step in itself would take a little time and, in the meantime, the consultant will give some talks on city planning. Ultimately, a city planning commission is authorized and organized.†

Under the terms of the ordinance the city engineer is made an *ex officio* member of the city planning commission. Contrary to the example previously cited, the City Planning Commission does not turn the job over to the consultant. On the contrary, it selects an efficient secretary—perhaps a stenog-

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 962.
† *Loc. cit.*, April, 1928, Papers and Discussions, p. 1012.

rapher in the office of the city engineer. She is to begin immediately to have charge of the data collected. The city engineer will probably need to employ a city planning assistant. Larger communities might need more than one, and there is the possibility that one of these might become secretary to the Commission. The Commission, again under the advice of its consultant, proceeds to make a comprehensive civic survey in such a way that every member will be familiar with it. They will be able to amend, extend, and perfect it and they will have all its details available for use for future years.

The next step undertaken by the Commission is to prepare a comprehensive city planning program. They will endeavor continuously to keep the program of planning up to date. As a whole, the program will need to be flexible enough to permit adjustments to new conditions. One important requisite is the contract drawn between the commission and the city planning expert. This should include such matters as fees, the time to be spent by the consultant, the extent to which he shall co-operate with the city engineer, and his duties in respect to directing the civic survey and any field investigations which may be necessary.

Suppose the Council should reject a street-widening proposition, thus upsetting the major street planning; or, suppose the people should vote against a bond issue for parks or a civic center. Every one knows that these are only temporary setbacks. Sooner or later, if the Commission's plans have followed comprehensive civic surveys and are really well considered, these projects will have to be carried out, perhaps in some modified form. If all the data are at hand and the consultant under the terms of his contract is also at hand, he can insure that these modifications are reasonable and in accord with the general program that the Commission has prepared. It is believed that this second proposed method of procedure will go far toward solving the problem of overcoming city-planning inertia which is often a serious matter.

MEMOIRS OF DECEASED MEMBERS

NOTE.—Memoirs will be reproduced in the volumes of *Transactions*. Any information which will amplify the records as here printed, or correct any errors, should be forwarded to the Secretary prior to the final publication.

GRENVILLE MELLEN DODGE, Hon. M. Am. Soc. C. E.*

DIED JANUARY 3, 1916.

Grenville Mellen Dodge was born on April 12, 1831, at Danvers, Mass. His parents were Sylvanus and Julia Phillips Dodge, people of gentle birth whose progenitors had come from England early in the Seventeenth Century, and had settled in the vicinity of Rowley, near Boston, Mass. His parents were poor, his father in later life becoming Postmaster of Danvers.

When the boy was thirteen years of age he went to work on the large farm of Mrs. Lander on the outskirts of Danvers, where he also assisted in his first surveying work. Through contact in this work with the son, Frederick Lander, the boy was fired with the ambition to prepare himself for college. He matriculated at Norwich University, Northfield, Vt., in the autumn of 1848, from which he was graduated in 1851, with the degree of Military and Civil Engineer. In 1892 he was further honored by the degrees of Master of Arts, Master of Military Science, and Doctor of Laws from Norwich University, and, later, the degree of Doctor of Laws which was conferred on him by Cornell College, Iowa, in 1904.

Immediately after his graduation Mr. Dodge went to Illinois, where he entered railroad work. In 1852, he served as Assistant Engineer on the Illinois Central Railroad, on the survey of the line from LaSalle to Dixon, Ill. In 1853, Chief Engineer Peter A. Dey, of the Chicago and Rock Island Railway Company, gave him charge, as Principal Assistant Engineer, of a survey of the Mississippi and Missouri River Railroad (now a part of the Chicago, Rock Island and Pacific Railway System), across Iowa, from Davenport to Council Bluffs, and he determined the latter point as the proper crossing of the Missouri River for a transcontinental line.

At various times between 1853 and 1861 Mr. Dodge made explorations west of the Missouri River and in the Rocky Mountains for the purpose of finding the best route for a transcontinental railroad.

He was employed on the construction of the Mississippi and Missouri Railroad until 1857, when with the cessation of railroad activities, he became a member of the firm of Baldwin, Pegram and Company, engaged in banking and mercantile interests, and freighting across the plains to Denver, Colo. He continued with this firm until the opening of the Civil War in 1861.

Mr. Dodge had always been interested in military training and was Captain of the Council Bluffs Guard when he was offered an appointment as Captain in the Regular Army, which he declined in order to accept a commission as Colonel in the 4th Iowa Infantry, which entered service at Rolla, Mo., on August 24, 1861. He was assigned to a post at Rolla on October 9, and

* Memoir prepared by George H. Pegram, Past-President, Am. Soc. C. E.

commanded expeditions to Houston and Salem, Mo., on November 1. The enemy was defeated in both engagements.

Some of Colonel Dodge's further engagements were as follows: Assigned to command of 1st Brigade, 4th Division, Army of the Southwest, January 21, 1862; advanced to and occupied Springfield, February 13; in engagement at Sugar Creek, Mo., February 17, and Cane Creek, Mo., February 20; defeated Gates' command at Blackburn's Mills, February 27; Battle of Pea Ridge, March 6 to 8, during which he was wounded in the right side; commissioned Brigadier General of Volunteers, March 31; reported to Major-General Halleck at Corinth, Miss., June 6, and ordered by him to report to Major-General Quimby commanding the District of Columbus, Kentucky, to re-build the Mobile and Ohio Railroad; assigned to the command of the Central Division, Army of Tennessee, with headquarters at Trenton, Tenn., June 28; organized and mustered into service, 1st Tennessee Cavalry; finished re-building Mobile and Ohio Railroad, in August, and built stockades and earthworks at all its important bridges and stations.

During the time General Dodge was in command at Trenton, the captures of Dyersburg, Huntington, and O'Brien, Tenn., were made, and Villepigue was defeated on the Hatchie River.

On September 29, 1862, by order of Maj.-Gen. U. S. Grant, General Dodge was assigned to the command of the District of Columbus, Kentucky. Extensive fortifications and important works in and around Corinth were finished while he held that command.

On December 1, 1862, a combined movement was made from Holly Springs and Corinth during which the troops of General Dodge captured Tupelo and Okolona, Miss., defeating the enemy and taking the stores at those places. During the same month, by order of General Grant, he moved to Lexington and Spring Creek, Tenn., driving Forrest across the Tennessee River. In February, 1863, he attacked Van Dorn's column at Tuscumbia, Ala., and captured the place with its stores, artillery, etc.

In April, 1863, while in command of the 2d part of the 5th and a part of the Cavalry Divisions of the 16th Army Corps, he conducted an expedition in Northern Alabama, defeating the forces of the enemy in the Tuscumbia Valley. During the movement, the fights at Bear Creek, Cherokee, Burton Station, Leighton, and Town Creek occurred. Immense quantities of stores for Bragg's Army were captured and destroyed. During the same month, Chalmers and Ruggles were attacked and defeated at Tupelo. In June, General Dodge crossed the Tennessee River at Savannah, Ga., and moving into Van Dorn's rear, captured the Town of Florence, Ga., defeating its garrison. On June 19, he attacked Ferguson's command on Big Muddy and stopped the raid on the Memphis and Charlestown Railroad. The fighting was severe and the loss considerable. On July 5, after the capture of Vicksburg, Miss., General Dodge was recommended by General Grant as first for promotion to the rank of Major-General. On July 8, he was assigned to the command of the left wing of the 16th Army Corps, with headquarters at Corinth. On the 14th of that month he defeated a part of Roddy's command at Jackson, Tenn., and captured many prisoners. On August 15, he made

a raid on Granada, Miss., captured a large number of locomotives, cars, railroad stock, stores, etc., and defeated the enemy at Water Valley and Granada.

While at Corinth, General Dodge organized and mustered into service the 1st Alabama Cavalry, the 1st Alabama Colored Infantry, and several companies of colored troops for siege artillery. On October 30, his command was attached to that of Major-General Sherman, then moving into Middle Tennessee.

During November and December, 1863, he re-built the Nashville and Decatur Railroad, also pontoon bridges across the Tennessee River at Decatur, Ala., and the Duck and Elk Rivers, and constructed good and substantial earthworks and stockades at all the important bridges and points. During this time the command had several engagements with the enemy, and captured and fortified Decatur, Ala.

In January, 1864, General Dodge organized and mustered into service the 2d and 3d Regiments of the Alabama Colored Infantry. In May, he was in command of the 16th Army Corps in the field in the Atlanta Campaign, and took part in the battles and engagements at Ship's Gap, on May 6; Snake Creek, on May 9; Resaca, on May 11; Estonula River, on May 12; Kenesaw Mountain, Roswell, Dallas, and Decatur, on July 21; Atlanta on July 22; and Ezra Church on July 28. On June 7, 1864, he was commissioned Major-General.

He was wounded in the head on August 19, and, on October 14, was ordered to City Point, Va., to visit General Grant. On November 3, General Dodge was assigned to the command of the District of Vicksburg and of the left wing of the 16th Army Corps. On December 2, he was assigned to the command of the Department and Army of Missouri, and on the 9th of that month was commissioned Major-General of the Missouri State Militia.

On January 30, 1865, the Department of Kansas was added to the Department of Missouri. Gen. Jefferson C. Thompson surrendered the Confederate forces in Arkansas. During January, February, and March, General Dodge made Indian campaigns on the plains, opening up the stage lines, and rebuilding telegraph lines destroyed by the Indians.

During 1865 and 1866, in his Indian campaigns, extending from the Arkansas River on the south to the Yellowstone on the north, many severe battles were fought. Treaties of peace were made with the Comanches, Apaches, Southern Cheyennes, and other Southern tribes, and a Council with the Northern Cheyennes, Arapahoes, and Sioux at Fort Laramie established the basis for a treaty. For service in this campaign, General Dodge received the thanks of the Legislature of Iowa.

In his "Memoirs" General Grant states, in part: "General Dodge besides being a most capable soldier was an experienced railroad builder," and, referring to the re-construction of the railway between Nashville and Stevenson, he says further:

"General Dodge had the work assigned to him finished within forty days after receiving orders. The number of bridges to re-build was one hundred and eighty-two, many of them over deep and wide chasms. The length of the road repaired was one hundred and two miles."

On May 30, 1866, General Dodge resigned from the Army and immediately took charge, as Chief Engineer, of the construction of the Union Pacific Railroad. The transportation of supplies was one of the serious difficulties. No railroad had yet reached Council Bluffs, the initial point, and supplies had to be shipped from St. Louis and St. Joseph, Mo., by boats, then across the plains through a hostile Indian country, by teams.

The road was completed to a connection with the Central Pacific at Promontory Point, Utah, on May 10, 1869, 556 miles having been built in the last year—the most rapid railroad construction of record.

General Dodge realized the importance of intensive preliminary study of the ground. In the location of the Union Pacific Railroad, 25 000 miles of reconnaissance were made, and 15 000 miles of instrument lines were run. As a result the Union Pacific was originally so well located that subsequent improvements have shortened the road but thirty miles.

General Dodge was America's greatest railroad builder. He constructed the Union Pacific from Council Bluffs, Iowa, to Ogden, Utah; the Texas and Pacific from New Orleans, La., to El Paso, Tex.; the Fort Worth and Denver City, from Fort Worth, Tex., to Trinidad, Colo.; the Missouri, Kansas and Texas, from Fort Worth to Taylor, Tex.; the International Railroad of Texas from San Antonio to Laredo, Tex.; and the Des Moines and Northern (now part of the Chicago, Milwaukee and St. Paul Railway System). He was engaged as well in an advisory capacity on the construction of other lines. As President of the Mexican and Southern Railroad, he supervised the surveys from the City of Mexico through the State of Oaxaca, to the Pacific Ocean.

General Dodge at times was President of the following Railroad Companies: St. Louis, Des Moines, and Northern Railway; Des Moines Union Railway; Denver, Texas, and Fort Worth Railroad; Des Moines and Northern Railroad; and Union Pacific, Denver, and Gulf Railway. He was a Director of the Union Pacific Railway Company, and of the Wichita Valley Railway and Iron Steamboat Company.

General Dodge also had served, either as President, Director, or Chief Engineer, of the following companies: The Pacific Railway Improvement Company; the American Railway Improvement Company; the International Railway Improvement Company; the Texas and Colorado Railway Construction Company; the Oriental Construction Company; and the Colorado and Texas Construction Company, and the Western Industrial Company.

On the death of General Sherman in 1892, General Dodge became President of the Society of the Army of the Tennessee, which office he held until his death. He was Vice-President of the Grant Monument Association and Grand Marshal of the parade at the dedication of Grant's Tomb in New York, N. Y. He was also one of the organizers of the Loyal Legion and served as its Commander-in-Chief from 1907 to 1909.

In 1886-88, he represented the First Iowa District in Congress, declining re-nomination on account of the demands of his railroad work. In 1898, he was appointed by the President as Chairman of the Commission, known

as the "Dodge Commission," to inquire into the management of the war with Spain.

In addition to his frequent contributions to periodicals, General Dodge published "How We Built the Union Pacific Railway, and Other Railroad Papers and Addresses" (1911), and, also, "The Battle of Atlanta."

He was married on May 29, 1854, to Annie Brown, of Peru, Ill., who, with their three children, Lettie (Mrs. Robert E. Montgomery), Ella (Mrs. Frank S. Pusey), and Annie, survives him.

General Dodge was patient in planning, but prompt to decide and act. His keen glance and quick movements expressed the energy he possessed in an unusual degree. He was direct and frank. As an illustration of these characteristics, he organized and conducted the Spy System for Grant and Sherman's Armies on the basis that spies should give correct information to the enemy, if any. A personality that made friends and a loyalty that bound them contributed in no small measure to his success.

Of modest and retiring disposition, he was dragged forward or pushed ahead by his friends, as his intimate history reveals, and this, of course, was the reaction of a loyalty that knew no bounds. It illustrates Emerson's maxim: "If you want a friend, be one." If a friend asked assistance, it was not "I will see what I can do," but "What can I do *now?*"

The history of General Dodge shows in a unique manner the value of preparedness. Educated in a military school at a time when war was not contemplated, and persisting in military training until the outbreak of the Civil War, he was fitted for military service at the start. His training in railroad and bridge building fitted him for the remarkable work of that character which he accomplished during the Civil War. His experience in freighting through an Indian country across the plains fitted him for the Indian campaign at the close of the war. His surveys west of the Missouri River and his wide knowledge of conditions in the West enabled him to advise President Lincoln on the selection from several routes proposed for the first transcontinental line.

All these experiences peculiarly fitted General Dodge to build the Union Pacific Railroad through a hostile Indian country with the necessity of transporting all materials for construction and supplies over long distances by boats and teams.

He died on January 3, 1916, in his eighty-fourth year, at his home in Council Bluffs, where he is buried.

General Dodge was elected Honorary Member of the American Society of Civil Engineers on March 2, 1915.

WILLIAM FRANKLIN ALLISON, M. Am. Soc. C. E.*

DIED JULY 7, 1927.

William Franklin Allison was born in Elgin, Minn., on March 31, 1870, the eldest child of William and Kezia (Culbertson) Allison. His parents

* Memoir prepared by C. Edward Magnusson, M. Am. Soc. C. E.

moved with their family to South Dakota in 1877, where they encountered the usual hardships and difficulties of pioneer life in the Middle West. Here, Mr. Allison attended the rural schools during his early life, and, later, the grade and high schools in the villages adjoining the family farm.

He taught a number of terms in the district schools in order to provide the means for his attendance at the State College of Agriculture and Mechanic Arts, at Brookings, S. Dak. from which institution he was graduated with honors in 1895. In 1897, he received the degree of Bachelor of Science in Civil Engineering from Purdue University.

From Purdue Mr. Allison went back to his home where he was engaged in the practice of general surveying until the outbreak of the war with Spain, at which time he organized a company of infantry and was commissioned its Captain in the First South Dakota Volunteer Infantry Regiment. Soon afterward he was promoted to the rank of Major. His regiment was sent to the Philippine Islands early in the summer of 1898 and remained there for the duration of the war and during the Philippine Insurrection. Major Allison was the Engineer Officer of the Regiment (May, 1898, to October 1899), detailed for road and bridge building and for the laying out of earthworks—winning signal merit for his services. He was recommended for the Brevet Rank of Lieutenant-Colonel of Volunteers for distinguished gallantry.

After his return from the Philippines in October, 1899, Major Allison spent some time in general engineering in South Dakota and worked for a time for the Chicago and Northwestern Railway Company. Later, he took graduate work at the South Dakota State College and at Purdue University and spent a year in Cornell University. From Cornell he received the degree of Civil Engineer in 1904.

In September, 1904, he became an Instructor in Civil Engineering in the Colorado School of Mines, at Golden, Colo., and, in 1905, was advanced to the rank of Professor of Civil Engineering, and Director of the Department of Civil Engineering at this school, which position he filled for the ensuing seven years. During this period he was also engaged in considerable private practice, including municipal, highway, water-works, sewers, and irrigation projects involving an expenditure of approximately $1 500 000, besides the design and supervision of construction of a large testing plant for the School of Mines. He also served as City Engineer for Golden, from 1908 to 1911.

In 1912, Professor Allison resigned his position at the School of Mines and entered private practice as Consulting Civil and Sanitary Engineer, in Portland, Ore. where he remained until September, 1913, when he was appointed Professor of Hydraulics and Sanitary Engineering at the University of Oregon. In 1914, he left Oregon to accept the position of Professor of Municipal and Highway Engineering at the University of Washington, Seattle, Wash., which position he held until his death on July 7, 1927. From August, 1914 to May, 1917, he also served as Consulting Engineer for the State Board of Health of Washington.

From his early boyhood he was interested in military affairs. He enlisted in the National Guard of Dakota Territory at the age of sixteen, and served

continuously until the outbreak of the Spanish-American War when he entered service with the Volunteers. At the beginning of the World War in 1914, he was intensely interested and felt that all who had been trained in military affairs should be ready to serve if the need presented itself. It was with this idea in mind that he offered his services to the Government and received the rank of Major in the Reserve Officers Engineering Corps of the United States Army. On the entrance of the United States into the war, he was called into service (May, 1917), and was granted a leave of absence from the University of Washington.

As Major he went to France with the American Expeditionary Forces in September, 1917, and from November, 1917, to April, 1918, he was assigned to duty with the Sixteenth Engineers on general construction work. From April to November, 1918, he was Chief of the Construction Staff at Is-sur-Tille, France, and during November and part of December, he was Consulting Engineer on water supply at Liffol-le-Grande. During the remainder of December and through January, 1919, he was Engineer with the Peace Commission in Paris, France. He was discharged from service in Washington, D. C., February 17, 1919, and returned to his duties at the University of Washington in March of that year.

Professor Allison took an active part in his profession as well as in church, business, and civic affairs. He served as President of the Western Washington Section of the American Society of Civil Engineers in 1923, and was a member of the Special Committee on Irrigation Hydraulics from 1922 to 1927. He was the author of a number of papers and bulletins on various phases of Irrigation and Sanitary Engineering. He was a member of the American Public Health Association; the Society for the Promotion of Engineering Education; the University Faculty Men's Club; the American Association of University Professors; the American Legion; and the Presbyterian Church.

On August 25, 1904, he was married to Mabel Hegeman, the daughter of Mr. and Mrs. P. J. Hegeman, of Brookings, S. Dak. He is survived by his widow; three children, Katherine, William and Muriel; his mother, Mrs. Emma Allison, all of Seattle, and two brothers, Dr. H. T. Allison, of Stanwood, Wash., and A. C. Allison, of Lake Stevens, Wash., and a sister, Mrs. C. E. Winegar, of Sequim, Wash.

He was a peace-loving, genial, whole-souled man, loved and respected by all who were privileged to know him.

Professor Allison was elected an Associate Member of the American Society of Civil Engineers on October 4, 1910, and a Member on October 1, 1913.

HENRY CLARKE ASH, M. Am. Soc. C. E.*

DIED JUNE 20, 1929.

Henry Clarke Ash, the son of John and Sarah (Clarke) Ash, was born on October 27, 1864, at Preemption, Ill. He was of Scotch-Irish ancestry.

* Memoir prepared by a Committee of the Duluth Section consisting of W. A. Clark, M. Am. Soc. C. E., and G. H. Butler, Sr., and Sheldon B. Shepard, Assoc. Members, Am. Soc. C. E.

He attended Cornell College, at Mount Vernon, Iowa, where he received the degrees of Bachelor of Civil Engineering, in 1890, and Civil Engineer in 1897.

On the completion of his studies in 1890, Mr. Ash went to work with the Chicago and Northwestern Railway Company in the capacity of Rodman and Leveler, but left this Company in 1891 to become Assistant City Engineer, of Duluth, Minn. While serving in this position, he was in charge of the office and field work in the West End Branch of the City Engineering Department, from which he supervised the customary engineering work necessary to such an office. He carried on the construction of about nine miles of street paving, six miles of sewers, two pile and timber trestles, two miles of rubble masonry retaining walls, and a great deal of ordinary minor city construction, such as sidewalks and curb and gutter installation.

In 1896, Mr. Ash joined the engineering forces of the Duluth and Iron Range Railroad Company, and was in charge of the field party at Two Harbors, Minn. He laid out the new yard system for handling and classifying the ore trains for the docks and located the "Loop Line" from the yards to the main line. He also re-surveyed and placed monuments for North Two Harbors and, later, made a complete survey of all the property owned by the Railroad Company in Duluth. In addition, he located the piers for the construction of the Duluth-Superior Interstate Bridge and inspected the foundation work for this structure.

In 1896 he returned to Cornell College and for a year was an Instructor in "Adjustment, Use, and Care of Instruments", and in "Field Practice". During this period Mr. Ash carried on the necessary post-graduate work to receive his Civil Engineer degree which was presented to him in June, 1897.

On examination he became eligible for appointment to the grade of United States Junior Engineer, and in June, 1897, received his assignment as United States Inspector in the Chicago District under Maj. W. L. Marshall, Corps of Engineers, U. S. A. From June, 1897, to the fall of 1898, Mr. Ash was engaged as Inspector of Dredging in the vicinity of Chicago, Ill. He was then transferred to Sterling, Ill., for work on the Illinois and Mississippi Canal. He carried on the office work and surveys for about one year and then became Resident Engineer on the Canal in charge of the Tampico Office and Receiving Station. During this time, approximately eleven miles of the Feeder Canal were completed, including foundations for siphon culverts, iron and concrete culverts, bridges, and bridge approaches.

In 1902, Mr. Ash became associated with the Great Northern Power Company (now the Minnesota Power and Light Company of Duluth) for which he made an examination and report on a water power proposition in Michigan. Following this work he made the topographical surveys, current meter and weir measurements of the St. Louis and Cloquet Rivers, and the necessary calculations of flow in rivers, canals, and pipes with the fluctuations of head incidental to preparations for the design of the Thompson Dam and Reservoir at Thompson, Minn. Mr. Ash designed the dam for this Company and it was his privilege to supervise the construction work on the Thompson Reservoir consisting of: An earth dam, ½ mile in length, faced with rip-rap; a 700-ft. rock-

fill dam with a concrete core wall; a concrete arch dam, 55 ft. high, with a 100-ft. radius; a concrete over-fall dam, 370 ft. long, which was 45 ft. high; 1 000 lin. ft. of concrete retaining dams ranging from 10 to 50 ft. in height; reinforced concrete sluice-gate house and head-gate house; 1 mile of canal; concrete and steel wagon bridges; an 80-ft. steel girder railroad bridge; and supervision of a complete laboratory for testing materials for the entire development.

In 1928, Mr. Ash prepared a paper* for the Twenty-fifth Convention of the American Concrete Institute held in Detroit, Mich., February 12 to 14, 1929, covering this work. Mr. M. B. Lagaard, of the Portland Cement Association, made an examination of this concrete work during 1928, and presented a discussion† on the paper by Mr. Ash, pointing out the fact that the thoroughness of the methods used by him as Resident Engineer was accountable for the excellent condition of all the concrete structures.

From 1907 to 1910, Mr. Ash was with the Minnesota and Ontario Power Company, following which he was Consultant on the Rice Lake Dam. In the winter of 1910 and 1911, he was in Beaudette, Minn., in charge of relief work following the disastrous forest fires. From 1915 to 1921, he was engaged as Assistant Engineer with the Duluth, Missabe and Northern Railway Company on valuation work. He then became Executive Secretary for the City Planning Commission of Duluth.

Mr. Ash had an unfailing sense of humor, which made him a delightful companion and associate. He was in very poor health during the last two years of his life, and while suffering and sick, as he knew himself to be, his usual cheerful smile gave an outward sign of content and happiness. His life was a continuous portrayal of that which man should be toward his fellows—considerate of others even to the neglect of self. It may be compared with that in which it was most useful, a piece of construction in which all success depends upon the foundation. The life of Mr. Ash on this earth constitutes the foundation upon which the superstructure of the most marvelous future may be erected without fear of deterioration due to faulty substructure.

Mr. Ash was a member of the Glen Avon Masonic Lodge, and of the Scottish Rite Fraternity. He was also a member of the Glenn Avon Presbyterian Church, in which he took an active interest.

On January 11, 1899, he was married to Frances C. Hoadley, of Boston, Mass., who, with a son, James H., and a daughter, Katherine C., survives him.

Mr. Ash was elected a Member of the American Society of Civil Engineers on October 3, 1906.

FRANK FOY AXTELL, M. Am. Soc. C. E.*

DIED AUGUST 14, 1929.

Frank Foy Axtell, the son of Thomas Jefferson, and Edna Ann Axtell, was born at Washington, Ind., on August 18, 1867. His father was a successful

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* "The Thompson Dam and Reservoir" (Minnesota), *Proceedings*, Am. Concrete Inst., Vol. XXV, 1929, p. 29.

† *Loc. cit.*, p. 39.

‡ Memoir prepared by Charles Warren Hughes, Assoc. M. Am. Soc. C. E.

business man, having come to Indiana from his native State, Pennsylvania, as a young man. The mother was from one of the Indiana pioneer families long resident in that part of the State and their son grew up with many early advantages in a material and social way.

Following his graduation from the local High School, Mr. Axtell entered Indiana State University, from which institution he received his degree in 1891. In September following, he was temporarily employed as Rodman on a precise level survey of the Missouri River. He left that work in the spring of 1892 to go as Recorder on a topographical survey of the Mississippi River, under the Mississippi River Commission, at St. Louis, Mo., becoming Field Draftsman in 1894. In that same year he left the work of the Commission for a time, engaging in the survey of suburban property and on an interurban railway in St. Louis, with F. B. Maltby, M. Am. Soc. C. E.; in 1895 he went with Mr. Maltby to Uvalde and Eagle Pass, Tex., on an irrigation project. On returning to the Mississippi River Commission in 1896, he was, successively, Recorder, Draftsman, Leveler, and Surveyor. He was promoted to the grade of U. S. Surveyor in 1897, and given charge of a Topographical Party.

As early as 1899, however, it became so advisable for Mr. Axtell to seek a milder climate on account of his health, that January, 1900, found him in New Orleans, La., attached to the U. S. Engineer Department, under the late Lt.-Col. H. M. Adams, Corps of Engineers. He was sent to Calcasieu Pass, La., to supervise the reconstruction and extension of the jetties at that place, covering mattress foundation and rip-rap stone work, which involved considerable expense. On the completion of this work, he was for a time Assistant to the late Mr. C. Donovan, at Port Eads, La., at the mouth of the Mississippi River. In the fall of that year Mr. Axtell transferred to Pilot Town, La., where he supervised the construction of a submerged dam across Pass á Loutre, one of the mouths of the Mississippi River, a work of many difficulties requiring the expenditure of $175 000.

On the completion of this work, Mr. Axtell was transferred back to the U. S. Engineer Office, at New Orleans, and advanced to the grade of U. S. Junior Engineer. He was placed in charge of the improvement of a number of small streams, including the dragging and dredging of these and the planning of improvements of a permanent nature. Then, for a time, pending the passing of the River and Harbor Bill, he was employed in the Division Engineer's Office of the Louisville and Nashville Railroad Company, at Montgomery, Ala. Following the passage of the River and Harbor Bill in June, 1902, Mr. Axtell returned to New Orleans and in December, 1902, he was transferred to Sabine, Tex., where he was given local supervision of the improvement of Sabine Pass Harbor and Jetties, Sabine Lake, and the Sabine and Neches Rivers. In 1906, the U. S. Government acquired title to the Port Arthur Ship Canal and Harbor, and the supervision of those works was turned over to Mr. Axtell. In 1908, the office maintained by him at Sabine was moved to Port Arthur, Tex.

From 1902 to 1912 his duties included the following: (1) Complete triangulation, topographic and hydrographic surveys of Sabine Pass, Sabine Lake, and Sabine and Neches River; (2) the preparation of plans and specifica-

tions and the supervision of jetty work at Sabine Pass, in 1903, 1905, 1908, and 1910, to the amount of $716 000, the work including rip-rap stone work foundation, and granite and concrete capping; (3) the surveys for, and the preparation of plans and specifications, and the supervision of the work of excavating a canal connecting the Port Arthur Ship Canal and the mouths of the Sabine and Neches Rivers, known as the Sabine-Neches Canal (this canal had a depth of 10 ft., was 100 ft. wide on the bottom, and 16 miles long when first completed, but it has been widened and deepened several times since as appropriations became available); (4) the supervision of the dredging and the repairing of the U. S. hopper, sea-going dredge, *Sabine,* and the U. S. hydraulic dredge, *Port Arthur;* and (5) the supervision of the surveys for the Intracoastal Canal from (*a*) Calcasieu River, Louisiana, to Sabine River, Texas; and (*b*) Sabine Pass, Louisiana and Texas, to Galveston Bay.

In 1912, Mr. Axtell went with the Texas Company at Port Arthur, as Construction Engineer, in which position he supervised the construction of concrete retaining walls, docks, warehouses, and oil pipe lines. In 1913, he was sent to Tampico, Mexico, where as Chief Engineer for the Panuco Transportation Company he was engaged in land surveys, hydraulic dredging, and the construction of docks, warehouses, and dwellings. When the Panuco Transportation Company was merged into the Texas Company of Mexico, he was made Chief Engineer for the construction of oil pipe lines, boiler and pump houses, docks and dwellings, hydraulic dredging, and filling—a total of more than $1 500 000. He remained in Mexico seven years and returned in 1920, as at this time his two sons were of an age to need the educational opportunities of the United States.

In 1921 Mr. Axtell became allied with the Gulf Refining Company, of Port Arthur, reputedly the largest refinery in the world. Dredging, the filling in of a bayou, a large expansion and construction program carried on by the Gulf Company, were projects in which he had a part; later, the land maps and records were handled. In spite of his failing health, for more than a year, he kept actively engaged with whatever came in his line of duty until July 19, 1929, when it became impossible for him to continue. He went home a sick man, indeed, and was unable to leave his bed from that time. Four weeks from that day, on August 16, he was laid to rest in Greenlawn Cemetery, Port Arthur, the city which had been his home for more than 20 years, except the 7-year period he spent in Mexico.

Mr. Axtell was a man of rare personality, loyal to his friends, yet in his judgment charitable toward all. He was a devoted family man—a man of high ideals. He was married on September 15, 1904, to Harriet Duncan, of Washington, Ind., who, with their two sons, Edwin Duncan and Frank F., survives him. He is also survived by one sister, Mrs. George Kramer.

He became a member of the Presbyterian Church in early manhood and continued in that faith to the end of his life. He took great interest in civic affairs as long as his health permitted, having served as a member of the local Chamber of Commerce and other organizations.

His technical training made him accurate in his methods, and he was unsparing of himself as an Engineer. He was generous, to a fault, toward

those with whom he was associated in various engineering projects, and this to such an extent that often others reaped where he had sown.

Mr. Axtell was elected an Associate Member of the American Society of Civil Engineers on June 6, 1911, and a Member on November 26, 1918.

EDWIN DURYEA, JR., M. Am. Soc. C. E.*

DIED MARCH 31, 1930.

Edwin Duryea, Jr., was born at Craigville, N. Y., on July 12, 1862. He was the son of Edwin and Hannah (Rumsey) Duryea. His family were Huguenots from Burgundy, France, who came to the United States in 1675 and settled in Long Island. Their descendants fought in the French and Indian Wars and, later, in the American Revolution.

Mr. Duryea spent his boyhood in New York State, attending the public schools and Cornell University, at Ithaca, from which institution he was graduated in 1883 with the degree of Bachelor in Civil Engineering. In 1890, he received the degree of Civil Engineer from the same institution. His first engineering work was on the Northern Pacific Railroad Bridge over St. Louis Bay at Duluth, Minn., under the late E. H. Beckler, M. Am. Soc. C. E., Assistant Engineer, and the late Alfred Noble, Past-President, Am. Soc. C. E., General Assistant Engineer in charge of bridges.

From March, 1886, to April, 1889, Mr. Duryea was employed under the late George S. Morison, Past-President, Am. Soc. C. E., as Assistant Engineer on the Rulo Bridge at Rulo, Nebr. (with the late B. L. Crosby, M. Am. Soc. C. E., as Resident Engineer), and as Assistant Engineer in charge of borings, surveys, etc., for the location of the Memphis Bridge over the Mississippi River, at Memphis, Tenn.; he also acted as Assistant Engineer in charge of river protection and reclamation for the Nebraska City Bridge over the Missouri River at Nebraska City, Nebr., and as Resident Engineer on the new Omaha Bridge (Northern Pacific Railway) over the Missouri River at Omaha, Nebr. He was also employed as Resident Engineer on the Cairo Bridge (Illinois Central Railroad) over the Ohio River, at Cairo, Ill., this too under the late Mr. Noble.

During 1889 and 1890, Mr. Duryea was engaged on the location of the Hutchinson, Oklahoma and Gulf Railroad, in Kansas; on the Chicago and Western Michigan Railroad, and the Detroit, Lansing and Northern Railroad, Grand Rapids, Mich., and as Assistant Engineer on the construction of the Traverse City Extension of the Chicago and Western Michigan Railway.

From January, 1891, to May, 1893, he served as Contractor's Engineer with Washburn, Shaler and Washburn, Civil Engineers and Contractors, in charge of topographic surveys at the lower end of Lake Michigan, harbor studies, etc., as well as on studies and plans for stockyards, packing houses, and railroad layout, and water supply and sewerage systems near Chicago, Ill. Under the same employment he was also engaged on computations for bids for the construction of the New Croton Dam, New Croton Aqueduct, for the New York

* Memoir prepared by Charles D. Marx, Past-President, Am. Soc. C. E., and Charles B. Wing and H. L. Haehl, Members, Am. Soc. C. E.

City water supply, and on the construction of the Third Avenue Cable Railway, in New York City, as well as in serving as Contractor's Engineer and Assistant Superintendent on a part of the New Croton Aqueduct. From May, 1893, to September, 1895, he acted as Superintendent for the same firm of contractors on the construction of the dams of the New Croton Aqueduct.

Beginning in September, 1895, Mr. Duryea was engaged on the design and construction of the New East River (Williamsburgh) Bridge, New York City, including investigations for live load, grades, tower foundations, etc., and the design of anchorages and similar structures. He was later Resident Engineer in charge of the Brooklyn half of the New East River Bridge until 1900, after which time he was engaged in private practice in New York City.

In 1902, he went to San Francisco, Calif., as Chief Engineer of the Bay Cities Water Company, and its allied interests, being engaged on investigation and design for city water supply and power development in connection with that Company until 1910.

Beginning in 1907, Mr. Duryea became associated (as Senior member) with the engineering firm of Duryea, Haehl and Gilman, of San Francisco. He continued in that connection until 1920. His work during this period involved the construction of the system of the South San Joaquin Irrigation District in California, the completion of dams and power development on the Rio Conchos in Mexico, and many other important engineering works in connection with water supply, irrigation, and power development in California. During this time he was a member of the "Committee of Forty" appointed by the Mayor of San Francisco to advise on the rehabilitation of the city after the great fire of 1906.

Owing to ill health, Mr. Duryea retired from active practice in 1920 and remained thereafter at his home in Palo Alto, Calif.

He was always an active and enthusiastic member of the Society and participated whole-heartedly in its activities. He took his profession very seriously, was especially jealous of its honor, and punctiliously observant of its ethics. He was one of the Charter Members of the San Francisco Section of the Society, having served as its President during 1906. He was also a member of the Western Society of Engineers, of Chicago, and an Associate Member of the Institution of Civil Engineers, of London, England.

Mr. Duryea was a man of unusual force and personality, whose gentlemanly conduct and consideration of others impelled the respect and admiration of all those who knew him. The high order of his engineering work, the great interest which he took in his fellow-engineers, and the unselfish assistance which he rendered, particularly to young engineers with whom he came in contact, won for him the unreserved loyalty and friendship of all in his profession.

In December, 1888, he was married to Roberta Taylor, in Ithaca, who, with five children, Robert, Margaret, Anne, Philip and Helena, as well as a sister, Mrs. Julia Tuthill, and a brother, Mr. Howard Duryea, survives him.

Mr. Duryea was elected a Junior of the American Society of Civil Engineers on February 2, 1887; an Associate Member on September 4, 1895; and a Member on February 2, 1898. He served as a Director from 1916 to 1918.

THOMAS NIXON GILMORE, M. Am. Soc. C. E.*

DIED JANUARY 17, 1930.

Thomas Nixon Gilmore was born in Manhattan, Ind., on December 10, 1863.

In 1880, Mr. Gilmore entered the service of the Vandalia Railroad Company, a part of the Southwest System of the Pennsylvania Railroad Company, as Special Apprentice in the Mechanical Department. He continued with this railroad until 1902, being employed, successively, as Draftsman, Engineer of Motive Power, Round-House Foreman, Division Foreman, and also, on various assignments, including studies in connection with fuel, steam heat, and oil.

In 1902, he entered the service of the Terminal Railroad Association of St. Louis, Mo., as Mechanical Superintendent in charge of the Mechanical and Car Departments. This position carried with it responsible charge of the design, construction, and equipment of extensive mechanical improvements preparatory to the World's Fair which was held in St. Louis in 1904. These improvements included locomotive shops and roundhouses, coaling plants, power houses, etc.

Mr. Gilmore was next engaged as Assistant General Superintendent of Motor Power for the Chicago, Rock Island and Pacific Railway Company during 1904 and 1905. In this position a large part of his time was devoted to improvements in standards of motor power facilities, in addition to the regular routine work connected with operation.

In 1905, he entered the service of Westinghouse, Church, Kerr and Company, as Engineer in charge of specific undertakings. The scope of this organization is well known and covers a great variety of engineering and construction operations, including railway lines, power facilities, industrial plants, and public utilities. The work under Mr. Gilmore's direction covered a large number of railway shops and terminals, including the West Springfield Shops of the Boston and Albany Railroad Company; the improvement and electrification of the Depew Shops of the New York Central and Hudson River Railroad Company; locomotive shops and power-house and incidental work at Silver Grove, Ky., and other points, for the Chesapeake and Ohio Railroad Company; shops, power-house, and water supply at Havelock, Nebr., and West Burlington, Iowa, for the Chicago, Burlington and Quincy Railroad Company; the Ogden Shops of the Canadian Pacific Railway Company, near Calgary, Alta., Canada; and the Communipaw engine terminal at Jersey City, N. J., for the Central Railroad Company of New Jersey. His other activities included inspections and reports on the improvement of shops and terminal facilities for a number of trunk-line railroads.

Subsequently, Mr. Gilmore became Managing Engineer, giving both detailed and general supervision to works similar to those outlined. Later, he became Vice-President and Chief Engineer, and also a Director, of Westing-

* Memoir prepared by James C. Boyd and Edwin J. Beugler, Members, Am. Soc. C. E.

house, Church, Kerr and Company, Incorporated. The activities of this Company were accented considerably during the period of the World War, and Mr. Gilmore's position as Chief Engineer contributed toward the expansion of the engineering and construction organization, which covered a large amount of industrial plant construction, warehouses, and special work, much of which was done directly for the United States and Canadian Governments in co-operation with the Army officers. A citation given to the Company after the war by the United States Government was due in a large degree to the efficiency of the organization developed under Mr. Gilmore as Chief Engineer.

In 1920, Westinghouse, Church, Kerr and Company was merged with Dwight P. Robinson and Company, continuing along the same lines of endeavor, and Mr. Gilmore became Consulting Engineer in the combined organization, giving particular attention to railroad shops, locomotive terminals, and power stations. In 1921, he entered private practice as Consulting Engineer, specializing on the lines in which he had been most actively engaged and thus continued until the time of his death.

Mr. Gilmore's contact with a great number of railway systems in the United States and Canada and his habits of observation and analysis fitted him for rendering unusual service in the development of designs and construction of railway facilities for any given location. Moreover, his faculty of co-operation with the various departments of the railroads for which he was doing work, produced results which reflected the best ideas of all concerned.

He was a member of the American Society of Mechanical Engineers, and the Structural Engineers' Association of Illinois. His connection with social organizations included the Engineers' Club of New York City, and the Echo Lake Golf Club of New Jersey.

Mr. Gilmore was elected a Member of the American Society of Civil Engineers on April 7, 1915.

ALBERT BANKS HILL, M. Am. Soc. C. E.*

DIED JANUARY 2, 1930.

Albert Banks Hill and his twin brother, Arthur Bradley Hill, were born at Redding, Conn., on May 28, 1847, the youngest of seven children of Bradley Hill and Betsy (Banks) Hill. The family was of old New England stock, having descended from the William Hill, who emigrated from England in 1632, twelve years after the sailing of the *Mayflower,* and finally settled in Fairfield, Conn. It is recorded that "he was a man of note among the Colonists." One of Mr. Hill's grandmothers was a niece of Joel Barlow, LL.D., author, poet, and diplomat.

Mr. Hill attended the common schools of Redding and prepared himself for college with the aid of one term at a private school. He entered the Sheffield Scientific School of Yale University and was graduated in the Class of 1869, with the degree of Bachelor of Philosophy. One year later he received the degree of Civil Engineer. During that year, in addition to studying, he

* Memoir prepared by E. M. T. Ryder, M. Am. Soc. C. E.

acted as Instructor in Mechanics and Surveying in the Scientific School, and also began his practical professional work as Assistant on construction of water supply reservoirs for the Town of Danbury, Conn. Mr. Hill was also an Instructor at Yale for the following year, during which period he was at one time Chainman on a construction party for the New York and Boston Air Line Railroad and at another time in charge of a party on preliminary survey for the Saugatuck Valley Railroad.

These connections with the building of original railroad lines show how far back in the engineering history of the United States Mr. Hill's work began. It provides the background for the fact that during the last years of his life, he typified to his younger colleagues the Profession of Civil Engineering of which profession he was regarded as the Dean in the State of Connecticut. He was practically the last of that distinguished group of engineers who can never be replaced, whose practice covered nearly the entire field of Civil Engineering. Mr. Hill had a fine record of accomplishment in Railroad, Highway, Hydraulic, Structural, and Sanitary Engineering fields which one or two generations ago would have been covered to a considerable extent by a single person, but which have now expanded so that each has developed into a specialty that can well absorb the life work of an individual engineer.

In 1871, Mr. Hill gave up teaching to enter the Engineering Department of the City of New Haven, Conn., being at first in charge of survey work and then, in the following year, in charge of the Sewer Department. He continued in the City employ and from 1883 to 1892 was City Engineer of New Haven. In 1892 he took up private practice which he continued for nearly forty years, the entire span of his engineering work having covered more than sixty years.

During this period Mr. Hill was active in the civic life of Connecticut and became one of the State's most respected and useful citizens. Particularly marked was the effect of his kindliness and high standard of ethics upon the men in his own profession. As one prominent Connecticut engineer stated, he did "more than any one in this State to elevate the profession of Civil Engineering in the eyes of his fellow citizens."

Some of the works designed by Mr. Hill in Connecticut and executed under his direction as Engineer, were: The swing bridge over Norwalk River, at South Norwalk; the steel arch bridges over Mill River, New Haven; over Lake Whitney, Hamden; and over Lieutenant River, Lyme; the suspension bridge over Lake Whitney (270-ft. span), for the New Haven Country Club; the stone arch bridges, East Rock Park; reinforced concrete arches in Cheshire, Hamden, Waterbury, and over Ash Creek, Bridgeport; park drives; the East Rock, West Rock, and Beacon Hill Park Drives, New Haven public park system; part of the New Haven Sewerage System; sewerage systems for Danbury and Shelton, Conn.; disposal works for Litchfield, Conn.; outfall system for Greenwich, Conn.; electric railways Norwalk to South Norwalk, South Norwalk to Roton Point, Norwalk to Winnipauk, New Haven to Bridgeport, Bridgeport to Fairfield and Southport, New Haven to Derby, New Haven to Cheshire, Cheshire to Waterbury, Cheshire to Milldale, New Haven to Walling-

ford, New Haven to East Haven, and Palmer to Ludlow, Mass.; and the Manufacturers Electric Freight Railroad, New Haven.

As Consulting Engineer he designed the reservoirs, dams, standpipes, and pipe lines for the New Haven Water Company; the Bridgeport Hydraulic Company; the Birmingham Water Company; the Greenwich Water Company; the New Canaan Water Company; the Noroton Water Company; the Stamford Water Company; and the Water Department of the City of Meriden, Conn.

In 1905 he was President of the Connecticut Society of Civil Engineers and, later, was made an Honorary Member of that organization. He was a member of the following societies and clubs: Graduates Club, New Haven; Chamber of Commerce, New Haven; Connecticut Academy of Arts and Sciences; New England Water Works Association; and of the Finance Committee of the Organized Charities Association of New Haven.

Lacking a family of his own, Mr. Hill devoted almost his entire efforts to engineering work and to his friends in the Engineering Profession. He was one of only two Connecticut citizens who have served as Directors of the American Society of Civil Engineers.

The late Albert L. Webster, M. Am. Soc. C. E., a life-long personal friend, wrote of him:

"His one and only passion was devotion to honest constructive engineering work. The magnitude and importance of the task in hand had no influence on his attitude toward what lay before him for solution. His concentration was on the most faithful and best solution of his problem be it large or small. It was the integrity of his nature practically applied to daily tasks with deliberation and calm. Spectacular projects had no appeal to him. He was always busy, but never hurried, and the long list of his accomplishments bears record of unremitting industry and love of his profession. Vacations to him were but interruptions to the life he loved best, so he took only some few holidays from his office as he felt needful for recreation of body and mind and such as he took were spent in study and contemplation of the engineering work of others akin to some problems of his own or to tramping the countryside in review of possible expansion of water supply services with which he was associated.

"He looked upon engineering as a great and almost sacred calling to be devotedly upheld by serious and honest industry and integrity. Any other point of view he regarded as a traitorous denial of faith.

"His long line of Puritan ancestry with their devotion to duty and the dictates of conscience was reflected in his simple adherence to the faith of his fathers and he retained to the end membership in the Congregational Church of his native town of Redding, Conn., the town of his fathers, and while in New Haven was a consistent attendant at the Congregational Center Church on the green, whose echoes still recall the sturdy preaching of that staunch devotee of the Faith, Dr. Leonard Bacon."

At his death the editorial in the *New Haven Courier Journal*, of January 8, 1930, accurately reflected the feeling of the community. It reads as follows:

"A Faithful Servant and Citizen

"The passing of Albert Banks Hill from the active life of New Haven ought not to be lost in the mere occurrences of the day. His career as a civil engineer is solidly written into the history of New Haven both as it reflects the activities of the local government during many years and as well the activities of private interests who found his talents indispensable. But it is not his professional ability which calls from us a tribute to his memory. That lives in

certain achievements of a practical and enduring character. It is to the nature of the man that we desire to pay a delayed compliment. He was a quiet spoken, modest, and sincere man of affairs. What he was called upon to do for public or private interests he did thoroughly. In that respect he was a craftsman of the old school. He threw into his work both as teacher and practitioner the joy of the task. It fed his imagination and gave a personality to his achievement. The generation in which he was educated and trained is passing and the mass principle is affecting the life of to-day. Both as a public servant and a private citizen he possessed virtues which were translated into modest but effective service."

Mr. Hill's death occurred on January 2, 1930, in the eighty-third year of his age. He was active to the last, and had planned to go to his office as usual on the day when he passed on suddenly and quietly.

Mr. Hill was elected a Junior of the American Society of Civil Engineers on February 2, 1876, and a Member on March 5, 1884. He served as a Director of the Society in 1892.

HENRY HOLGATE, M. Am. Soc. C. E.*

DIED JANUARY 22, 1930.

Henry Holgate was born at Milton, Ont., Canada, on September 14, 1863, the son of John and Jane (Browne) Holgate, who were of English descent.

His early education in the public Schools and in the High School in Toronto, Ont., Canada, prepared him for an apprenticeship to the Northern Railway Company of Canada, under the direction of F. W. Cumberland, Chief Engineer and Managing Director, from 1878 to 1883.

On the completion of his service, Mr. Holgate was placed in charge of the Northern Railway and the Hamilton and North Western Railway, both of which were absorbed by the Grand Trunk Railway Company of Canada, under the general management of Sir Joseph Hickson, President of the Grand Trunk, with headquarters in Montreal, Que., Canada, to whom Mr. Holgate reported directly. During the tenure of office under Mr. Holgate, these railways were practically re-built, permanent structures being erected in place of the temporary works. The gauge of the Northern Railway was changed to the standard 4 ft. 8½ in., from the former 5 ft. 6 in., and yards and depots were gradually improved to a fairly high standard. This period extended from 1883 to 1892.

From 1892 to 1894, Mr. Holgate acted as Engineer for the Central Bridge Company at Peterborough, Ont., Canada, designing and building highway and railway bridges and various structures, including the Union Station at Toronto. During the two years (1896 to 1898), he served as Chief Engineer and Manager of the Montreal, Park, and Island Electric Railways, a suburban system of Montreal, and as Construction Engineer for the Company on suburban electric railways.

From 1898 to 1900, he was Chief Engineer and Manager of the West India Electric Company, of Kingston, Jamaica. He developed power on the Rio Cobre, transmitted it to Kingston, built and equipped twenty-seven miles

* Memoir prepared by M. J. Butler, M. Am. Soc. C. E.

of electric railway, and also operated it. In 1900, he entered general engineering practice, maintaining an office in Toronto. He built the hydro-electric plant for the Canadian General Electric Company, at Peterborough, and constructed the hydraulic work for the Seven Islands Power Company, on the north shore of the St. Lawrence River, 320 miles northeast of Quebec, having previously explored the region and selected the site for the piers, power plant, etc.

From 1901 to 1911, Mr. Holgate was in partnership with Mr. Robert A. Ross, an eminent Electrical Engineer. During this period the firm built a great number of hydro-electric plants, notably for the International Nickel Company, on the Spanish River; the West Kootenay Power Company, in British Columbia; the Sherbrooke Light and Power Company, the Disraeli, the Farnham, and others. In 1907 Mr. Holgate was appointed by the Government of Canada as Chairman of a Commission to report on the causes that led to the Quebec Bridge disaster. He and his associates produced a fearless and masterly report.

When the partnership with Mr. Ross was terminated in 1911, Mr. Holgate again engaged in private practice. His next work of importance was the 180 000-h.p. Cedar Rapids Development, on the St. Lawrence River. In 1915, he prepared and presented a paper on this work* before the Canadian Society of Civil Engineers (now the Engineering Institute of Canada), for which he was awarded the Gzowski Medal. He also was engaged on a number of other works, his most recent outstanding service as a Consulting Engineer having been a report, made in collaboration with Mr. J. A. Jamieson, on the St. Lawrence Deep Waterway Project.

Mr. Holgate was a gentleman of courage and ability, and his death from a sudden attack of angina pectoris, is a loss to the Engineering Profession. While he did not confine himself to hydro-electric development, he was internationally known as an expert on that subject, and was much sought as a Consultant.

He was a member of the Engineering Institute of Canada, American Society of Mechanical Engineers, and the Engineers Club of Montreal, of which he was a Past-President.

He was married at Milford, Del., on September 19, 1888, to Bessie Bell Headley who, with two sons, Henry W. and Edwin H., survives him.

Mr. Holgate was elected a Member of the American Society of Civil Engineers on October 2, 1901.

RICHARD LEVERIDGE HOXIE, M. Am. Soc. C. E.†

DIED APRIL 29, 1930.

Richard Leveridge Hoxie, the son of Joseph and Jacqueline (Barry) Hoxie, was born in New York, N. Y. on August 7, 1844. He came of Colonial

* "The Hydro-Electric Power Plant of the Cedars Rapids Hydraulic Manufacturing Company, at Cedars, P. Q." *Transactions*, Canadian Soc. of Civ. Engrs., Vol. 29 (1915), Pt. 1, p. 440.

† Memoir prepared under the direction of Maj.-Gen. Lytle Brown, Chf. of Engrs., U. S. A., Washington, D. C.

stock, both his great-great-grandfather, Lt.-Col. Gideon Hoxie, and his great-grandfather, Lieut. Peleg Hoxie, having served in the Revolutionary Army. He attended schools in New York, Pennsylvania, and Italy, from 1851 to 1858, and was a student at Iowa State University from 1858 to 1861.

At the outbreak of the Civil War Mr. Hoxie enlisted in the First Iowa Cavalry and served as Bugler, Private, and Corporal in Company F of that Regiment from June 13, 1861, to June 9, 1864. While he was still in the Army, he received an appointment as a Cadet at the United States Military Academy, at West Point, N. Y., from Iowa. He entered that institution on July 1, 1864, was graduated third in his class on June 15, 1868, and was appointed on that date as a Second Lieutenant in the Corps of Engineers, U. S. Army. He began his commissioned service with the Engineer Battalion at Willets Point (now Fort Totten), N. Y. where he remained until 1870, when he was transferred to duty on the public works at Boston Harbor, Massachusetts.

In 1872, Lieutenant Hoxie was assigned to duty under the orders of Lieut. George M. Wheeler, Corps of Engineers, U. S. Army, then in charge of the geographical survey of the United States west of the 100th meridian; he served as Assistant on this work until 1874.

In 1874, Lieutenant Hoxie was appointed Engineer to the old Board of Commissioners of the District of Columbia and served in this capacity until 1878, when under the new District of Columbia Government he became Assistant to the Engineer Commissioner. He was promoted to the grade of Captain, Corps of Engineers, in 1882, and remained on duty as Assistant to the Engineer Commissioner of the District of Columbia until 1884. During the last two years of this service he was also on duty in connection with the extension of the Washington Aqueduct for the improvement of the water supply of the National Capital.

In 1884, Captain Hoxie was assigned to duty at Montgomery, Ala., where he remained until 1889 in charge of various river and harbor improvements in Alabama, Georgia, and Florida, and also of the defensive works at Pensacola, Fla. He was then transferred to duty at Willets Point, where he served as Instructor at the Engineer School of Application and with the Battalion of Engineers, until 1893. His next station was at Pittsburgh, Pa., where he was in charge of the improvement of the Allegheny and Monongahela Rivers until 1897. He reached the grade of Major in the Corps of Engineers in 1895.

In 1897, Major Hoxie was transferred to duty at Portland, Me., in charge of river and harbor and fortification works in Maine and New Hampshire, where he remained for about one and one-half years. In 1898 he was assigned to duty as Engineer Secretary of the Lighthouse Board, in Washington, D. C.

He served with the Lighthouse Board until 1901 when he was sent to St. Paul, Minn., in charge of the works of river and harbor improvement in that District. In 1903, Major Hoxie was transferred to Baltimore, Md. His duties in Baltimore consisted of river and harbor improvements and fortification work and, in addition, he served as Engineer of the Fifth Lighthouse District and as a member of the Board of Engineers for Rivers and Harbors. He reached the grade of Lieutenant-Colonel, Corps of Engineers, in 1904,

and that of Colonel, in 1907. On August 7, 1908, while still on duty at Baltimore, having reached the statutory age of retirement—sixty-four years—he was placed on the retired list of the Army, with the rank of Brigadier-General.

General Hoxie's service to his country was practically continuous over a period of nearly half a century. It began when as a boy, at the outbreak of the Civil War, he responded to the call for volunteers. He was the last member of that group of officers of the Corps of Engineers who had active service in the field during the great conflict of 1861-65. He participated in the explorations which gave to the people of the United States the first accurate information of the territories of the Far West, a vast area that sixty years ago was practically *terra incognita*. His services as City Engineer in Washington, then at the beginning of its development, were characterized by foresight and engineering skill. Much of the sewer and water systems, and many of the streets which serve the people of the Capital City to-day, were planned and built by him more than fifty years ago.

In 1873, General Hoxie made a visit to Florida and at that time realized its advantages as a winter resort. He was a close friend of Mr. Henry M. Flagler, who later developed the east coast of Florida and built the Florida East Coast Railway.

After his retirement, General Hoxie spent part of each year in Washington, in his boyhood home in Iowa City, Iowa, and in Miami, Fla., where he maintained a winter home. He died in Miami, and his remains were brought back to Washington, where, after a simple service in his home, his ashes were placed in the Arlington National Cemetery, the last resting place of an ever-increasing company who have served their country loyally and well.

General Hoxie was married on May 28, 1878, to Vinnie Ream, the celebrated sculptress, who died on November 20, 1914, leaving one son, Richard Ream Hoxie. On April 30, 1917, he married Mae Ruth Norcross, who, with his son, survives him.

General Hoxie was a member of the Grand Army of the Republic, the Sons of the American Revolution, the Loyal Legion, and of the Metropolitan, Army and Navy, and Cosmos Clubs of Washington. At one time he belonged to the Duquesne Club of Pittsburgh and the University Club of Baltimore.

General Hoxie was elected a Member of the American Society of Civil Engineers on June 2, 1886.

JOHN EDWARD JENNINGS, M. Am. Soc. C. E.*

DIED MARCH 27, 1929.

John Edward Jennings was born in Brooklyn, N. Y., on April 18, 1875, the son of John S. Jennings and Mary A. (Sigsworth) Jennings.

Mr. Jennings attended the Brooklyn Public Schools until he reached the age of fifteen. At this time he secured a position in the Construction Department of the Central Railroad Company of New Jersey. While so employed he

* Memoir prepared by a Committee consisting of Messrs. Theodore M. Avery, and R. D. Coombs, E. M. Van Norden, M. Am. Soc. C. E., and E. J. Staubitz, Assoc. M. Am. Soc. C. E.

continued his studies at Cooper Union, from which institution he was graduated in 1896. His efforts resulted in a rapid promotion, and when he left the employment of the railroad (in the year of his graduation from Cooper Union), he held the position of Rodman.

Subsequently, Mr. Jennings was employed in the Drafting Rooms of the Berlin Iron Bridge Company, the Elmira Bridge Company, and the Dominion Bridge Company; he also was associated with the firms of Post and McCord and Levering and Garrigues. In 1898, he entered the employ of Milliken Brothers, Incorporated, as Draftsman and Checker. His ability soon placed him at the head of a squad having complete charge of detailing many large hotels, office buildings, and similar structures. His supervision also included mill buildings of various kinds, including some of the largest sugar-mills in the Hawaiian Islands. In 1901, Mr. Jennings received further recognition by being placed in complete charge of the Designing Division of the Company. He continued in this capacity until 1906, during which period he supervised and controlled the design of many large structures. From 1906 until 1908, he held the position of Assistant Chief Engineer.

In 1908, steel towers for supporting high-tension transmission lines came into prominence. The investigations, testing, and developments of these structures were put under the supervision of Mr. Jennings. His investigations and testing of full-sized structures led to the conclusion that the accepted methods of design were unsatisfactory. He introduced many new ideas in reference to loadings, safety factors, and working stresses to govern the design. While progress has demanded some modifications, it is a tribute to his exceptional ability and far-sightedness that his fundamental principles still govern the design of many of the present-day transmission-line structures.

When radio became a commercial possibility, Mr. Jennings quickly realized the necessity of structures for supporting the antennæ. With his wide experience in the transmission field he was in an excellent position to undertake the studies of these structures. He developed a standardized design of tower which proved highly satisfactory, and is now used at many of the important wireless stations throughout the world, including many of the broadcasting stations in the United States.

In 1918 he became Vice-President and Chief Engineer for Milliken Brothers Manufacturing Company, Incorporated. While in this position he was frequently consulted by many of the large power ompanies and syndicates, as well as by the Government of Norway. His work in connection with the tower designs for the Norwegian Government called him from the United States in 1920 and resulted for him in a trip around the world for a study of transmission-line conditions and construction. His travels led him to Australia, where his skill made itself evident in the design of the 100-mile transmission line extending from Melbourne to the Morwell brown-coal fields. In addition to work in the transmission and radio tower field, he was also responsible for the design of towers used for supporting the cableways in Colombia.

A few of the outstanding operations with which Mr. Jennings was connected were: Transmission towers for the Great Western Power Company; the

Southern California Edison Company (Big Creek Line); the Utah Power and Light Company; the Pacific Gas and Electric Company; the Montana Power Company; Southern Power Company; the American Gas and Electric Company; the Nevada-California Power Company; the Ebro Power Company, in Spain; Victoria Falls, South Africa; the Norwegian Government; the Nipon Electric Power Company; the São Paulo Tramway, Light and Power Company, in Brazil; and many others.

He was also responsible for major engineering work in connection with the electrification of the South African Railways; the Great Indian Peninsular Electrification Scheme; the Perak River Hydro-Electric Development; and the Central Scotland Electricity Development. Other important work under his charge included the Gammarra Cucuta Cableways, Cable del Norte, and Cable del Choco, all of which are in Colombia, South America.

Mr. Jennings was held in high esteem by his associates and employees. His fine character and ability resulted in a world-wide circle of friends. His passing has left a deep sense of loss to those who have been privileged to be associated with him in any way during his career. He was an active member in the Brooklyn Engineers' Club until his work required extensive travel abroad. He was also a member of Kane Lodge, F. and A. M., as well as the Royal Automobile Club of London.

Mr. Jennings was elected an Associate Member of the American Society of Civil Engineers on November 30, 1909, and a Member on December 3, 1913.

PETER JUNKERSFELD, M. Am. Soc. C. E.*

DIED MARCH 18, 1930

Peter Junkersfeld was born at Sadorus, Ill., on October 17, 1869. He was graduated from the University of Illinois in 1895, with the degree of Bachelor of Science in Electrical Engineering. His scholarship was excellent and he was made a member of Tau Beta Pi, the Engineering Honor Fraternity, Editor of *The Technograph*, and was generally prominent in student activities. In 1909, after presenting his thesis, he was awarded the degree of Electrical Engineer. From 1911 to 1913 he served as President of the University of Illinois Alumni Association.

Subsequent to his graduation Mr. Junkersfeld went to work in Chicago, Ill., for the Chicago Edison Company (now the Commonwealth Edison Company) with which he occupied a very insignificant position. He later became identified with the Engineering Department and was a Mechanical Engineer. Afterward, as Electrical Engineer, he was Assistant Vice-President in charge of engineering and construction.

The Commonwealth Edison Company, being a pioneer in the development of the steam turbine for use in central stations, sent Mr. Junkersfeld to England, in conjunction with the firm of Sargent and Lundy, to study the mechanical features of the steam turbine. He was closely identified with

* Memoir prepared by M. J. Whitson, M. Am. Soc. C. E.

this great development from its very beginning in the United States until the end of his activities.

When the Officers' Reserve Corps of the Army was organized Mr. Junkersfeld was one of the first to be commissioned as Major in the Army Engineers Corps and during the emergency in 1917 when the United States entered the World War, he was one of the first to be called into active service. He was assigned to the Construction Division of the Army, an emergency organization operating directly under the Secretary of War for the construction of camps, cantonments, embarkation camps, and army bases and terminals. He was promoted to the rank of Lieutenant-Colonel and, later, to that of Colonel. In August, 1918, he was awarded the Distinguished Service Medal for his services.

During his two years' service in the Army, due to his training, his superior judgment, his great energy, and lovable character, Colonel Junkersfeld was able to perform a service in the Construction Division which perhaps should be credited as the greatest performed by any single officer. The work of this Division in providing facilities and quarters for the training of the troops and for the assembling and expediting of supplies, was very great, and Colonel Junkersfeld was called upon to give the best that was in him in these emergencies.

In 1919, he was selected as Engineering Manager for Stone and Webster and was made Vice-President of the Division of Engineering and Construction. Two years later, with Mr. W. McClellan, he organized the engineering firm of McClellan and Junkersfeld. This firm did a large engineering and construction business, among its projects being the $35 000 000 Cohokia Power Plant, at East St. Louis, Ill.

During the last two years of his life, in addition to his association with McClellan and Junkersfeld, Colonel Junkersfeld was Vice-President of the Stone and Webster Engineering Corporation.

Colonel Junkersfeld was a Fellow of the American Institute of Electrical Engineers; a member and Past Vice-President of the Western Society of Engineers; President of the Association of Edison Illuminating Companies, in 1916-1917; and First President of the Construction Division Association. He was a member of the American Society of Mechanical Engineers, the American Institute of Electrical Engineers, the Franklin Institute, the National Electric Light Association, and the American Electric Railway Association, as well as of the following clubs: The Chicago Athletic Club, the University Club of Chicago, the Illini Club of Chicago and New York, the Engineers' Clubs of Chicago and New York, the New York Lawyers Club, and the Scarsdale Country Club, at Scarsdale, N. Y.

He was the author of many technical papers. Among these is included "General Review of Current Practice in Steam Generating Stations", of which George A. Orrok, M. Am. Soc. C. E., was co-author, and which was presented at the first World Power Conference in London, England, in 1924.

Peter Junkersfeld, born in the heart of the Middle West of sturdy and substantial parentage, learned the principles of perseverance and honor on an

Illinois farm. He sought higher education at the University, at Champaign, Ill., and afterward during his entire life he was identified with its activities and those of its alumni organizations. When his life was ended, the President and Deans of the University assembled as honorary pall bearers when he was taken to his final resting place at St. Mary's Cemetery, in South Champaign.

During his entire life Colonel Junkersfeld was extremely fond of engineering. He pursued and developed its technical phases, and applied it in its general scope to the large business enterprises with which he was connected.

Even above his love for his profession was his spirit of responsibility as a citizen. As stated, he left his responsibilities with the Commonwealth Edison Company to help the United States when the great emergency arose in 1917. Serving without personal consideration, he gave himself completely to the service of his country. His character was without blemish. To know him was to be his friend—he had no enemies.

On June 19, 1901, he was married at Champaign, Ill., to Anna C. Boyle, who, with two daughters, Florence Rita and Mary Josephine, survives him.

Colonel Junkersfeld was elected a Member of the American Society of Civil Engineers on June 16, 1919.

LAURENCE FRANCIS McCOY, M. Am. Soc. C. E.*

DIED JANUARY 13, 1930.

Laurence Francis McCoy was born on April 29, 1865, at Dumont, Iowa. He was the son of John and Anna (Coyle) McCoy, both of Irish extraction. His father was a Civil War veteran who engaged in farming after the war.

Mr. McCoy received his education in Iowa, having been graduated from Iowa State College in 1885. Following his graduation, he worked for three years on surveys in Illinois and Iowa. Becoming interested in the development of the Northwest, he went to Spokane, Wash., in 1888. Railroad location and construction were the phases of engineering he preferred, and his first position in the West was with the Seattle, Lake Shore, and Eastern Railway Company.

He worked on many of the D. C. Corbin lines, serving at various times as Locating and Chief Engineer. These railroads included: The Spokane Falls and Northern, in 1889 to 1891; the Nelson and Fort Shepherd, a Canadian continuation of the Spokane Falls and Northern, in 1893; the Columbia and Red Mountain, a line built in 1896 to facilitate the development of the Rossland mines; the Spokane International, in 1906; and the Flathead Valley, in 1910.

From 1891 to 1893 Mr. McCoy was employed by the San Francisco Bridge Company at Sandpoint, Idaho. During the time of slackened building construction in 1894 and 1895, he taught school in Whitman County, Washington.

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* Memoir prepared by Edward G. Taber, M. Am. Soc. C. E.

From 1901 to 1904, he worked on several divisions of the Oregon Short Line Railroad. On the Boise, Nampa and Owyhee Branch, he was in charge of both engineering and construction. In 1907, he was employed by the Idaho and Washington Northern Railroad; his work here continued for two years, during which time he served both as Locating and Chief Engineer.

Mr. McCoy went to Sudbury, Ont., Canada, in August, 1911, as Division Engineer for the Canadian Northern Railroad Company. The outstanding feature of this work was the great number of bridges and tunnels necessitated by the rough and irregular nature of the country. On the completion of the work in 1914, he returned to the West where he accepted the position of Chief Engineer for the Blackwell Lumber Company at Cœur d'Alene, Idaho, a position that he held until the time of his last illness. For this company, which was committed to a policy of railroad construction to get out its timber, a series of switchbacks on the side of a steep mountain was Mr. McCoy's first project. He later located and constructed many branch lines to the Company's scattered holdings. Subsequently, his duties grew to include the letting of all contracts and the determining of amounts to be paid. In consequence his reputation for honesty and accuracy was well established.

Although his work was in varying locations, Mr. McCoy was greatly interested in public affairs. He seldom failed to vote and frequently wrote articles on public questions for the newspapers.

He was accepted by all who knew him as a man of unquestioned integrity and the highest sense of honor. He was happy and proud of the fact that he numbered as firm friends the men who worked for him, as well as those for whom he worked.

He was married on November 23, 1893, to Crissie Rusch, who with two sons, Robert F. and Joseph P., and two daughters, Anna and Marie, survives him.

His private life was governed by the best of ideals, and his genial and sympathetic nature made him loved as well as respected by all those with whom he was associated.

The sons share their father's interest in engineering, Robert being employed since his graduation from Washington State College by the Plant Department of the Telephone Company, in Seattle, Wash., while Joseph is now a Junior in Engineering at the same school.

Mr. McCoy was elected a Member of the American Society of Civil Engineers on December 5, 1911.

LINTON WADDELL STUBBS, M. Am. Soc. C. E.*

DIED JANUARY 18, 1930.

Linton Waddell Stubbs, the son of Frank P. and Margaret (Linton) Stubbs, was born on October 5, 1855, at Thomasville, Ga. His early education was obtained in the local schools at Monroe, La., and at the Brothers' College, at

* Memoir prepared from information on file at the Headquarters of the Society.

Bay St. Louis, Miss. Later, he entered Rensselaer Polytechnic Institute, at Troy, N. Y., from which he was graduated in 1877.

On the completion of this course, Mr. Stubbs entered the service of the United States Government in its survey of the Missouri River. Later, in 1888, he became Resident Engineer of the Meridian-Shreveport Division of the then Queen and Crescent Railroad System, and continued in that position until 1901. During Mr. Stubbs' service with this System, the road-bed between Delta Point, and Monroe, La. was raised to a grade above the high-water mark in the territory at that time.

In 1902, Mr. Stubbs entered the service of the Southern Pacific Railroad Company. His employment with this Company embraced construction of works of great significance, chief among which was the important bridge across the Atchafalaya River at Morgan City, La.

In 1909, Mr. Stubbs resigned from the service of the Southern Pacific Railroad Company to become Engineer in charge of the construction of the Galveston (Tex.) Causeway, a large and important task which was completed in 1912. At this time he retired from the active practice of his profession. In later years, he devoted his time to the supervision of his landed and other interests which developed and grew steadily under his wise and capable direction.

Mr. Stubbs was an Engineer of marked ability and his service was ever characterized by fidelity and devotion to his trust. He was deeply interested in civic affairs and was the author of many forceful and interesting articles relating to the public interest.

Mr. Stubbs was a Mason, and a member of the Presbyterian Church. He leaves a widow, Mrs. Helen (Chase) Stubbs, of Shreveport, La., and two daughters, Mrs. Horace M. Witherspoon and Mrs. J. O. Crawford, both of Los Angeles, Calif.

Mr. Stubbs was elected a Member of the American Society of Civil Engineers on October 3, 1888.

FRANK MARTIN WILLIAMS, M. Am. Soc. C. E.*

DIED FEBRUARY 20, 1930.

Frank Martin Williams was born on April 11, 1873, in Durhamville, N. Y., the son of William and Ellen L. (Sterling) Williams. He was of Welsh and Mohawk Dutch descent. On his mother's side he was of the fifth generation removed from Henrich Starin, the first Judge of Herkimer County, New York, and an officer in the American Army at the Battle of Oriskany in the Revolutionary War.

Mr. Williams' schooling began in the district school at Durhamville. Afterward, he attended the Oneida, N. Y., High School, from which he was graduated in 1890. In the fall of 1891, he entered Colgate University where he became a member of the Phi Gamma Delta Fraternity. He was graduated from Colgate in 1895 with honors and with membership in Phi Beta Kappa.

* Memoir prepared by Roy G. Finch, M. Am. Soc. C. E.

After graduation, Mr. Williams accepted employment in the City of Oneida as an Engineer on Highway and Sewer Construction, and spent his spare time in the study of law in the office of the late J. T. Durham in that city. Believing that a full course in law would be of assistance to him in the Engineering Profession, he entered the College of Law at Syracuse University and was graduated with the degree of Bachelor of Laws in 1897, and admitted to the Bar.

In December, 1897, he entered the service of the New York State Engineer and Surveyor as Rodman and remained until April, 1898. In November, 1898, Mr. Williams became Resident Engineer for the Stanwix Engineering Company, of Rome, N. Y., in charge of the construction of water-works and electric light plant at Charlotte, N. Y.

In April, 1900, he again entered the engineering service of the State of New York, advancing, successively, from the grade of Rodman, through that of Leveler and Assistant Engineer to Resident Engineer in charge of maintenance and construction of highways in various counties of the State.

In November, 1908, Mr. Williams was elected State Engineer and Surveyor of New York State, serving a two-year term from January, 1909, to December, 1910, inclusive. During this term as State Engineer and Surveyor, certain canals of the State were in process of improvement, the project being commonly known as "The Barge Canal Improvement." Under his supervision and direction, plans and estimates were prepared and contracts were let for more than $30 000 000 worth of work; and $14 000 000 worth of construction was completed. During 1910 and 1911, Mr. Williams was also Chairman of the Barge Canal Terminal Commission and, in that capacity, in the summer of 1910, inspected the harbors of Europe.

In June, 1911, he became Chief Engineer of the Coleman-du Pont Road, Inc., in charge of surveys, plans, and estimates for the proposed du Pont Boulevard in the State of Delaware. In 1912, Mr. Williams was appointed Chief Engineer of the Portage County Improvement Association, with headquarters at Ravenna, Ohio. This work involved the supervision of an extensive mileage of highway improvement in Eastern Ohio.

In November, 1914, Mr. Williams was again elected to the office of State Engineer and Surveyor in New York and, with three subsequent re-elections, served from January 1, 1915, to December 31, 1922, inclusive. During these eight years, many of the most difficult problems connected with the Barge Canal construction were solved, including the settlement of the railroad-crossing question, and the location, design, and construction of terminals. The entire Barge Canal System was opened for service on May 15, 1918.

From January, 1923, to the date of his death, Mr. Williams was associated with Roy G. Finch and Edward H. Sargent, Members, Am. Soc. C. E., as Consulting Engineer, with offices in Albany, N. Y.

He was a Consultant for the New York State Bridge and Tunnel Commission and the New Jersey Interstate Bridge and Tunnel Commission on the construction of the Holland Vehicular Tunnel under the Hudson River, connecting New York, N. Y. with Jersey City, N. J. He was also Consultant for the Hudson River Regulating District on the construction of the Sacandaga

Reservoir; for the Water Service Commission of the City of Oswego, N. Y., on the construction of a hydro-electric development at Dam No. 6, Oswego River; and for various public utility corporations.

Mr. Williams was a member of the American Institute of Consulting Engineers, New York State Society of Professional Engineers, and an Honorary Member of the Albany Society of Engineers.

Colgate University, his Alma Mater, honored him in 1915 with the degree of Doctor of Science, and he was a Trustee of that institution for more than twelve years prior to his death; and, in addition, for three years, he served as President of Colgate University Alumni Association. In 1919, Syracuse University conferred upon him the degree of Master of Civil Engineering. His latest honor came only a short time before his death, when President Hoover appointed him a member of the Interoceanic Canal Board.

Mr. Williams was a staunch Republican, and his life was marked by clean achievement in that field. His personality and his conduct in office inspired confidence.

He was a member of the First Presbyterian Church, Albany, of the Masonic Order, in Goshen, N. Y., and of the Benevolent and Protective Order of Elks in Oneida.

In 1907, he was married to Lucy Mary Sterling, of Watkins Glen, N. Y. They had two sons, Frank Martin Williams, Jr., who died in infancy, and David Sterling Williams, 12 years of age at the time of his father's death.

The life of Frank M. Williams comprised a variety of interests, but whatever he attempted he did thoroughly. He was probably best known as the New York State Engineer and Surveyor who was elected to that office five times and during whose administration the greater part of the Barge Canal Improvement was planned, constructed, and completed.

His life has been well outlined and epitomized by one whose acquaintance and friendship covered college days and extended to the time of his death, Dr. A. W. Risley, Professor of History in the New York State College for Teachers, Albany, who is quoted,* as follows:

"As we think of Frank Williams, devoted son of Colgate, we are sure that three qualities jut out boldly. These are common sense, silent fighting purpose, and reliability. Of these, the last was most evident in his entire career. Diligence, more than brilliance. Reliability, not display."

Mr. Williams was elected a Member of the American Society of Civil Engineers on May 28, 1912. He served as Director of the Society from District No. 3 for the term, 1926-1928, and was also a representative from the Society on American Engineering Council.

WALDO JUSTIN BICKLE PORTER, Assoc. M. Am. Soc. C. E.†

DIED FEBRUARY 6, 1930.

Waldo Justin Bickle Porter was born at Crawford, Nebr., on May 9, 1895. His mother, Edith M. Porter, died when he was about four years old, after

* *Colgate Alumni News*, April, 1930.

† Memoir prepared from information on file at the Headquarters of the Society.

which time he lived with his grandparents, Mr. and Mrs. James Porter, in Moose Jaw, Saskatchewan, Canada. After graduating from the High School there, he attended Iowa State College, at Ames, Iowa, from which he was graduated as a Civil Engineer in 1916.

As soon as the United States entered the World War, Mr. Porter went in training at Fort Riley, Kansas, for a commission in the artillery, sailing for France as a Second Lieutenant in Battery C, Eightieth Field Artillery, of the Seventh Division, with which he served until the end of the war.

On his return to the United States, he entered the Department of Public Works of the State of Nebraska; he was employed in this office for about eight years. He was District Engineer with headquarters at Ainsworth, Nebr., when he resigned that position to accept another with the American Paving Corporation of Omaha as Construction Superintendent and Engineer. He had been in charge of that Company's construction work in Iowa, Kansas, Illinois, Oklahoma, Missouri, and Nebraska.

Mr. Porter was living at Plattsmouth, Nebr., at the time of his fatal illness. Three weeks elapsed before he was taken to the hospital in Lincoln, Nebr., where he died from a ruptured appendix on February 6, 1930; he was buried in Wyuka Cemetery on Sunday, February 9.

He has left a place in the hearts of his relatives, friends, and business associates that never can be filled. He was by nature somewhat quiet, reserved, and conservative in manner, but extremely courteous and considerate in his treatment of others. Naturally of a retiring disposition, his ability and personality were known only to those with whom he was associated, but by them he was held in the highest esteem. He was a member of the Masonic Order, and his religious affiliations were with the Baptist Church. He enjoyed his profession fully, and always took a keen delight in any unusual problems, no matter how difficult or tedious.

On May 8, 1926, Mr. Porter was married to Jean Karnes, of Lincoln. He is survived by his widow, his parents, Mr. and Mrs. J. E. Porter, and a brother, George, of Crawford, Nebr., and a sister, Mrs. Caroline Barnes, of March Field, Calif.

Mr. Porter was elected an Associate Member of the American Society of Civil Engineers on December 5, 1927.

CLARENCE MARVIN FOSTER, Affiliate, Am. Soc. C. E.*

DIED FEBRUARY 18, 1930.

Clarence Marvin Foster, the son of Maj.-Gen. Robert S. Foster and Margaret (Foust) Foster, was born in Indianapolis, Ind., on April 6, 1866. He completed his common and high school education in 1883.

From 1883 to 1887, Mr. Foster was associated with the firm of R. S. Foster Company, of Indianapolis, dealers in cement and building materials. In 1887, he started in business for himself as a General Contractor on concrete and masonry work. He continued in this business until 1892, when he dis-

* Memoir prepared by J. Edward Cassidy, M. Am. Soc. C. E.

posed of it to become associated with the firm of Buntin, Shryer and McGannon, of Indianapolis, Duluth, Minn., and Kansas City, Mo., with which he remained until 1895. During his services with these various organizations, Mr. Foster had qualified himself as an expert on matters pertaining to cement and work in which cement was involved. Recognizing his value as an expert in matters pertaining to this type of work, he was engaged by the Utica Cement Association, Manufacturers of Hydraulic Cement, as a Traveling Inspector. This work caused him to travel throughout the West.

In 1899, Mr. Foster left the Utica Cement Association to become Sales Manager for the Meacham and Wright Company, of Chicago, Ill. This Company was for many years one of the largest (if not the largest) jobbers of Portland cement in the United States. He continued in this capacity until the early part of 1914, when the Meacham and Wright Company discontinued handling Portland cement. He then became Manager of the Edison Portland Cement Company, at Philadelphia, Pa.

In April, 1915, Mr. Foster and J. Edward Cassidy, M. Am. Soc. C. E., organized the Foster Construction Company with headquarters in Newark, N. J., and engaged in the design and construction of reinforced concrete bridges. Mr. Foster, as President, was specifically charged with office administration and the purchase of materials and supplies for the operations of the Company. In the spring of 1917, when the United States entered the World War, the Foster Construction Company had a well-established business, and, at the time war was declared, had a number of bridges under construction. Mr. Cassidy, who was in charge of all the technical and construction work of the Company, was a Captain (later, Colonel) in the Engineer Reserve Corps and went on active duty the day following the finishing and acceptance of the last of the bridges under construction.

Although he was 51 years of age when war was declared, it was particularly fitting that Mr. Foster should take an active part in the service of the country, as his father, Maj.-Gen. Foster had had a very distinguished record in the Civil War serving in various capacities up to and including the command of a Division. His service further included the position of Chief of Staff of the 10th Army Corps, and afterward that of Major-General commanding the First Division, 24th Army Corps, Army of the James; this Division performed a 36-mile march in 24 hours to close the last gap around the Army of Northern Virginia at Appomattox, April 8 and 9, 1865.

Being especially fitted for work in connection with materials and supplies used in construction, Mr. Foster received an appointment in the Construction Division of the Army, was commissioned as a Captain, and, later, promoted to the rank of Major. He continued in the capacity of Purchasing and Contracting Officer, Construction Division, until October, 1919, when he left the service to become District Manager of the Barber Asphalt Company, with headquarters at Washington, D. C.; he remained with that Company until his death. During the period, 1919 to 1923, while his headquarters were in Washington, Major Foster compiled a great deal of valuable data on municipal work for which he received commendation not only from his own Company, but from a number of foreign countries.

In October, 1923, Major Foster was promoted to be District Manager of the Barber Asphalt Company on the Pacific Coast with headquarters at San Francisco, Calif., his territory covering a large group of Western States, the Hawaiian and Philippine Islands.

His death on February 18, 1930 occurred from a stroke of apoplexy. He had not been feeling well for days and on February 13, 1930, he came home early in the afternoon and shortly afterward suffered the stroke from which he never regained consciousness. He died five days later. Funeral services were held at his home in San Francisco, after which all that was mortal was taken to his old home in Indianapolis, and interred in Crown Hill Cemetery.

He was a member of the Military Order of the Loyal Legion, Indiana Engineering Society, and the U. S. Army Quartermaster Association.

Major Foster was a man of exemplary habits, and with his genial, pleasant disposition and courteous manner, had the faculty of making a host of friends wherever he went. His passing is a personal loss not only to his friends in many parts of the country, but to his numerous business associates with whom he was always a "hale fellow well met."

He was married on December 24, 1895, to Mary Margaret Roberts, of Indianapolis, who survives him, together with a daughter, Mrs. Maryanna Childs, of Detroit, Mich., and a son, Robert S. Foster, of San Francisco.

Major Foster was elected an Affiliate of the American Society of Civil Engineers on January 31, 1899.